# AI Problem Solving by Searching & Robot Path Planning

Lecture by Margrit Betke

Reading: Russell and Norvig, Chapter 3

Winston

# What you need to do to specify an agent-based AI Problem:

- Initial state that the agent starts in
- Actions available to agent
- Transition model & state space:  Path through state space = sequence of states = sequence of actions
- Goal test
- Path cost (e.g. sum of step costs)

# "AI Toy Problems" useful for learning concepts

- 8 Puzzle: Start state:

| | | |
|---|---|---|
| 7 | 2 | 4 |
| 5 | | 6 |
| 8 | 3 | 1 |

Goal state:

| | | |
|---|---|---|
| | 1 | 2 |
| 3 | 4 | 5 |
| 6 | 7 | 8 |

- 8 Queens Problem: 8x8 chess board

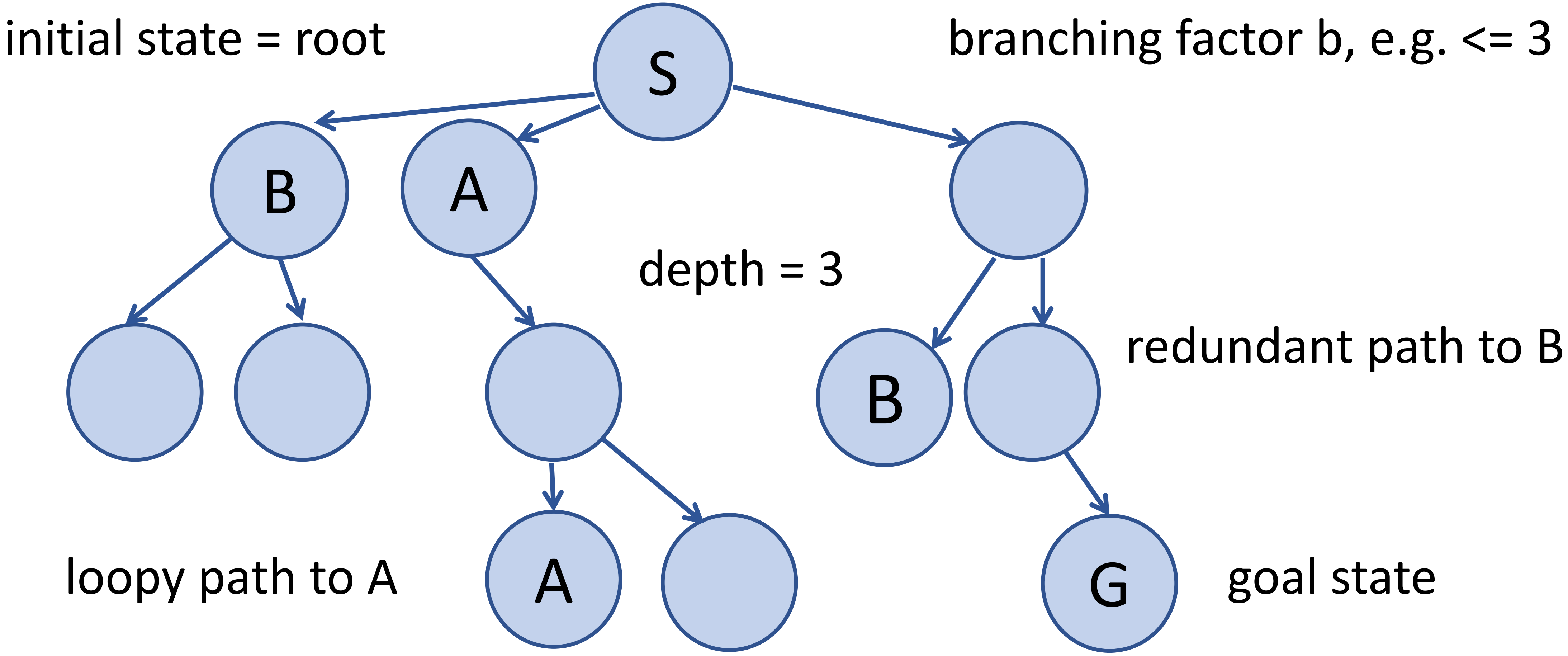
Place queens so that none attacks any other queen.

"Attack state" = 2 queens are on the same row, column or diagonal.

# Problem Solving by Searching: “Real-World Problems”

- Route finding problems
- Touring problems, e.g., Traveling Salesperson (efficient path for visiting every city once)
- VLSI layout
- Robot navigation
- Drone navigation
- Protein design
- Cancer detection

# Solution: Sequence of Actions = Path through a Search Tree



# Evaluation of Search Algorithm Performance

- Completeness:  Is it guaranteed to find a solution?
- Optimality:
  - Shortest path?
  - Lowest cost?
- Time Complexity
- Space Complexity

# Path-based Search Algorithms

Task: Find shortest path through a graph

Applications: Games, robot path planning

Lots of algorithms!

# Path-based Search Strategies

- Exhaustive search: Explore all paths

# Exhaustive Search

- Winston calls this strategy the “British Museum Procedure”
- Find all paths and select the shortest
- Search tree: root level 1 node

  2nd level b nodes

  3rd level b*b nodes

  4th level b*b*b nodes

  ….

  $d^{th}$ level $b^d$ nodes

If b=10, d=10:

$10^{10}$ = 10 billion paths

Too many to test!

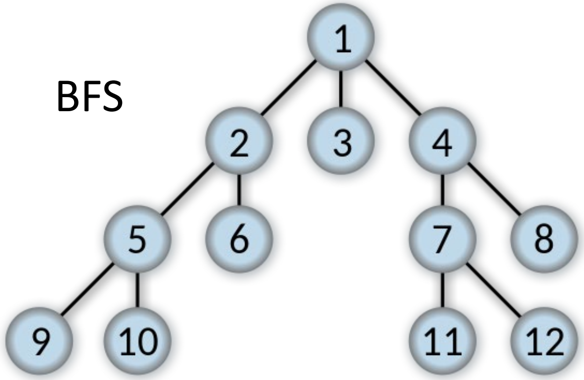
# Path-based Search Strategies

- Exhaustive search: Explore all paths
- Breadth-first search (BFS): Expand shallowest node
- Depth-first search (DFS): Expand deepest node

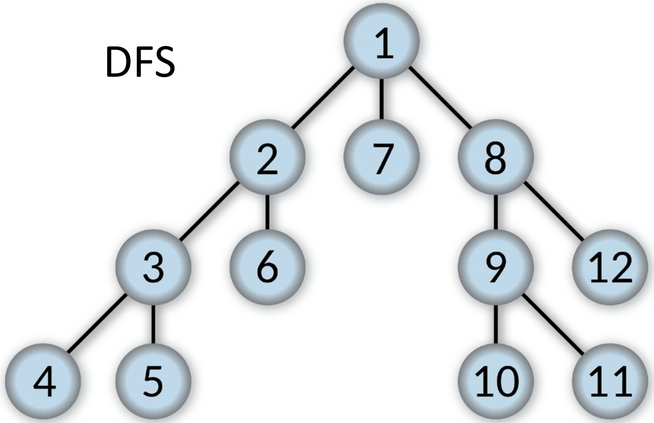
Cost per edge: 1

BFS & DFS typically covered in your previous classes.
If not, please read Wikipedia pages

BFS




DFS






Illustrations: Drichel, Wikipedia

# Path-based Search Strategies

- Exhaustive search:  Explore all paths
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- Uniform cost search:  Expand node with smallest cost
- Beam search:  BFS but only keep limited number of best nodes
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- Iterative Deepening:  Gradually increasing depth d = 0, 1, 2, …
- Bidirectional search: Search from start S and goal G nodes, hoping to meet

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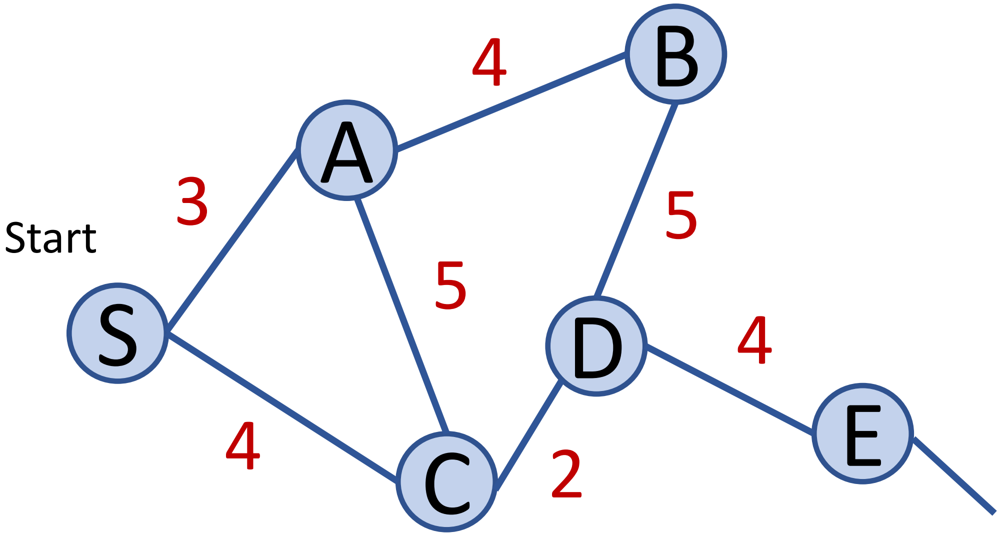
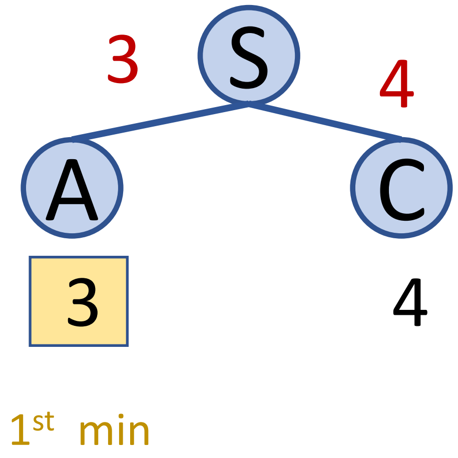
Used in AI when graphs are extremely large or infinite

# Example for Uniform Cost Search

- Exhaustive search: Explore all paths
- Breadth-first search (BFS): Expand shallowest node
- Depth-first search (DFS): Expand deepest node
- Uniform cost search: Expand node with smallest cost



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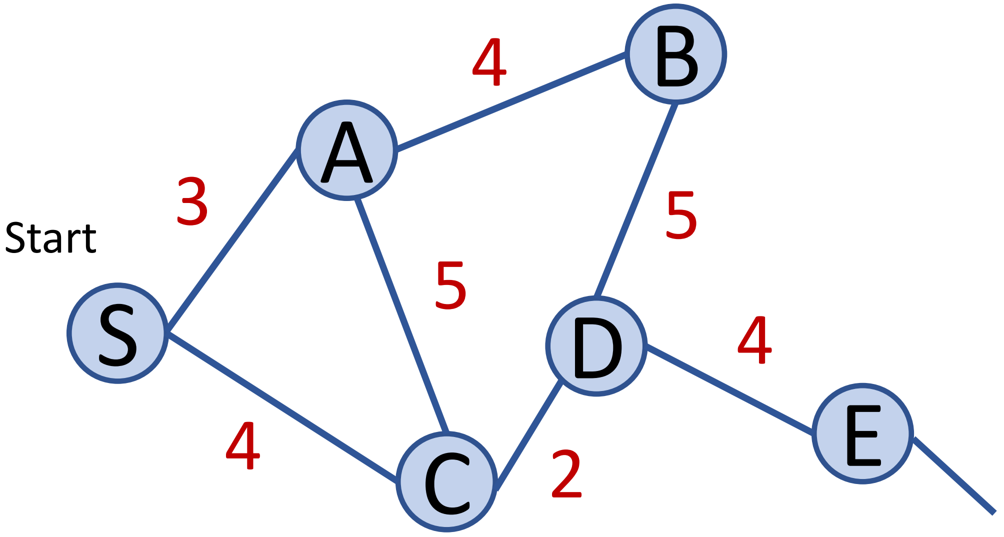


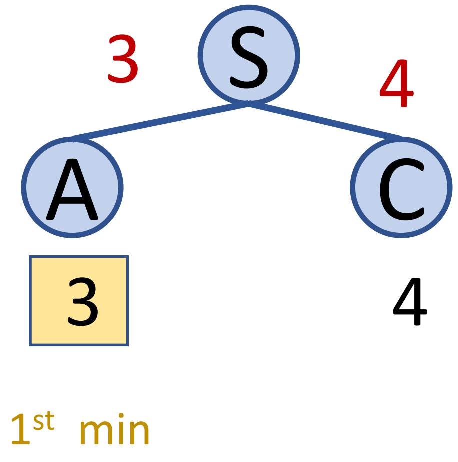


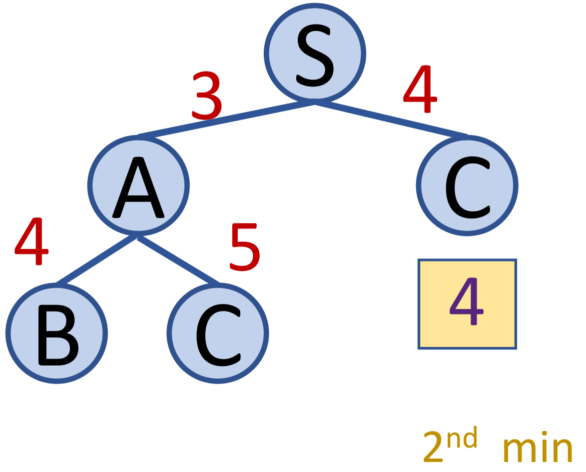

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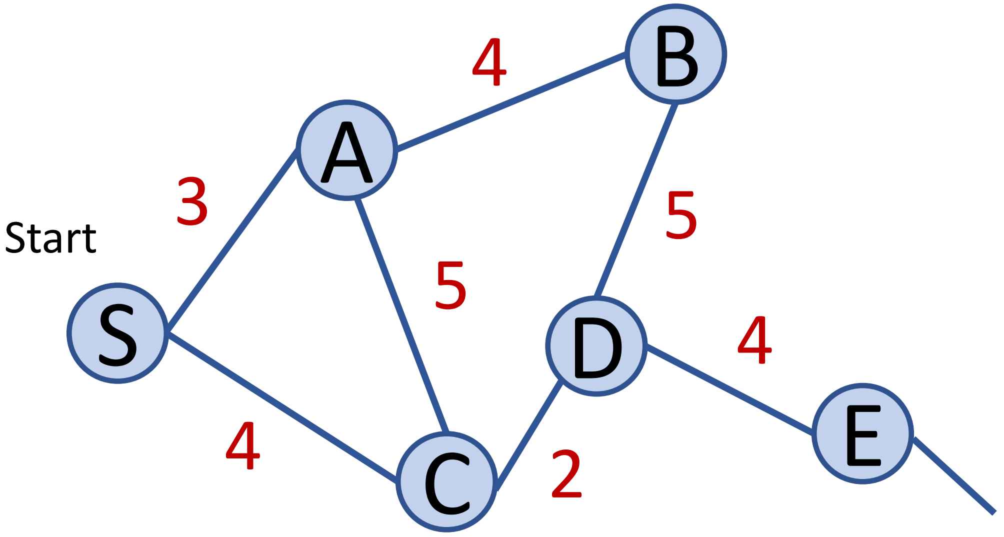


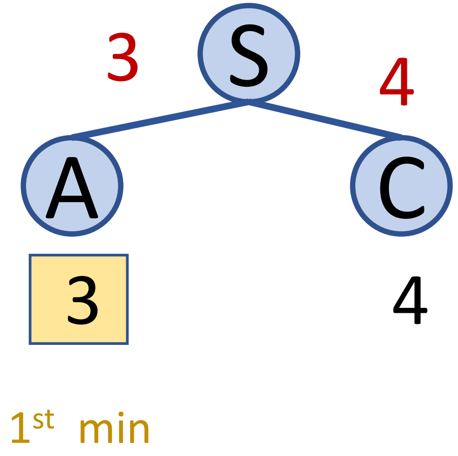


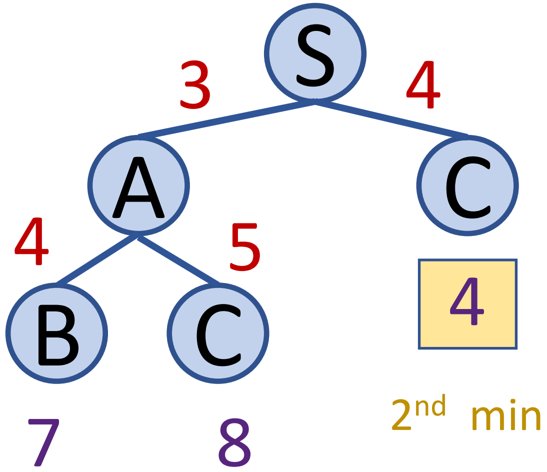


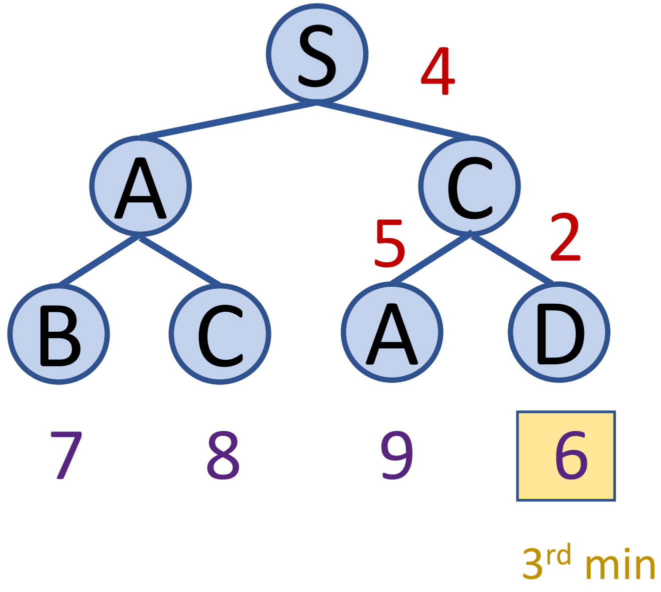

# Beam Search

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- Depth-first search (DFS): Expand deepest node
- Uniform cost search:  Expand node with smallest cost
- Beam search:  BFS but only keep limited number w of best nodes at each level, the beam width w
    - Same as BFS with w = infinite
    - The greater w is the fewer states are pruned
    - Useful in AI if BFS search tree is too large to fit in memory
    - Not guaranteed to find optimal solution

# Path-based Search Strategies

- Exhaustive search: Explore all paths
- Breadth-first search (BFS): Expand shallowest node
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- Uniform cost search: Expand node with smallest cost
- Beam search: BFS but only keep limited number of best nodes
- Depth-limited search: Predetermined depth limit

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- Beam search: BFS but only keep limited number of best nodes
- Depth-limited search: Predetermined depth limit
- Progressive Deepening (also called Iterative Deepening):

  Gradually increasing depth d = 0, 1, 2, …

  We have seen this algorithm used for adversarial game playing.

# Path-based Search Strategies

- Exhaustive search:  Explore all paths
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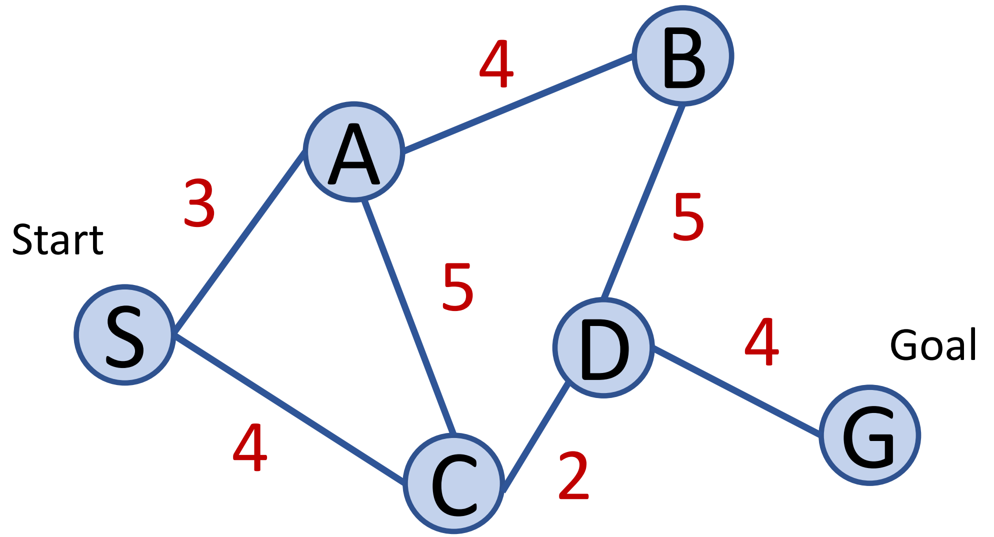
- Bidirectional search: Search from start S and goal G nodes, hoping to meet
- Greedy search = branch & bound search
- Greedy search with pruning
- A* = Greedy search with pruning and underestimates of remaining distance

# Greedy Search = Branch & Bound Search

Phase 1: Extend shortest partial path until goal is reached.

Reject loops.

Phase 2: Extend all partial paths
until their length >= complete path to goal

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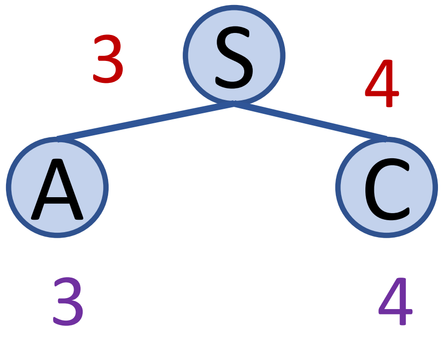

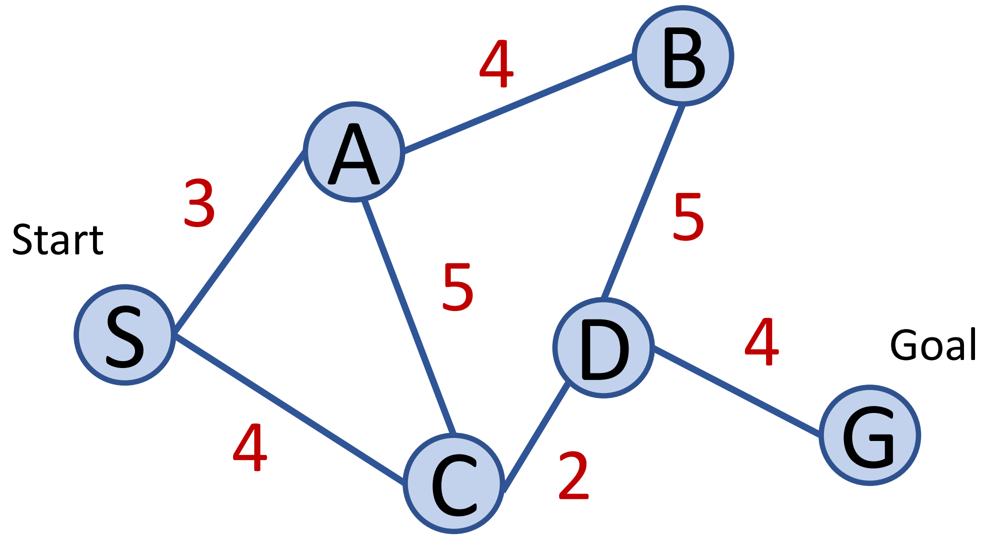


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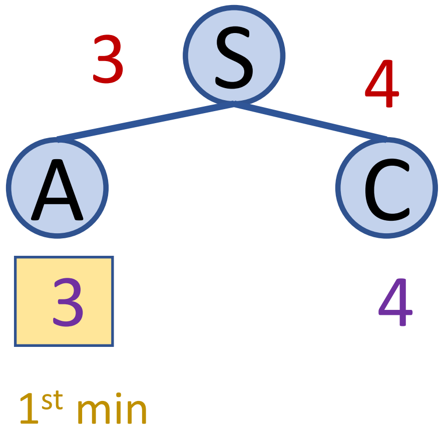

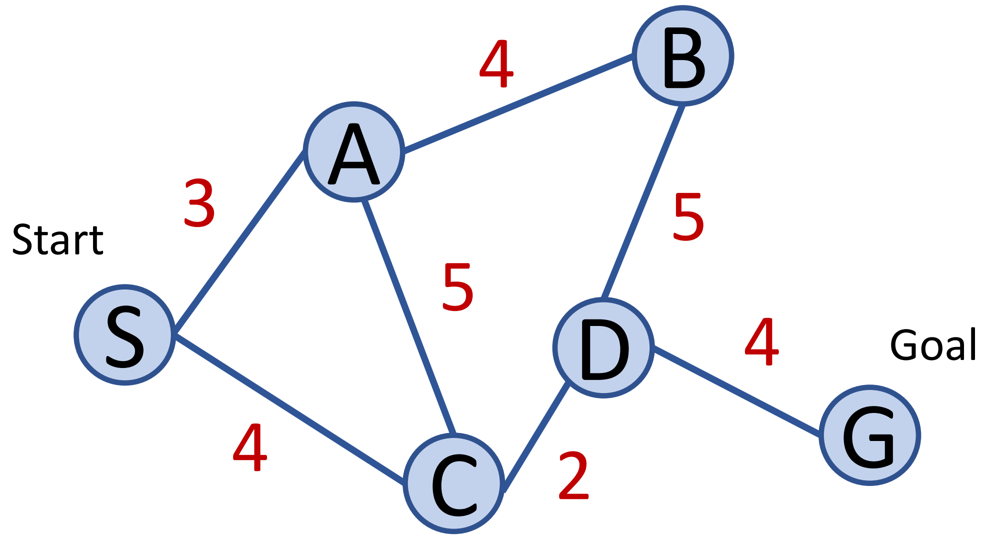


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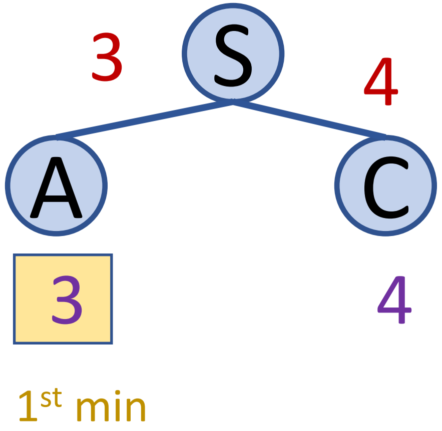


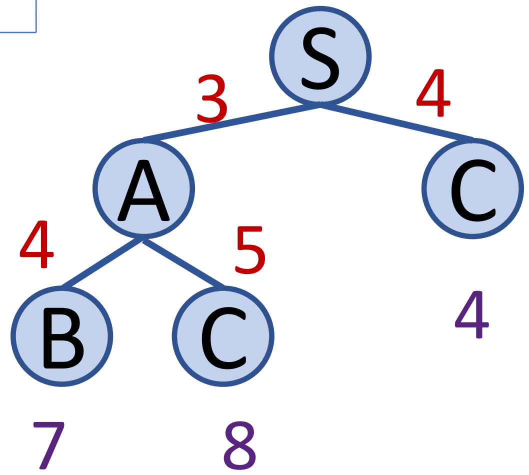

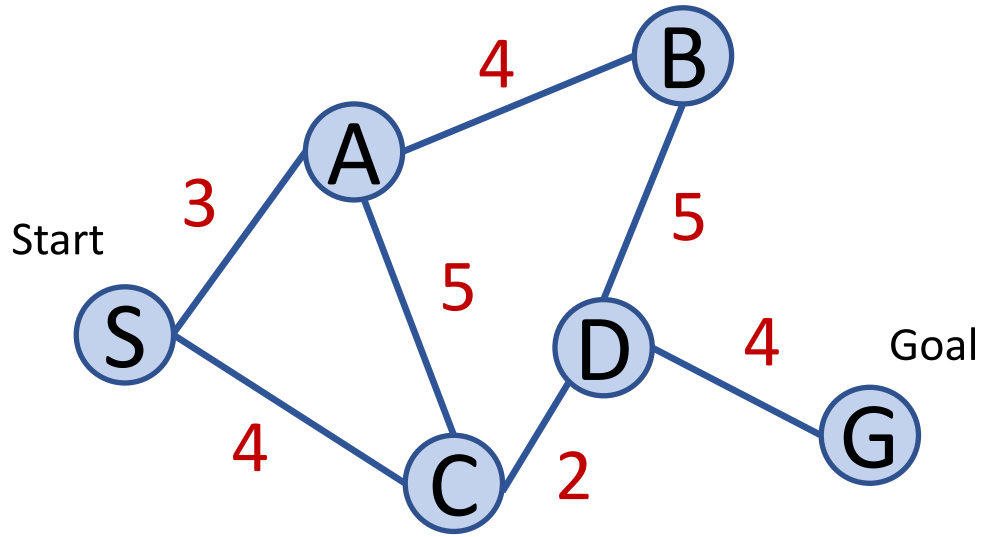


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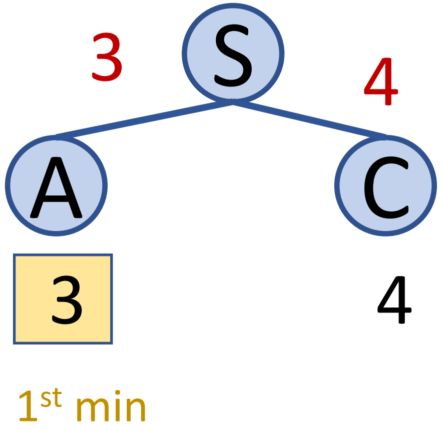


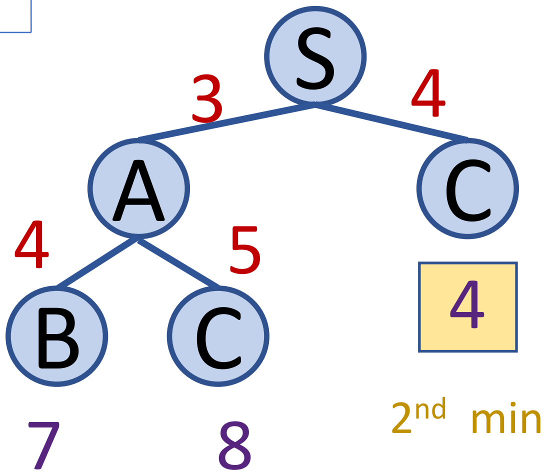

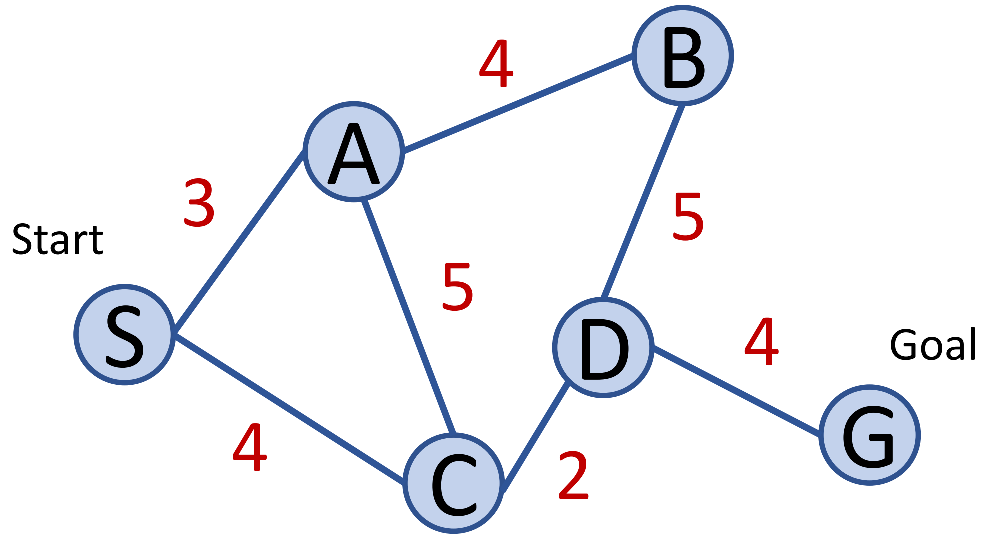


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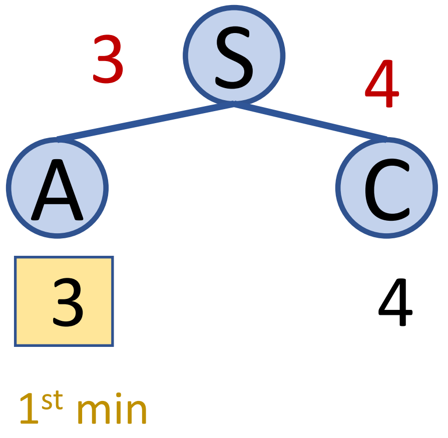


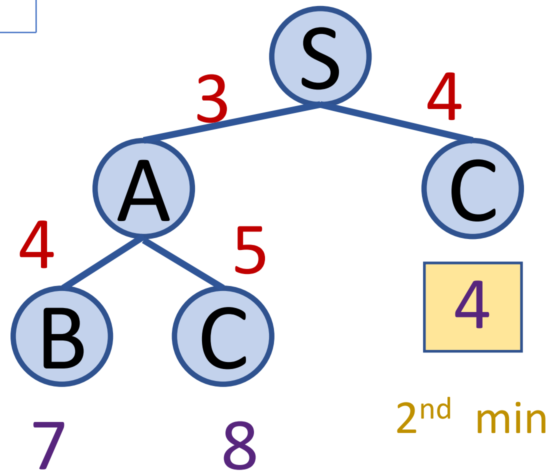


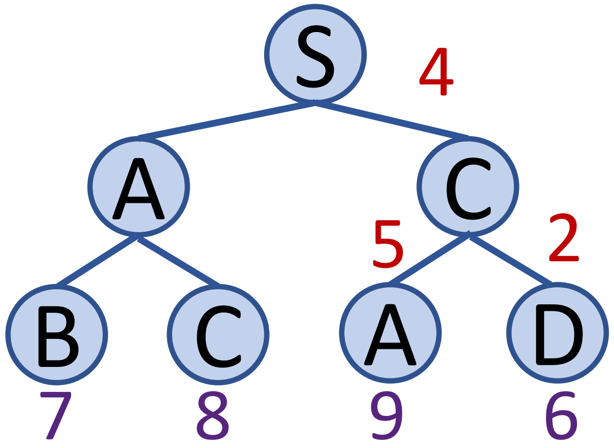

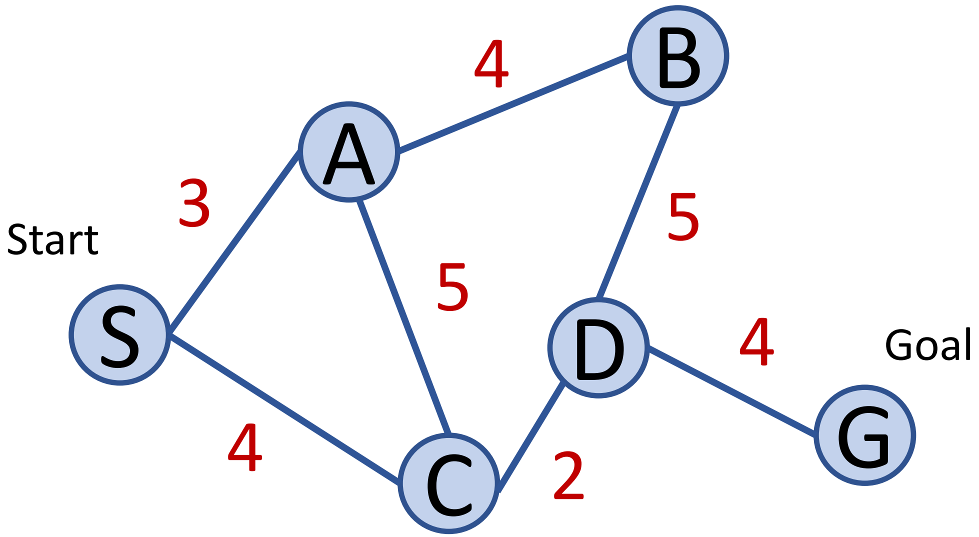


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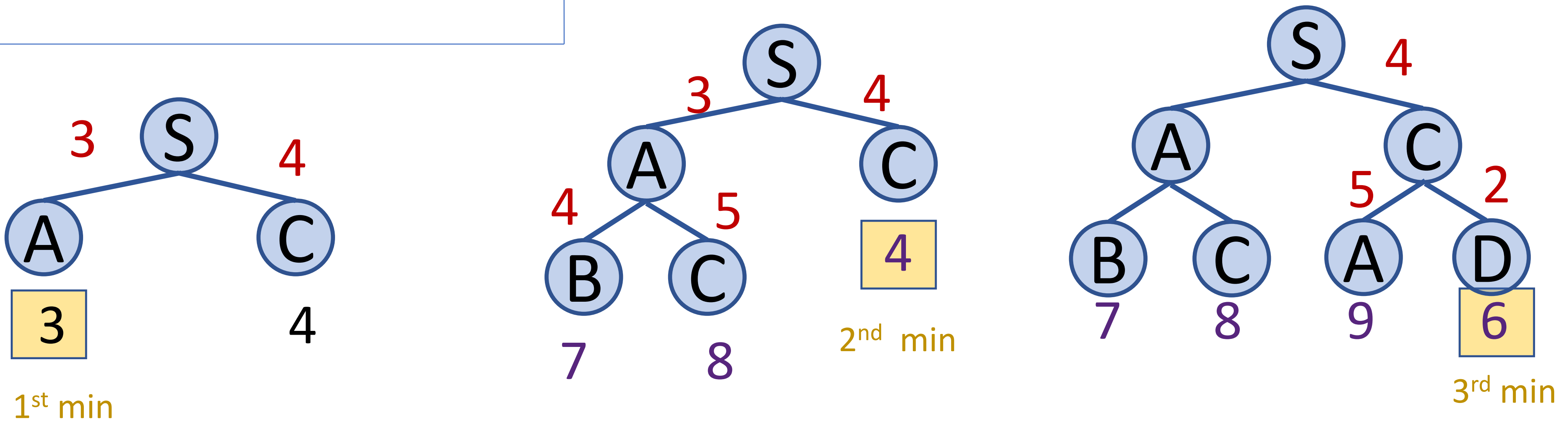

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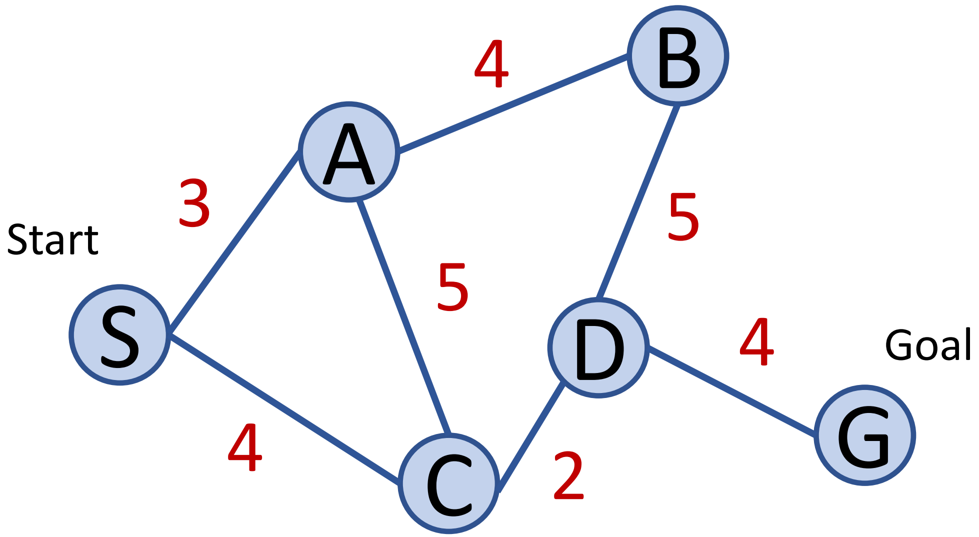


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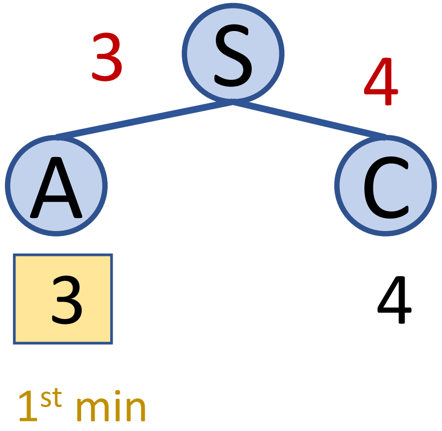
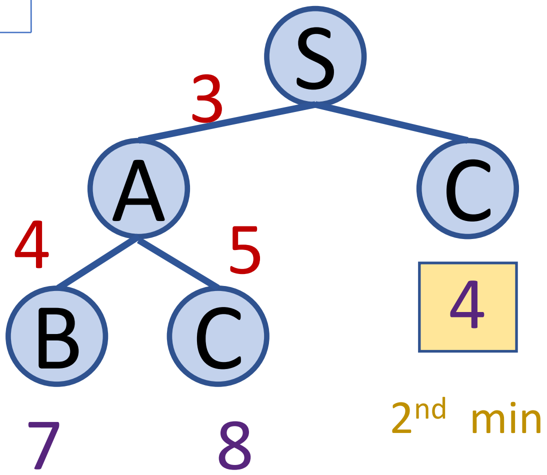
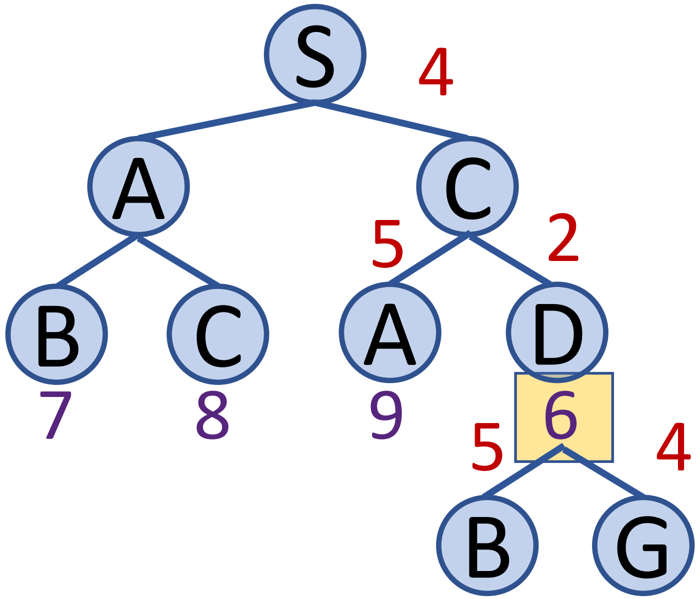
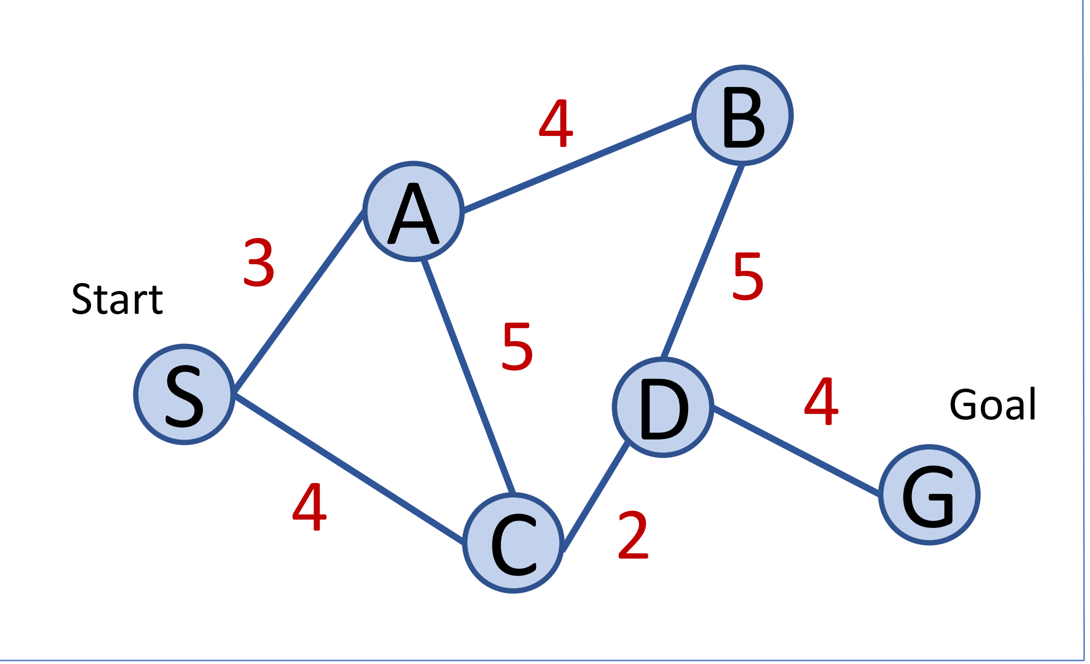


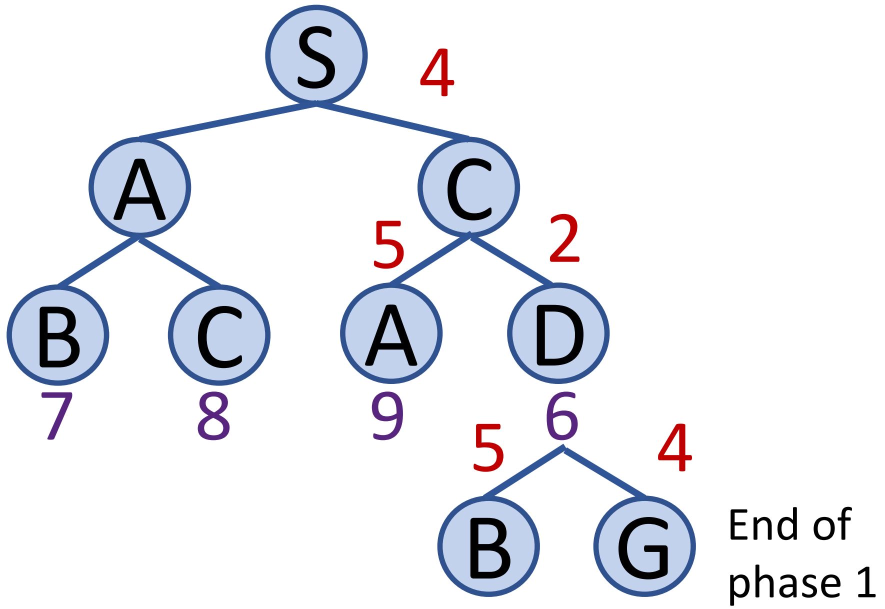



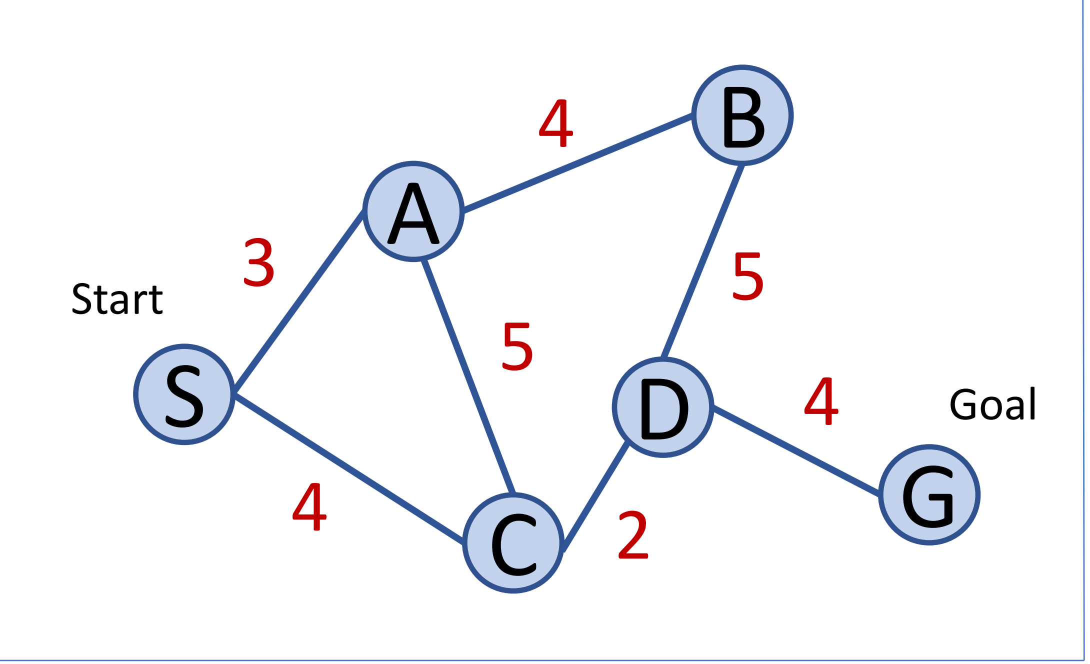

End of phase 1: G reached




Phase 2:




Phase 2: Extend all partial paths
until their length >= complete path to goal

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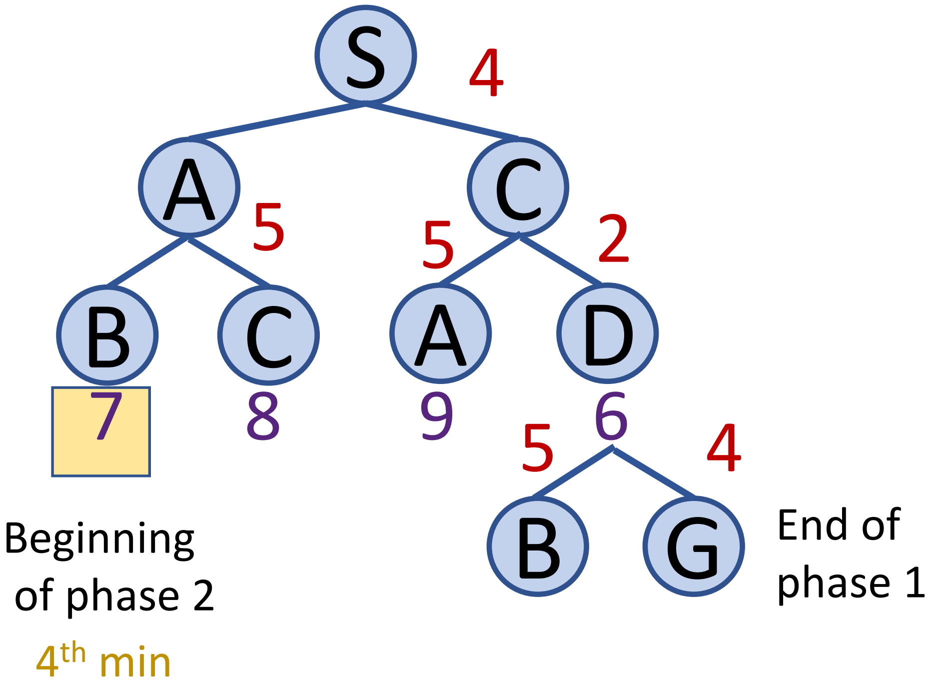


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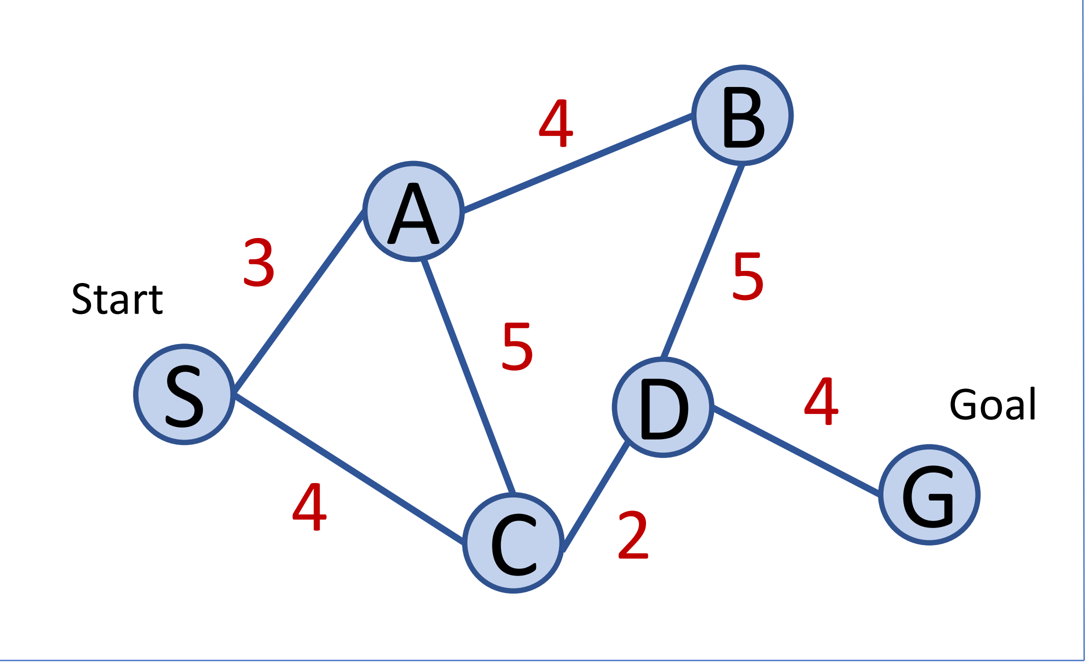


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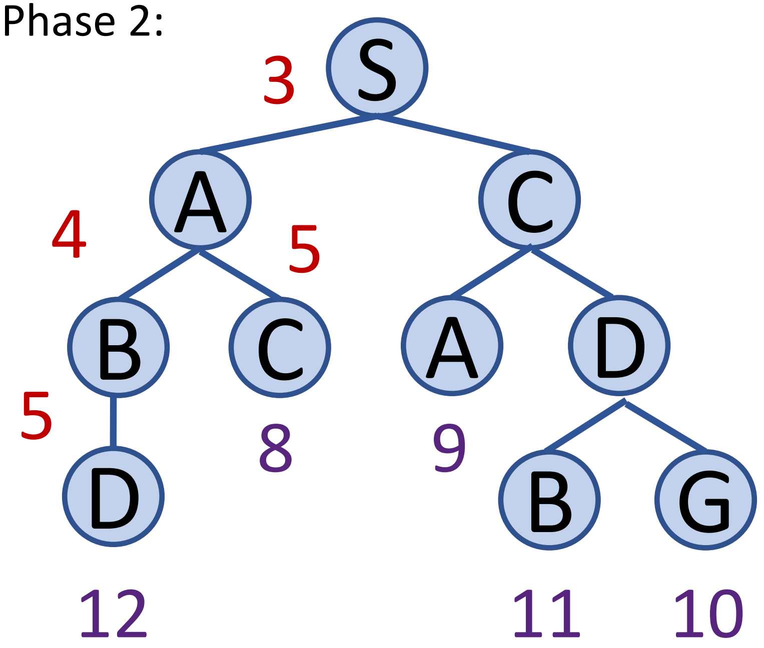


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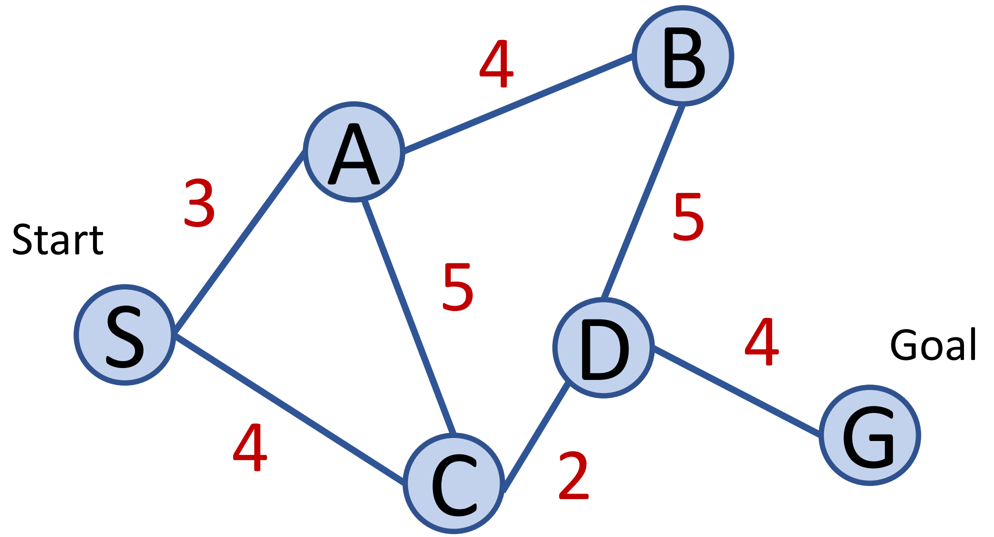


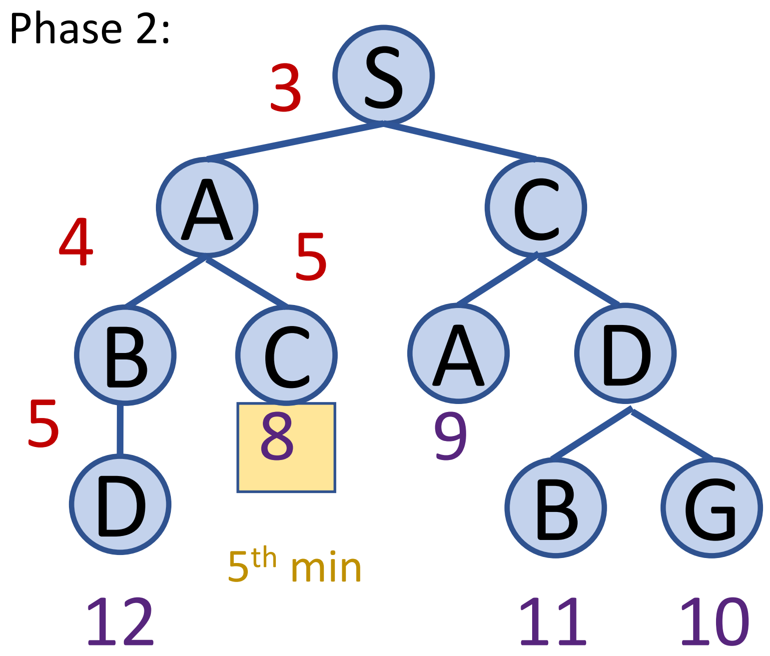


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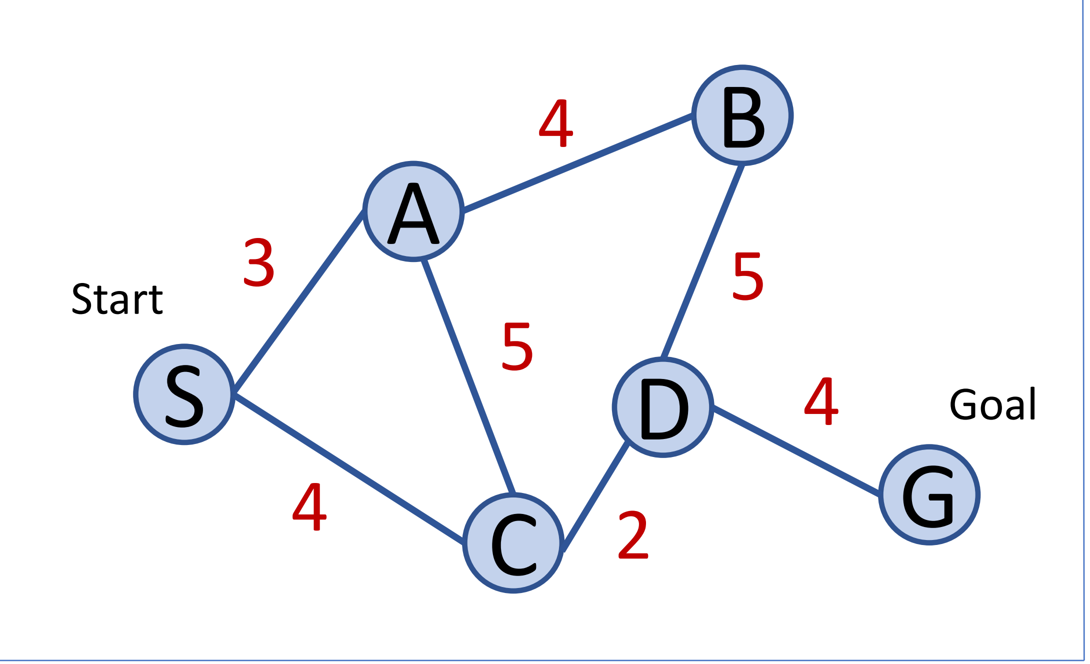


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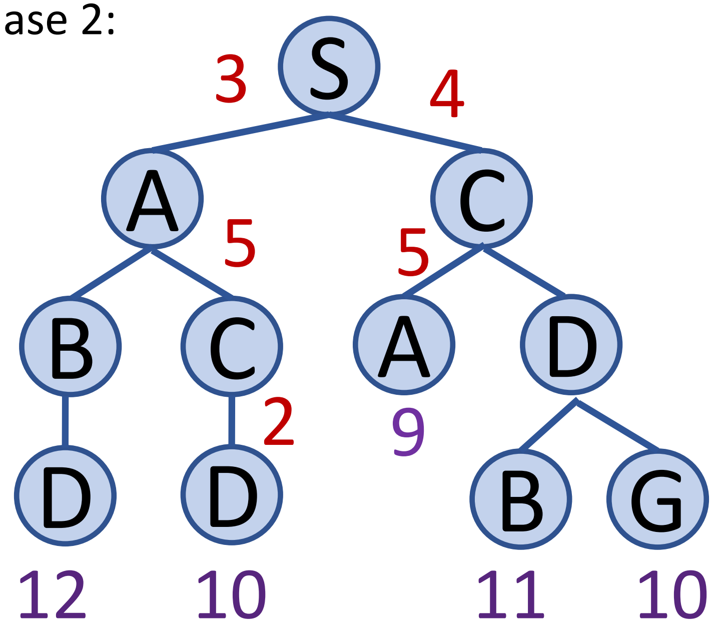


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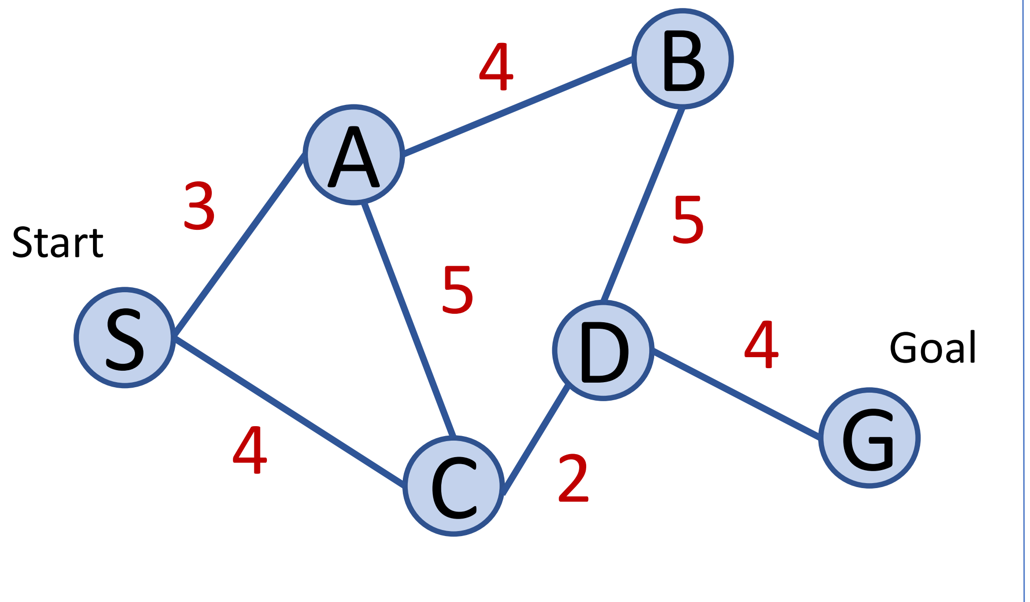


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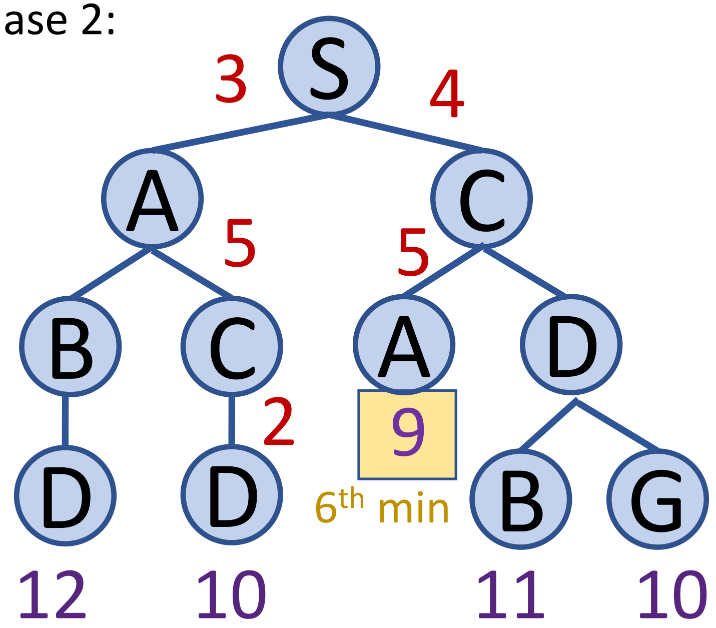


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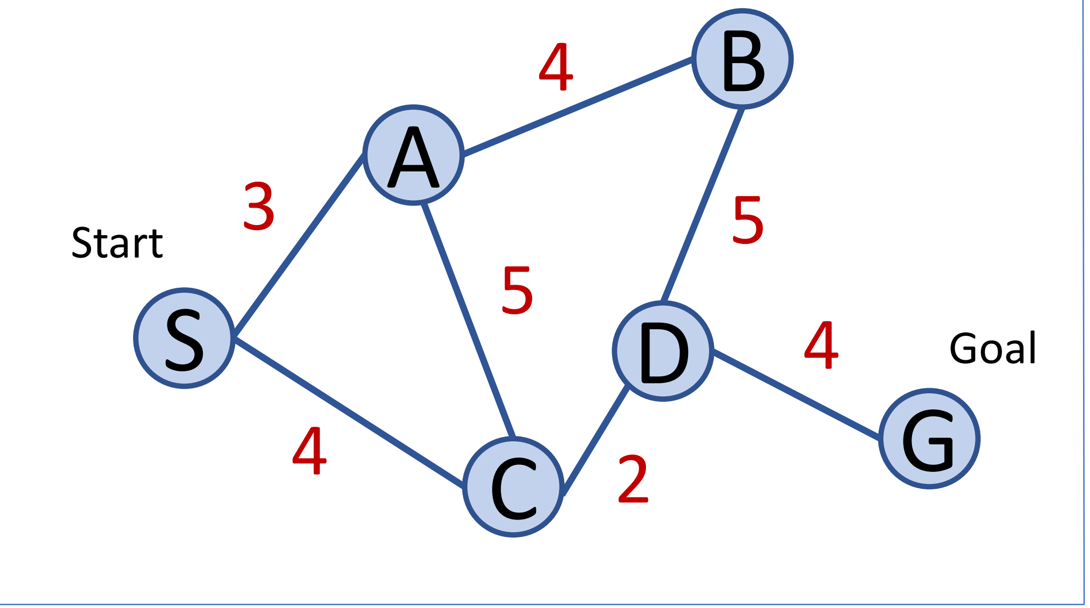


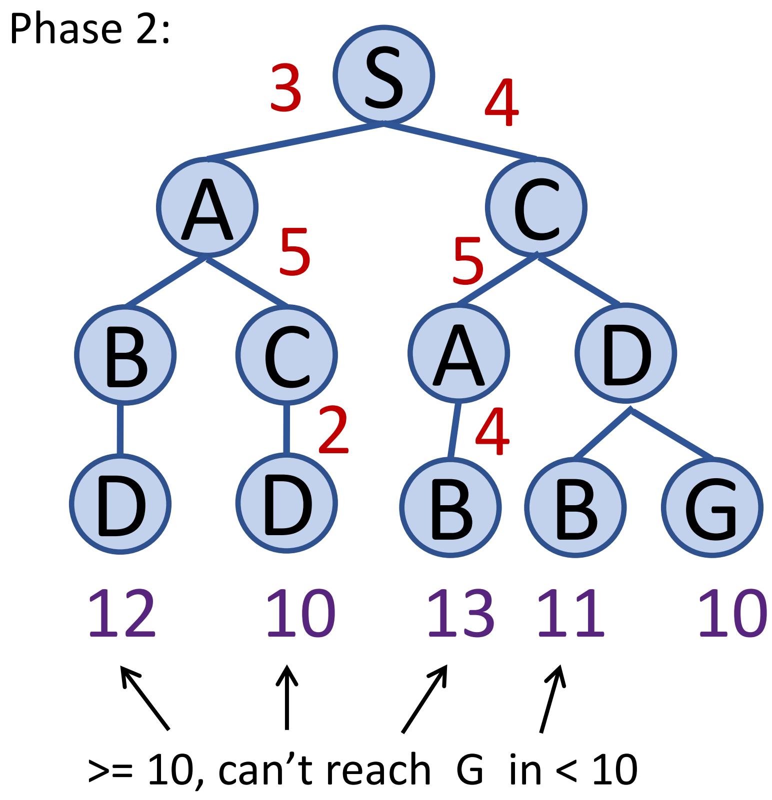


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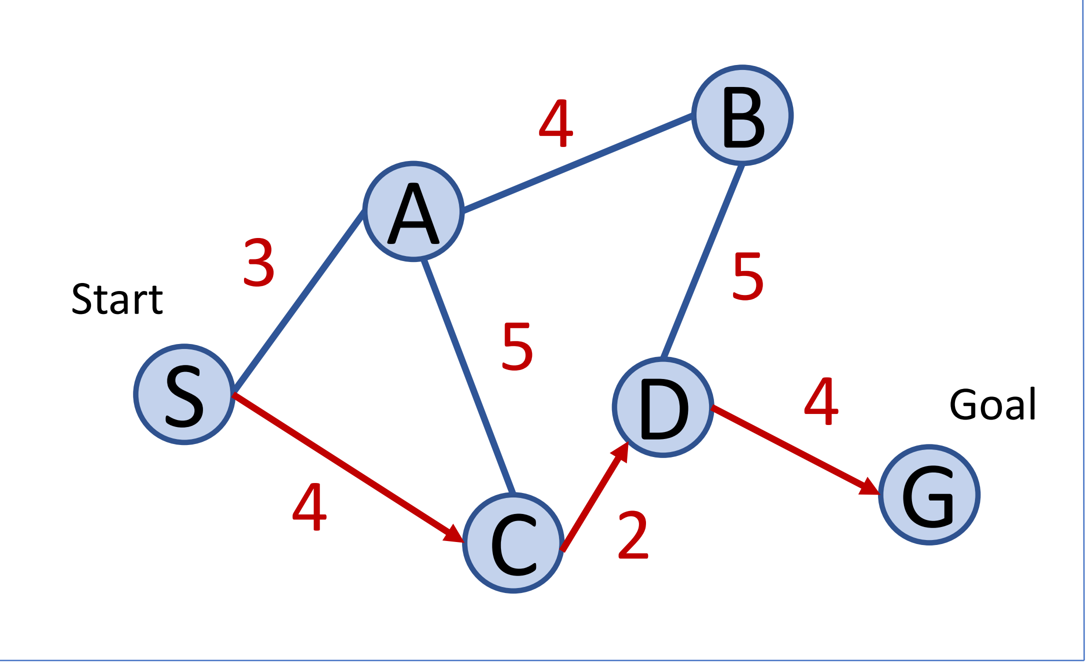


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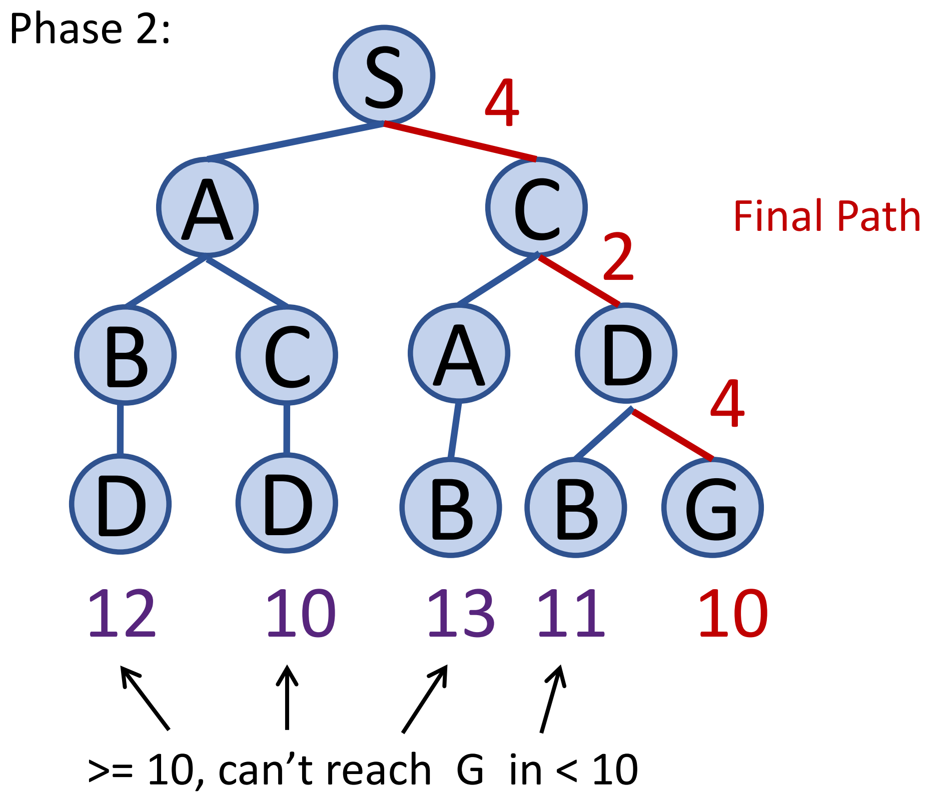
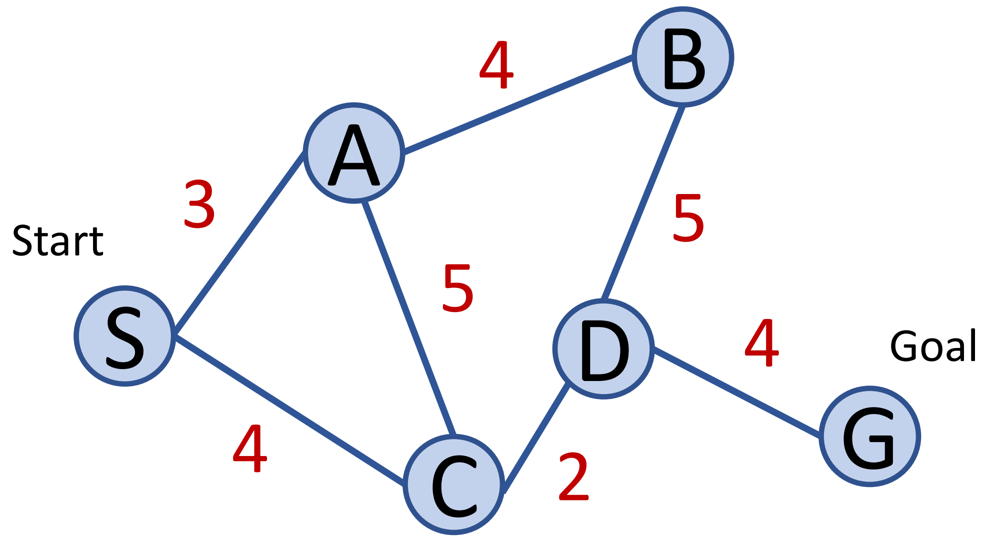


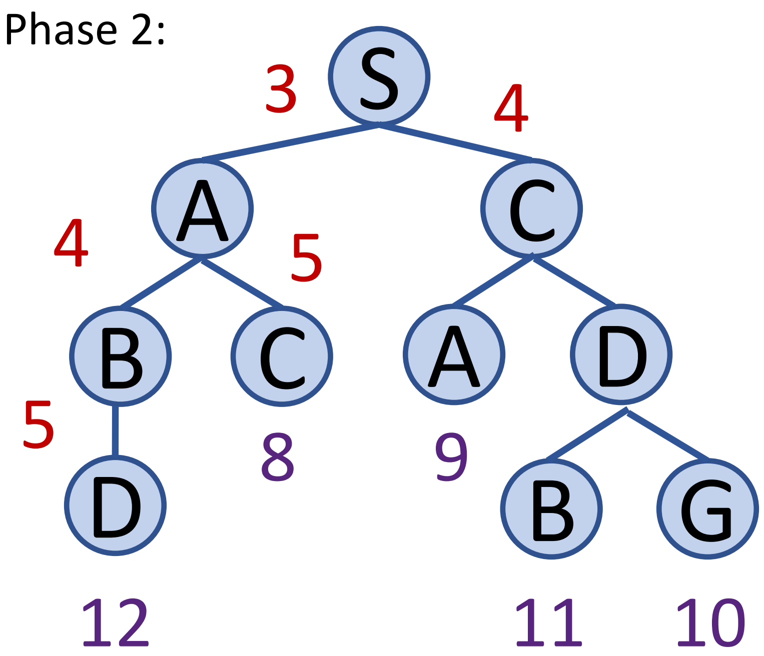
# Greedy Search with Pruning

Dynamic Programming Principle:

If two or more paths reach a common node, delete all paths except the minimum cost path.

# Greedy Search with Pruning




Phase 2: Extend all partial paths
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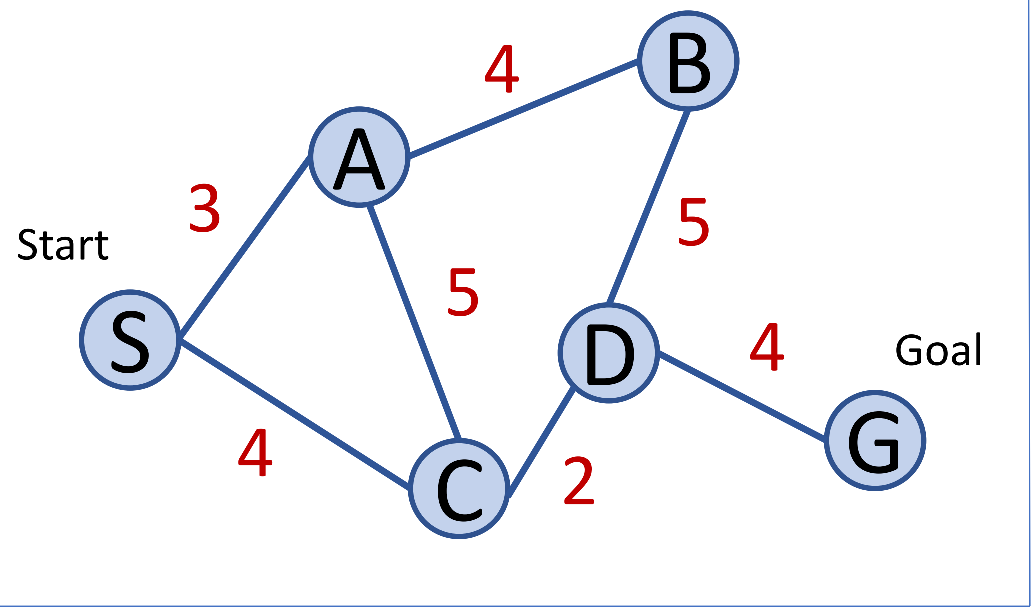
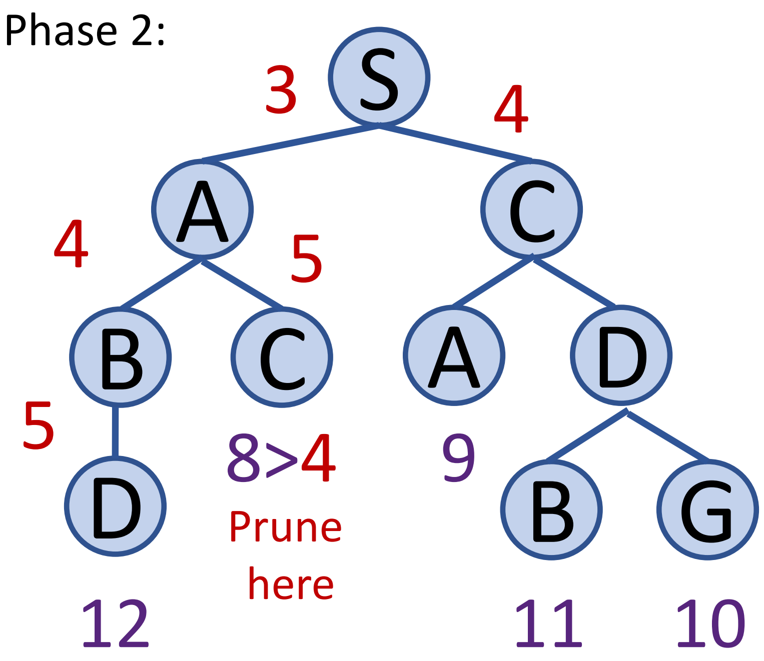
# Greedy Search with Pruning

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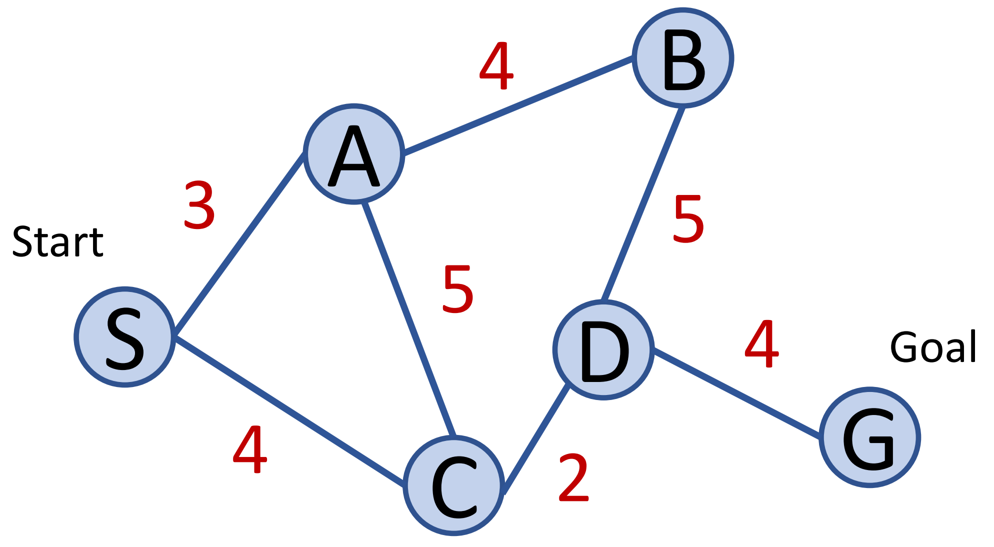


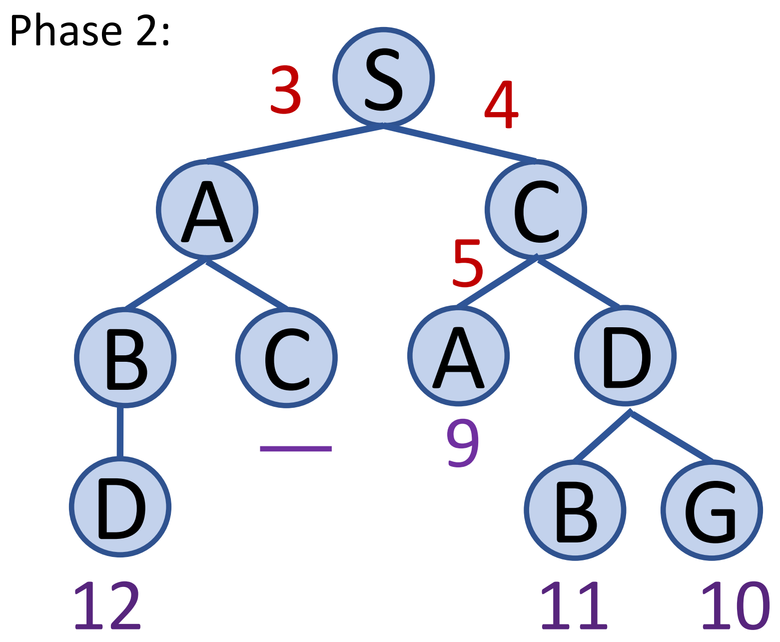


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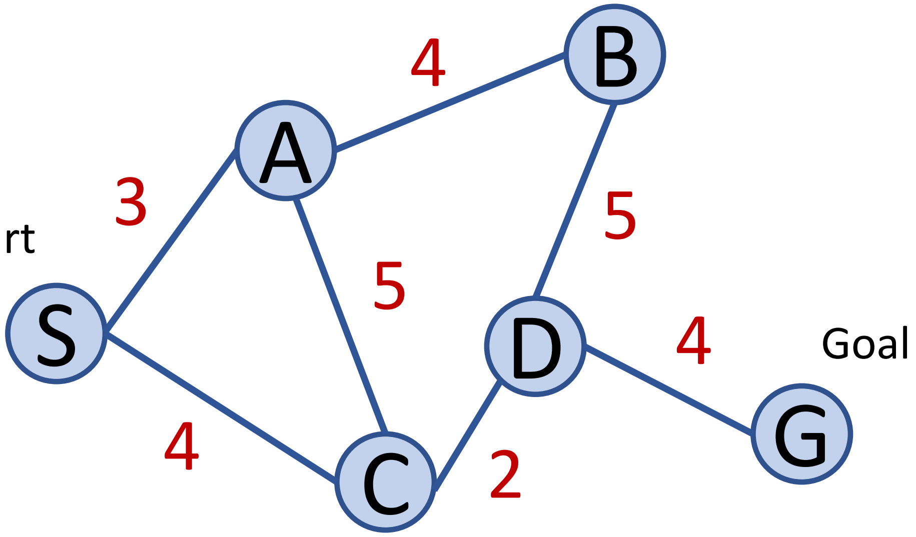
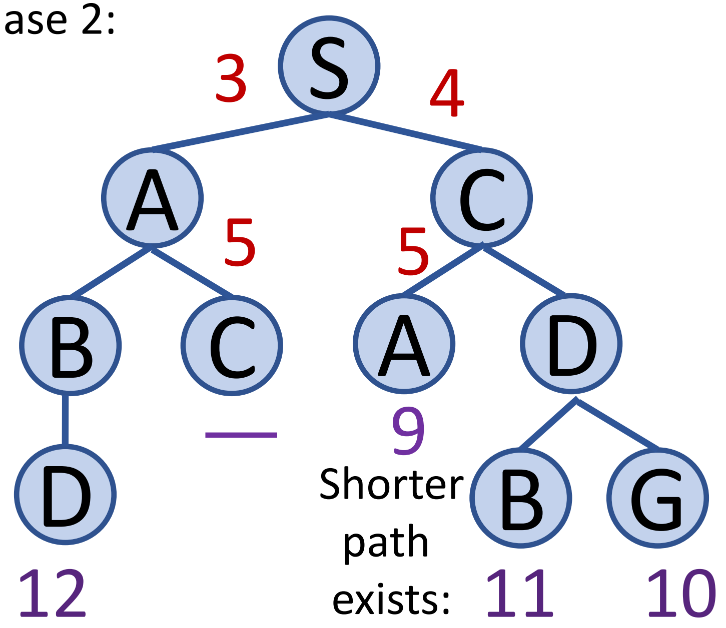


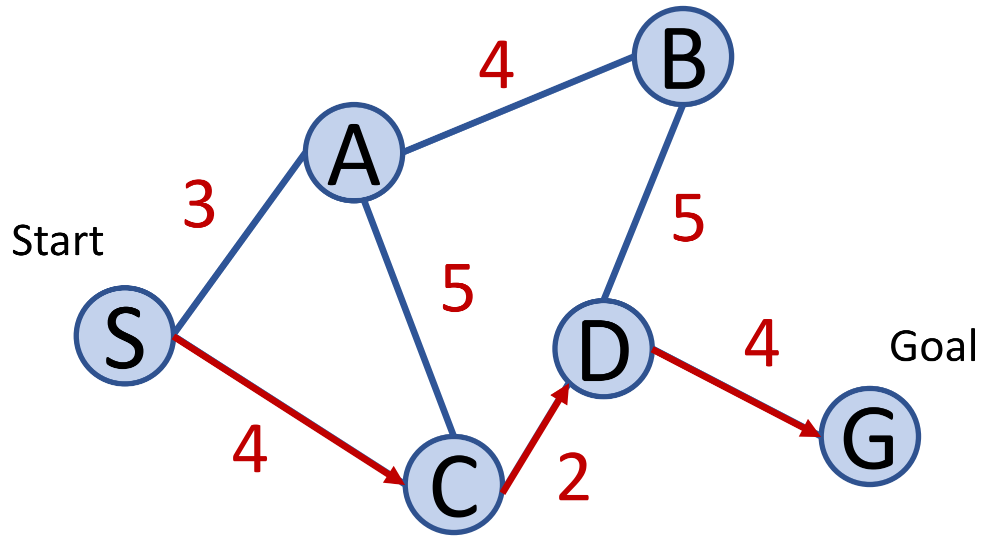

Phase 2:

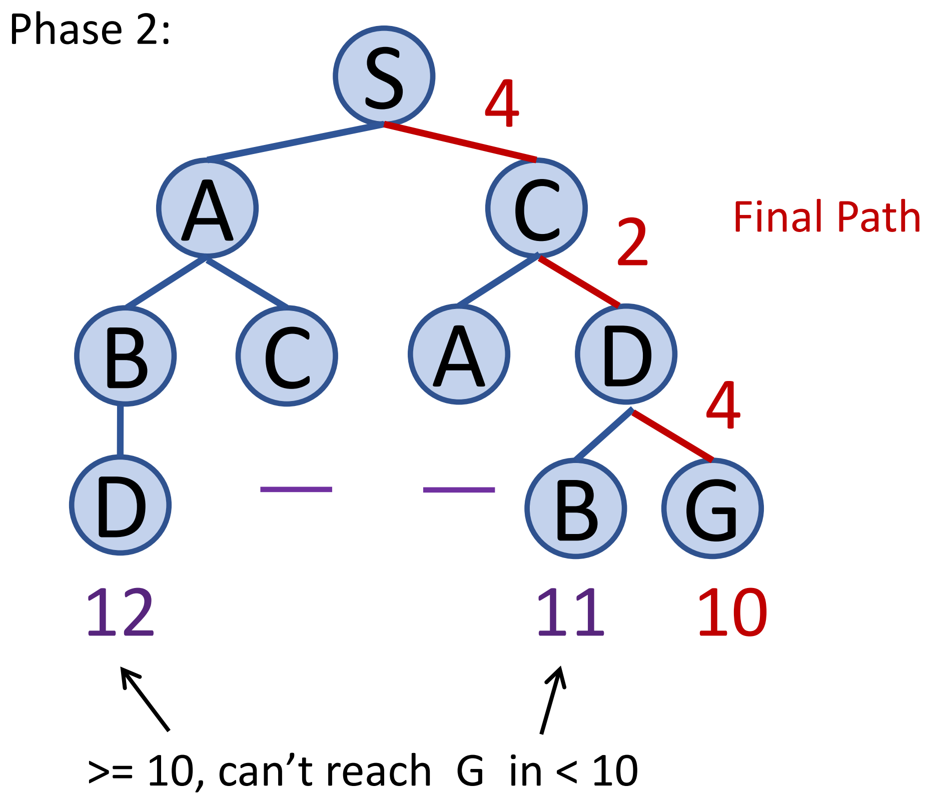



Prune!

Phase 2: Extend all partial paths until their length >= complete path to goal

# Greedy Search with Pruning




Phase 2: Extend all partial paths until their length >= complete path to goal

# Implementation of Greedy Search with Pruning

Data Structure: Queue
Elements of queue: Partial paths

Initialize: Place start node in queue
Until a path in queue reaches goal node or queue is empty:

- Remove 1st queue element & extend path to its children
- Reject loops
- Add new paths to queue
- Prune
- Sort

# Connection to Dijkstra's Algorithm

Same as Greedy Search with Pruning except

- Dijkstra's algorithm computes the all-pairs shortest paths
- while "Greedy Search with Pruning" computes the shortest path between a single start state and a single goal state.

# A* Algorithm = Greedy Search with Pruning and Underestimates of Remaining Distance

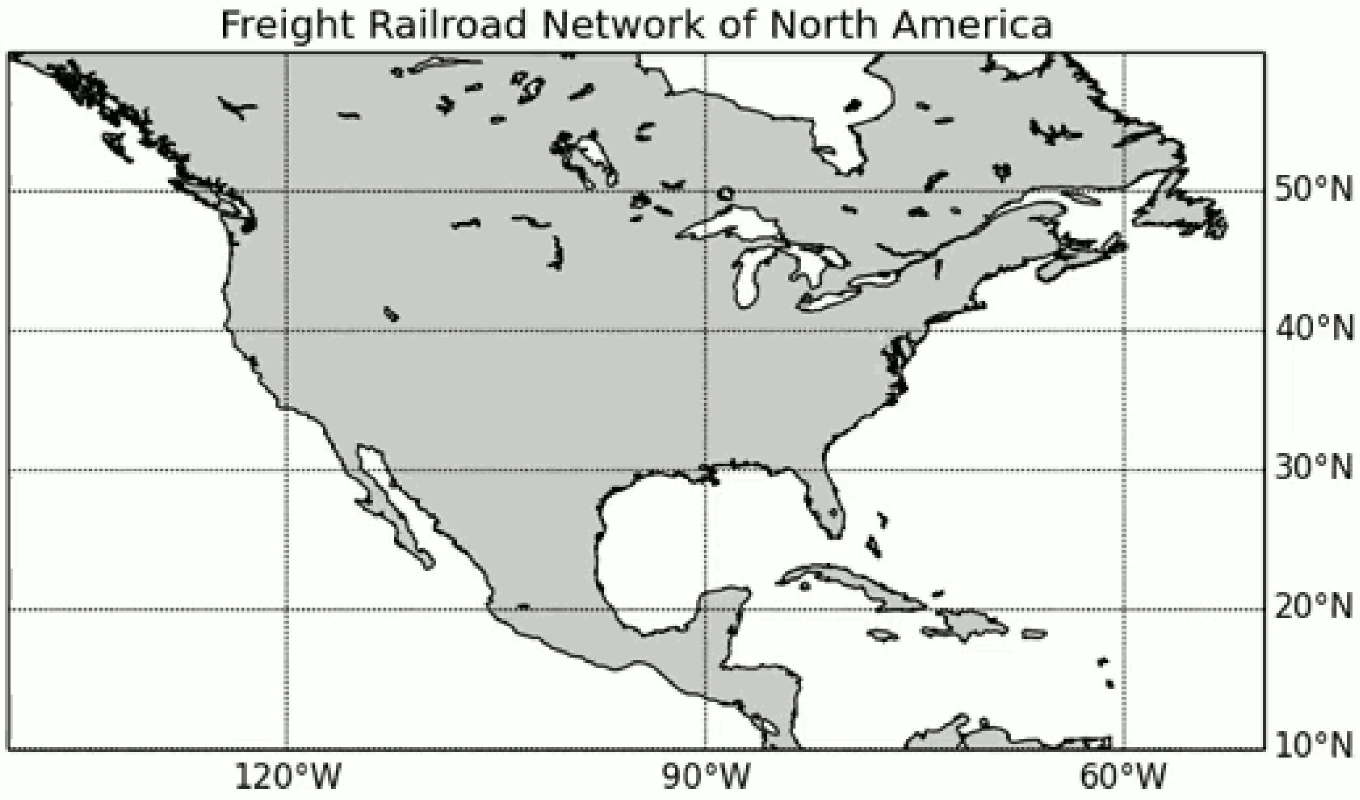
- Remaining distance = e.g., straight-line distance on a highway map
- In each step:

Estimate of total path length =

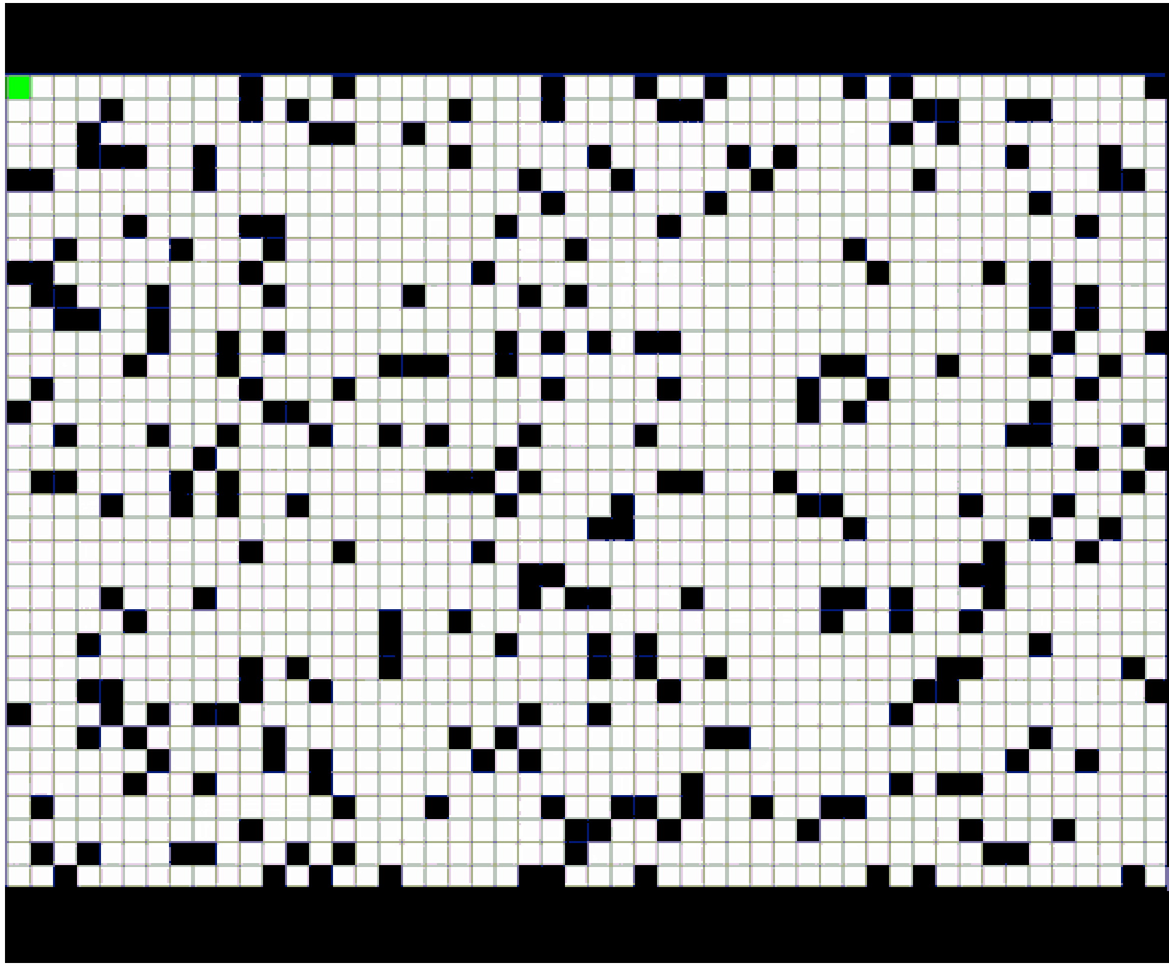
length of partial path + underestimate of remaining

"This path is at least this bad."

An animation of the A* algorithm as it explores the North American freight train network to find the optimum path between Washington, D.C. and Los Angeles.

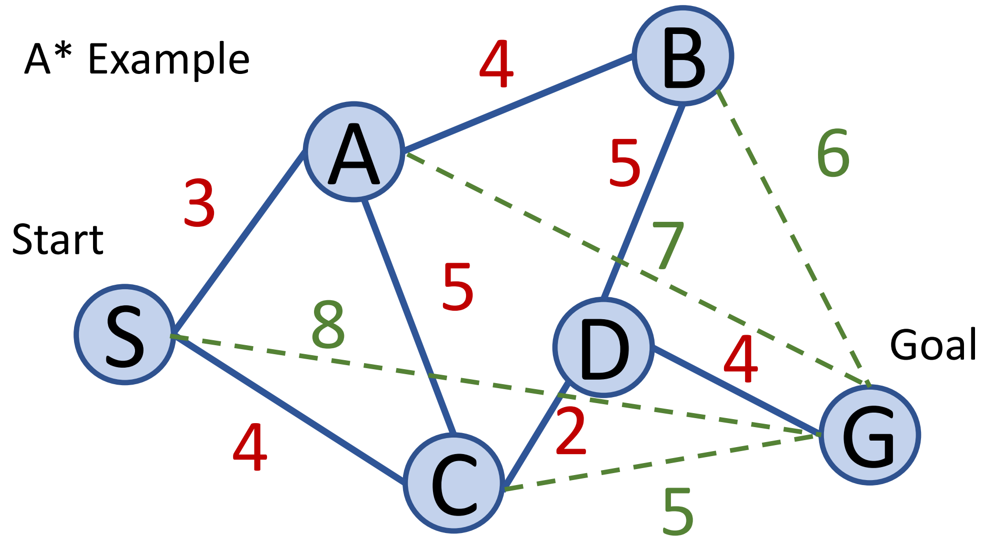



A* pathfinding algorithm navigating around a randomly-generated maze



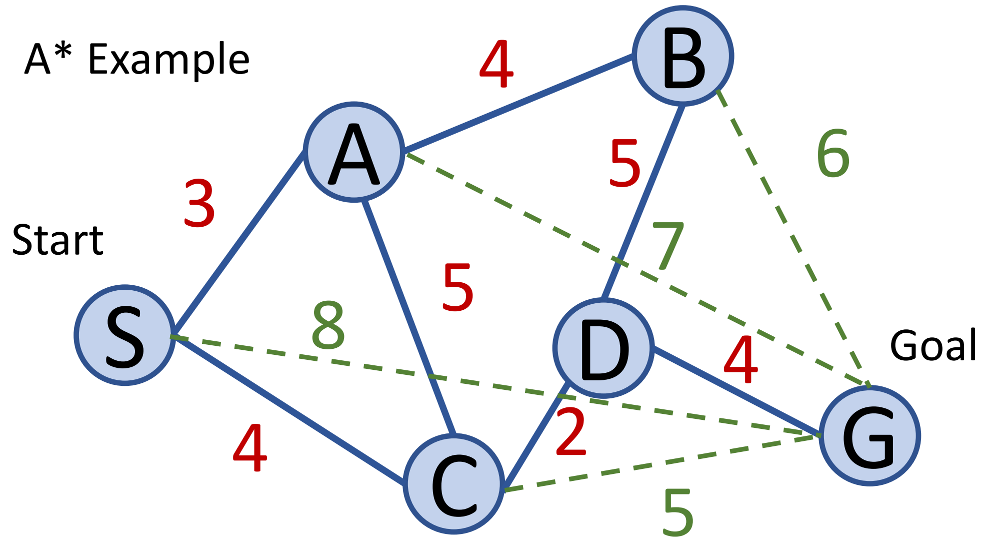


Source: Wikipedia, Srossd, Wgullyn




A* Phase 1: Extend shortest estimated partial path
(= length of partial path + underestimate of remaining)
until goal is reached.
Reject loops.

Phase 2: Extend all estimated partial paths
until their length >= complete path to goal

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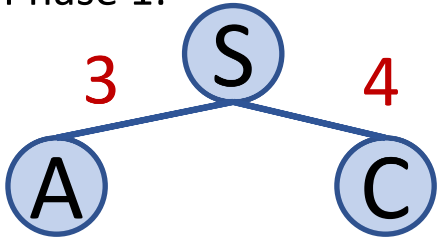
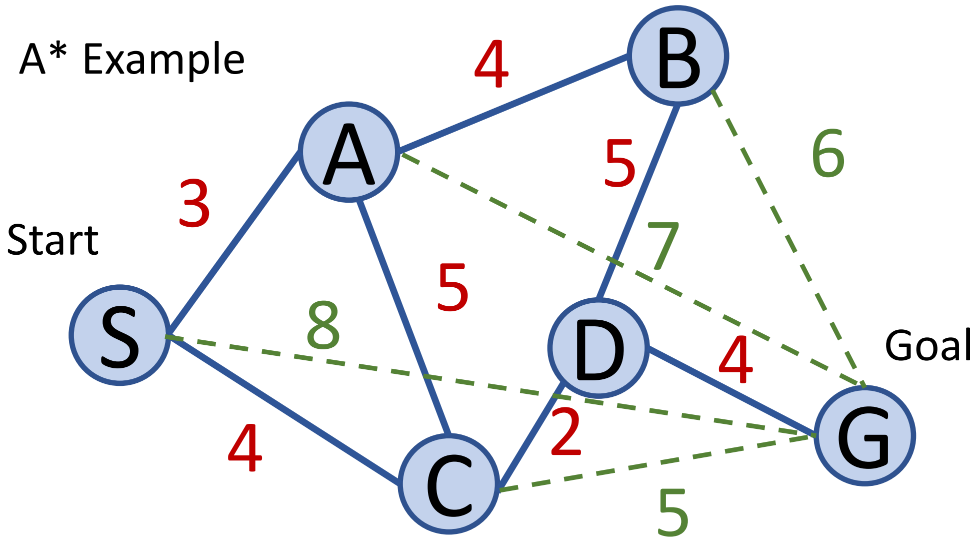


A* Example

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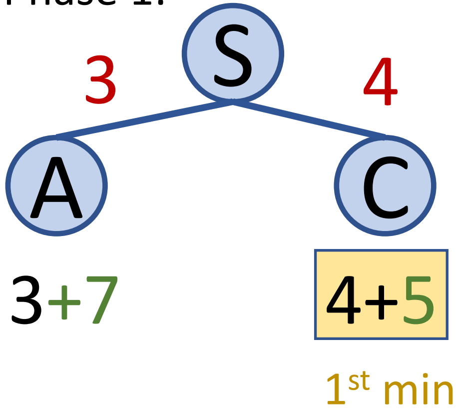

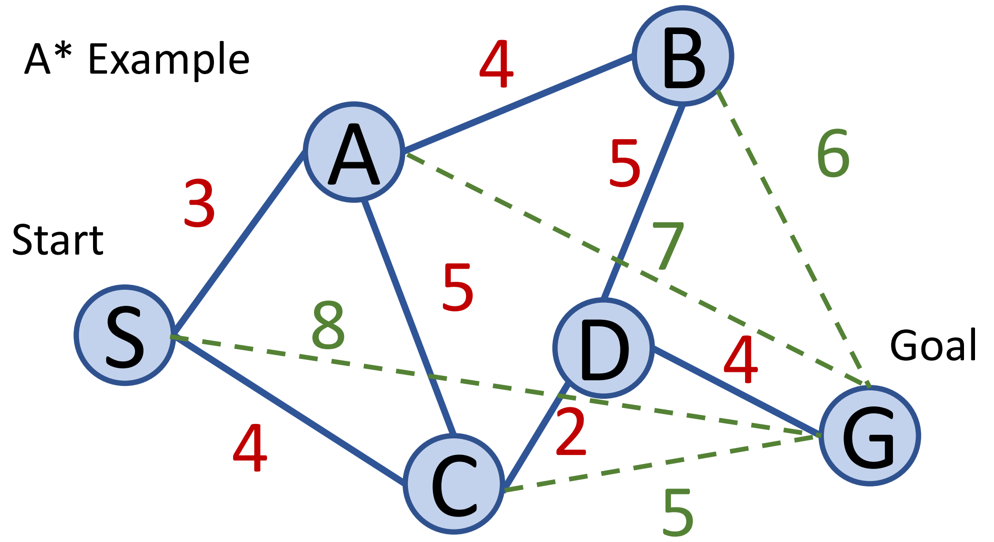


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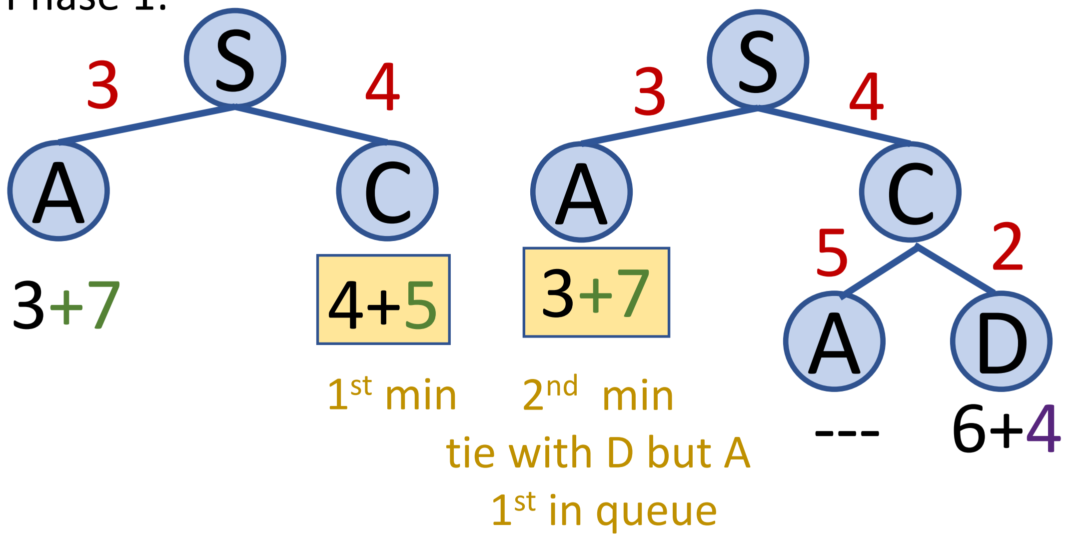
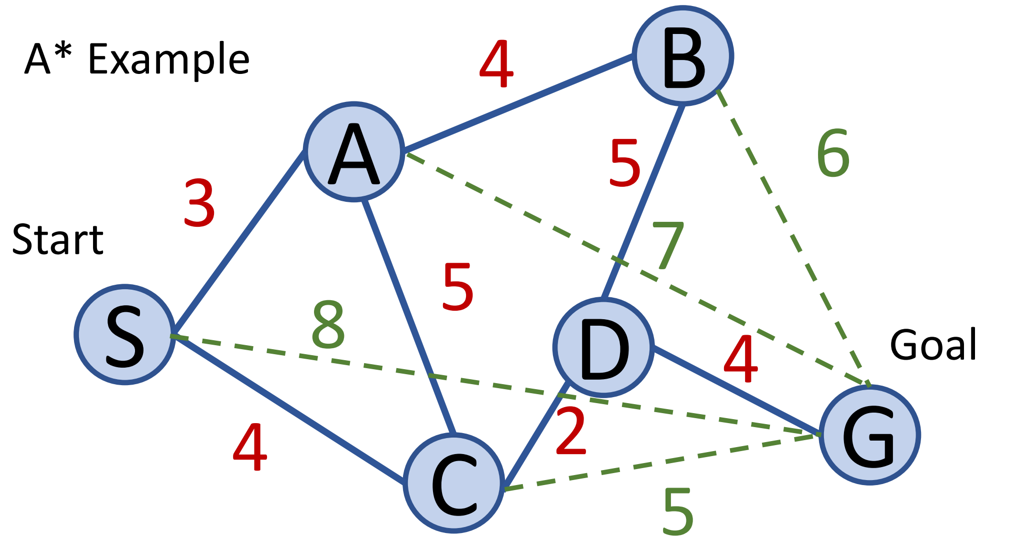


A* Example

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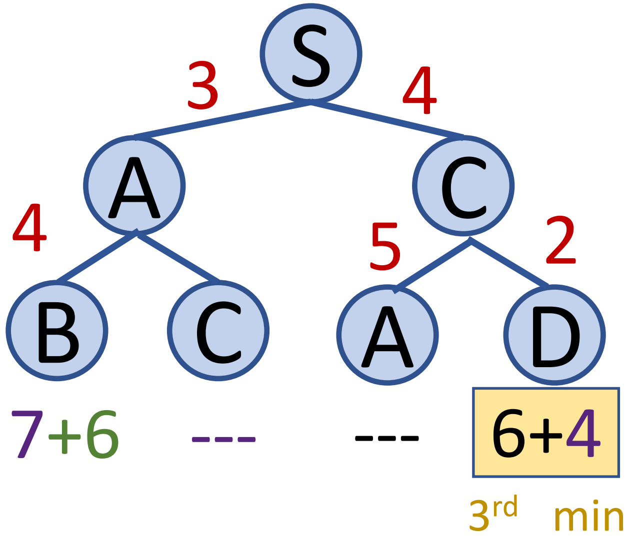


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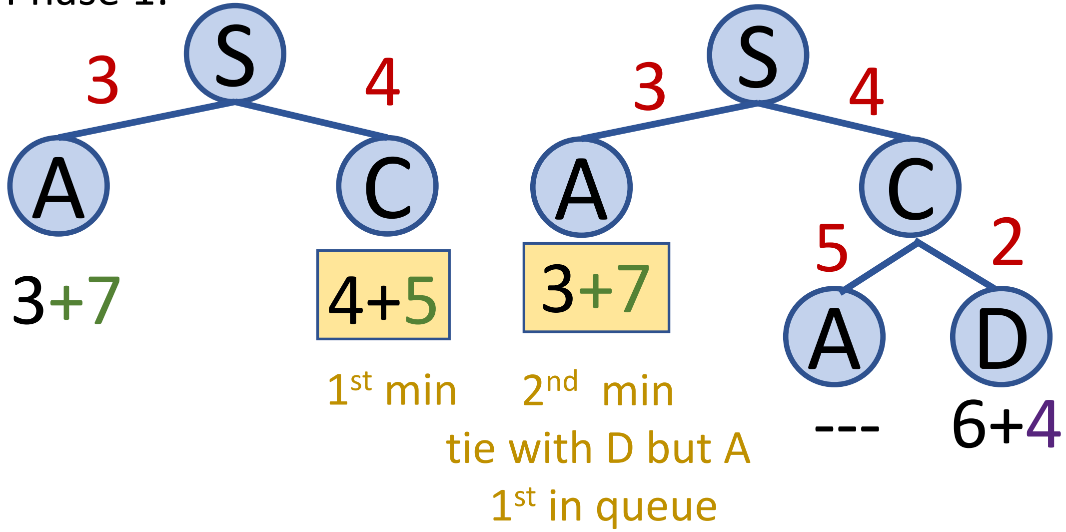
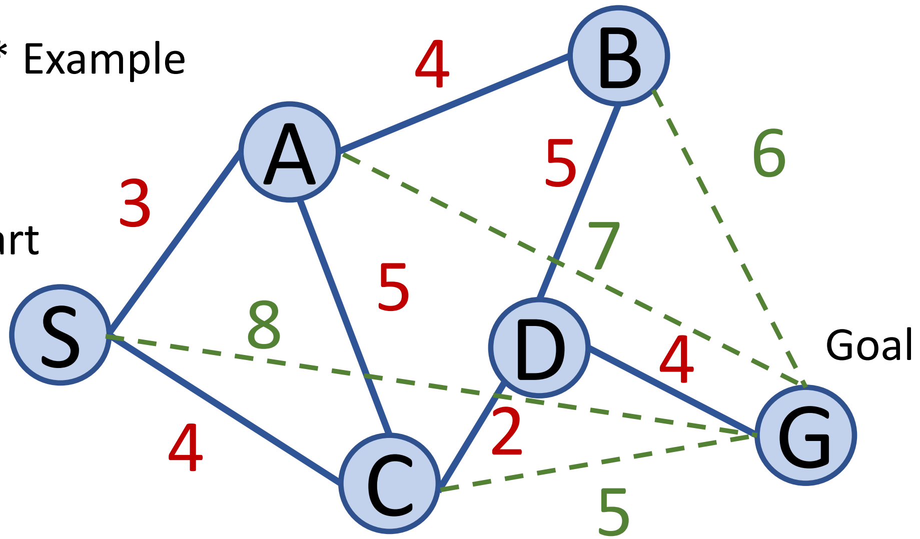


A* Example

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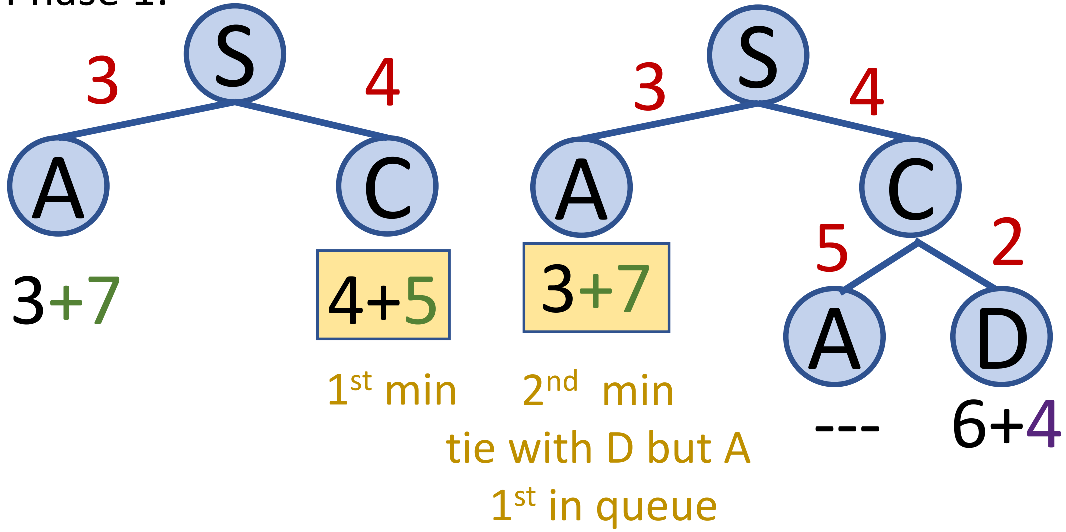


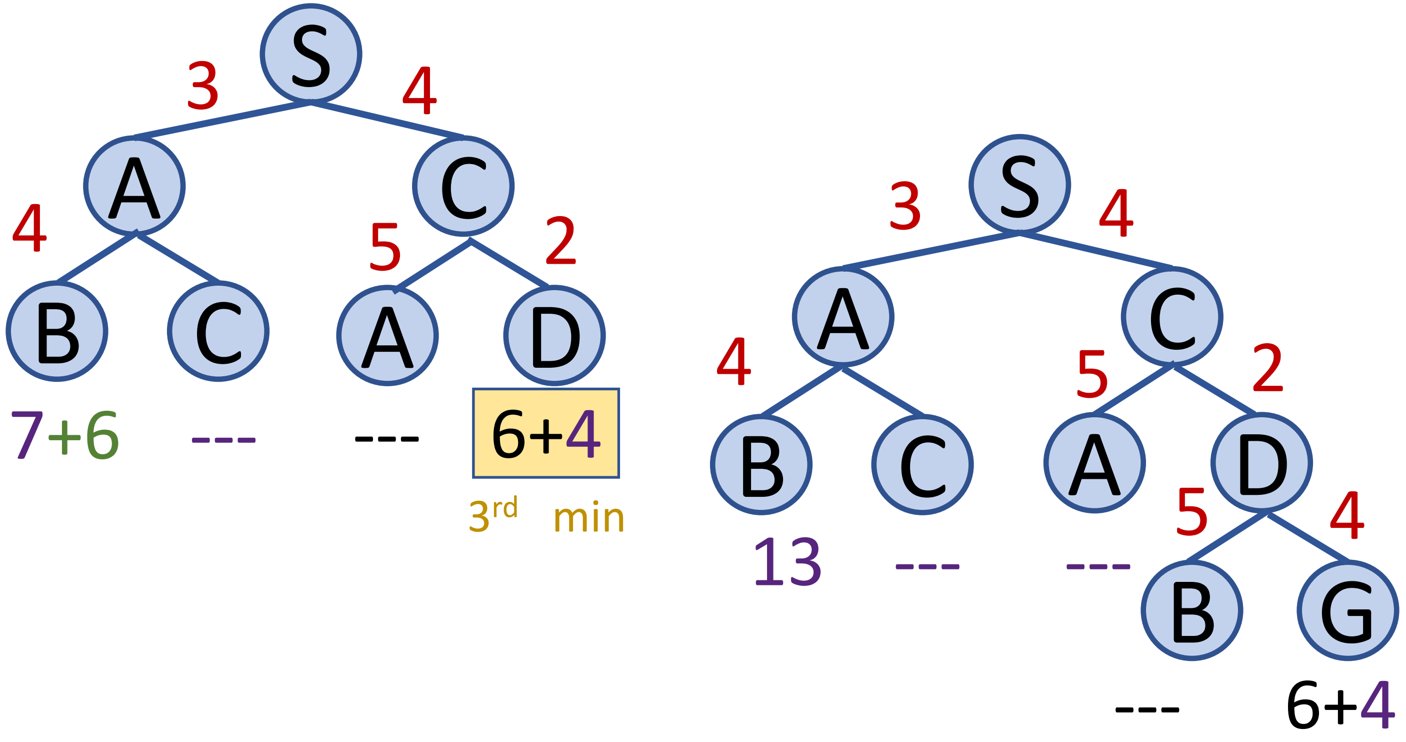

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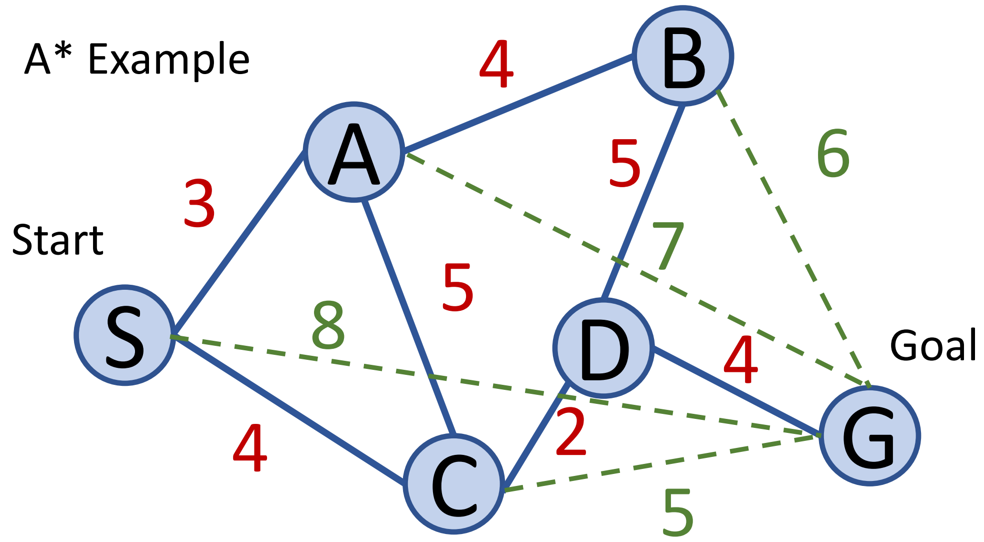


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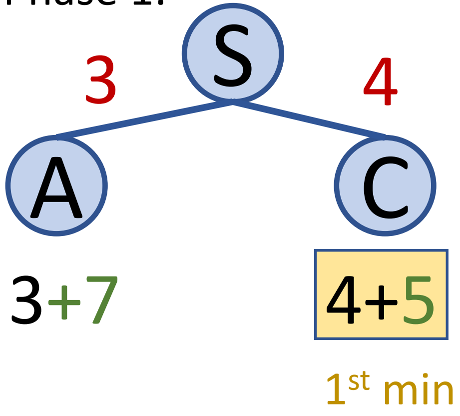
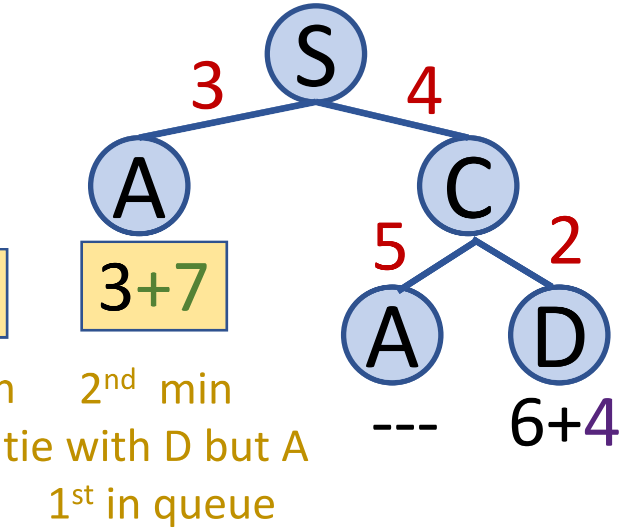
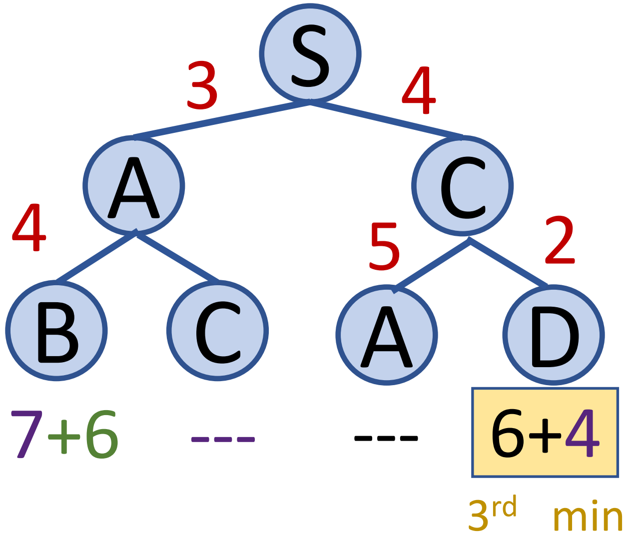
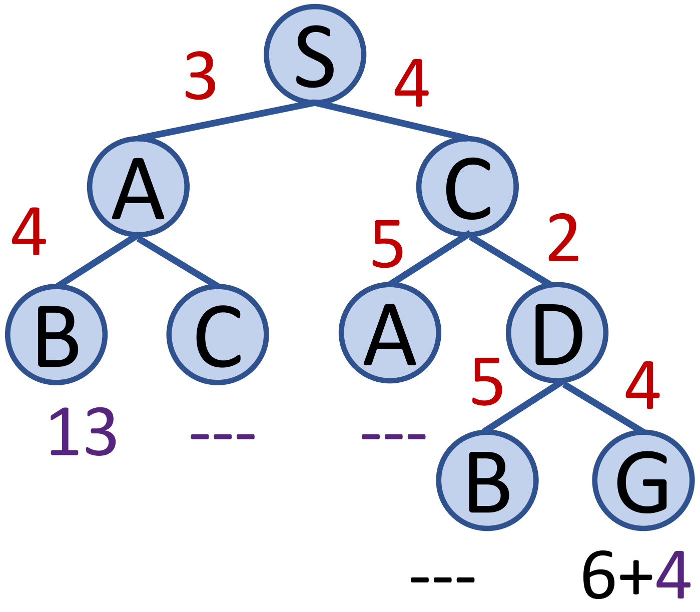


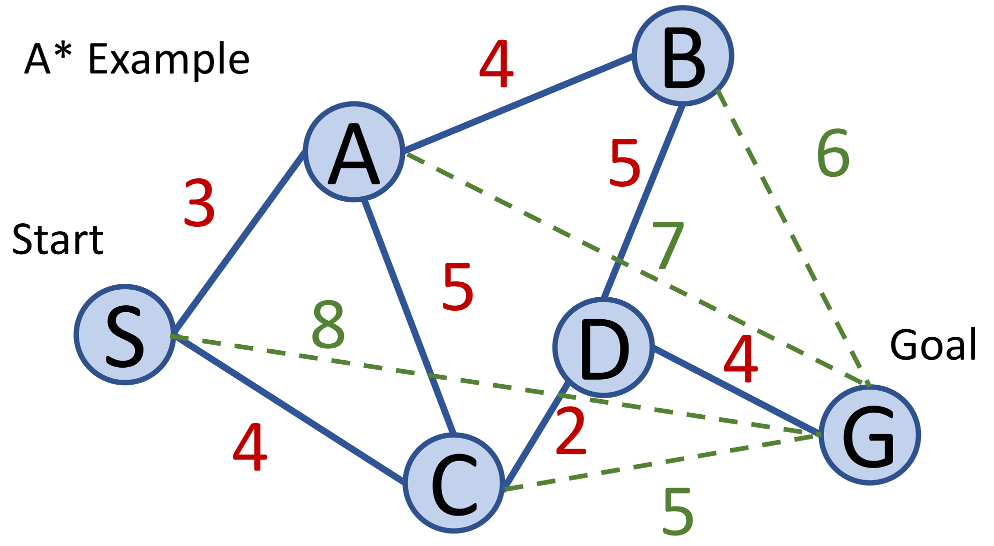




End of phase 1: G reached

A* Example

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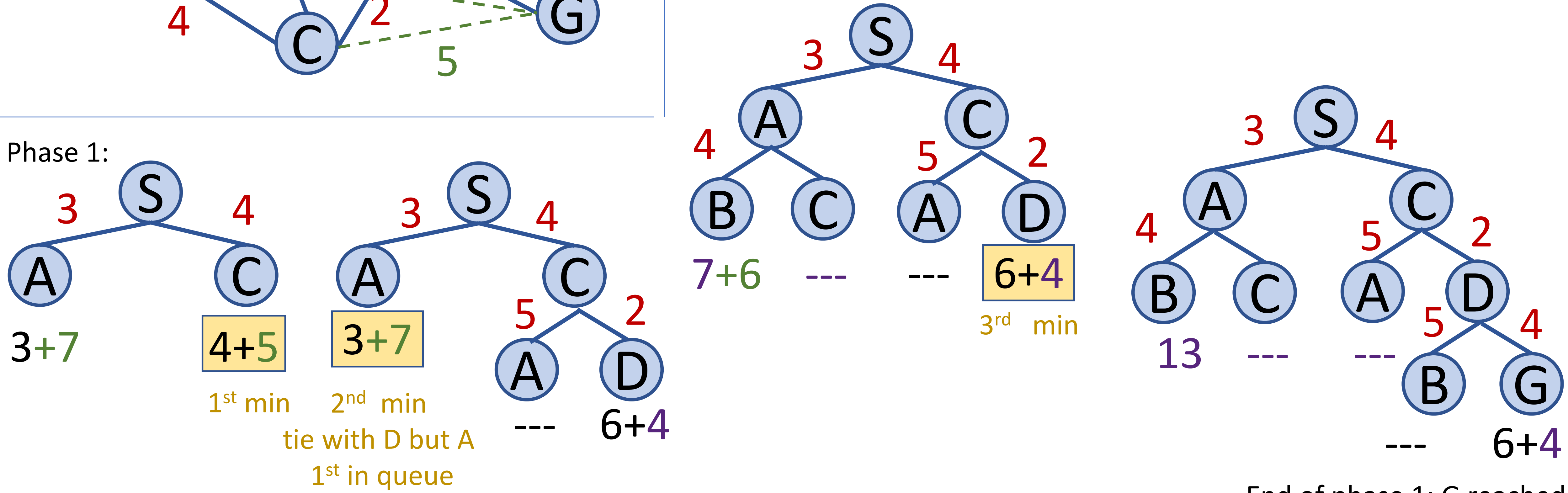


End of phase 1: G reached

End of phase 2: 13>10

# Admissible Heuristic

- If the heuristic function never overestimates the actual cost to get to the goal, then A* is guaranteed to return a least-cost path from start to goal.

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- If the heuristic function never overestimates the actual cost to get to the goal, then A* is guaranteed to return a least-cost path from start to goal.

# Time Complexity

- If goal state exists and is reachable from start state:

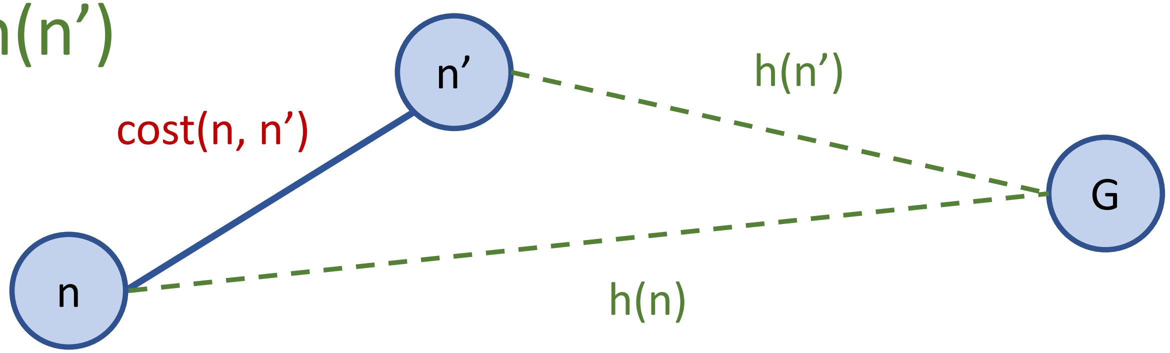
  Worst case $O(b^d)$ where d = depth(start,goal), b branching factor

- Otherwise, A* will not terminate
- A good heuristic function allows A* to prune away many of the $b^d$ nodes.

# Monotone Underestimates of Remaining Distance

Heuristic function h(n) is “monotone” if and only if it satisfies the triangle inequality:

- ❑ h(n) = 0 if n=goal state
- ❑ 0 <= h(n) <= cost(n, n’) + h(n’)




With a monotone heuristic, A* is guaranteed to find an optimal path without processing any node more than once.

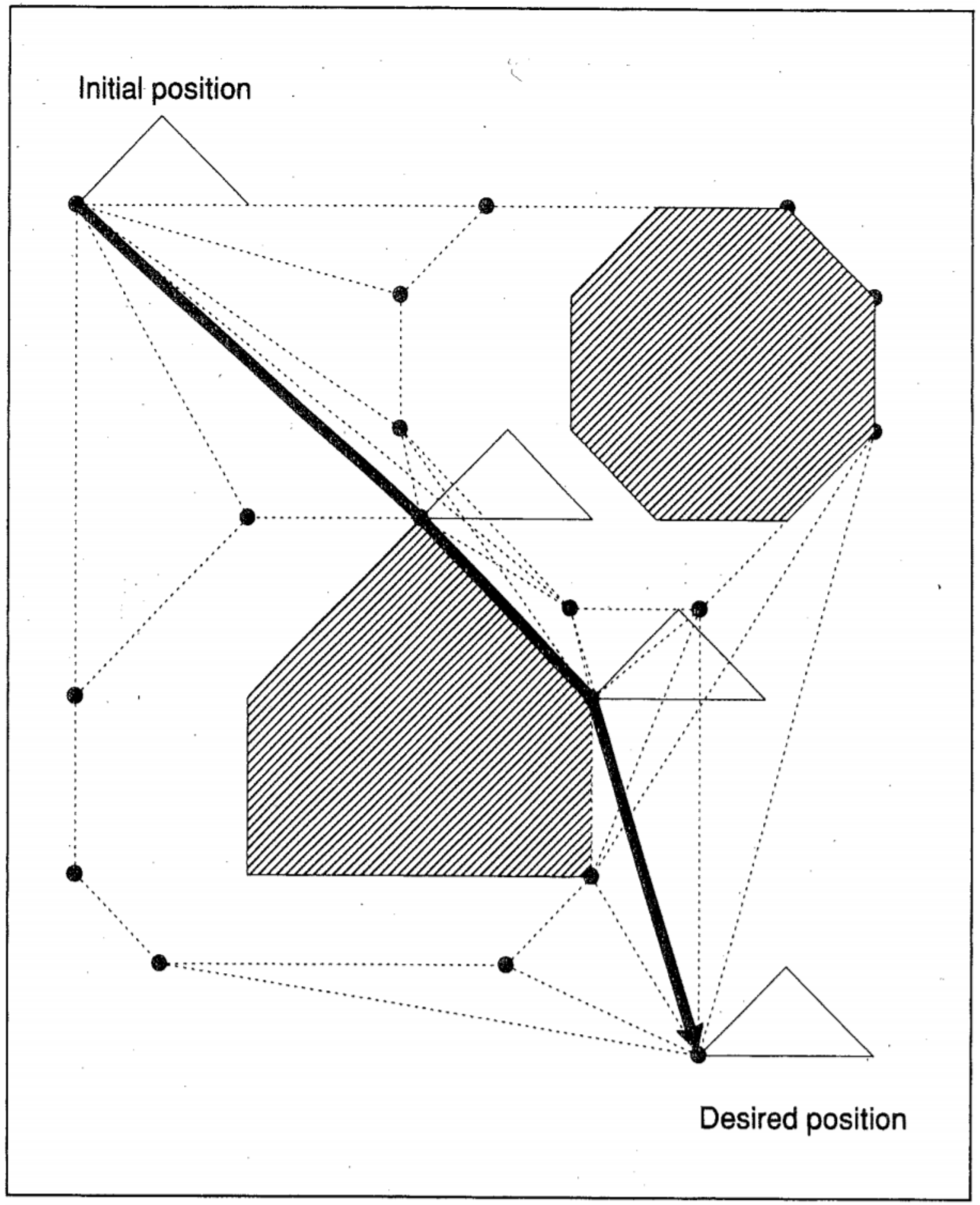
# Robot Path Planning with A*

- Convert 2D Map into

  "Configuration Space" C
  - Robot represented as point
  - Obstacles represented as obstacles + fence = O

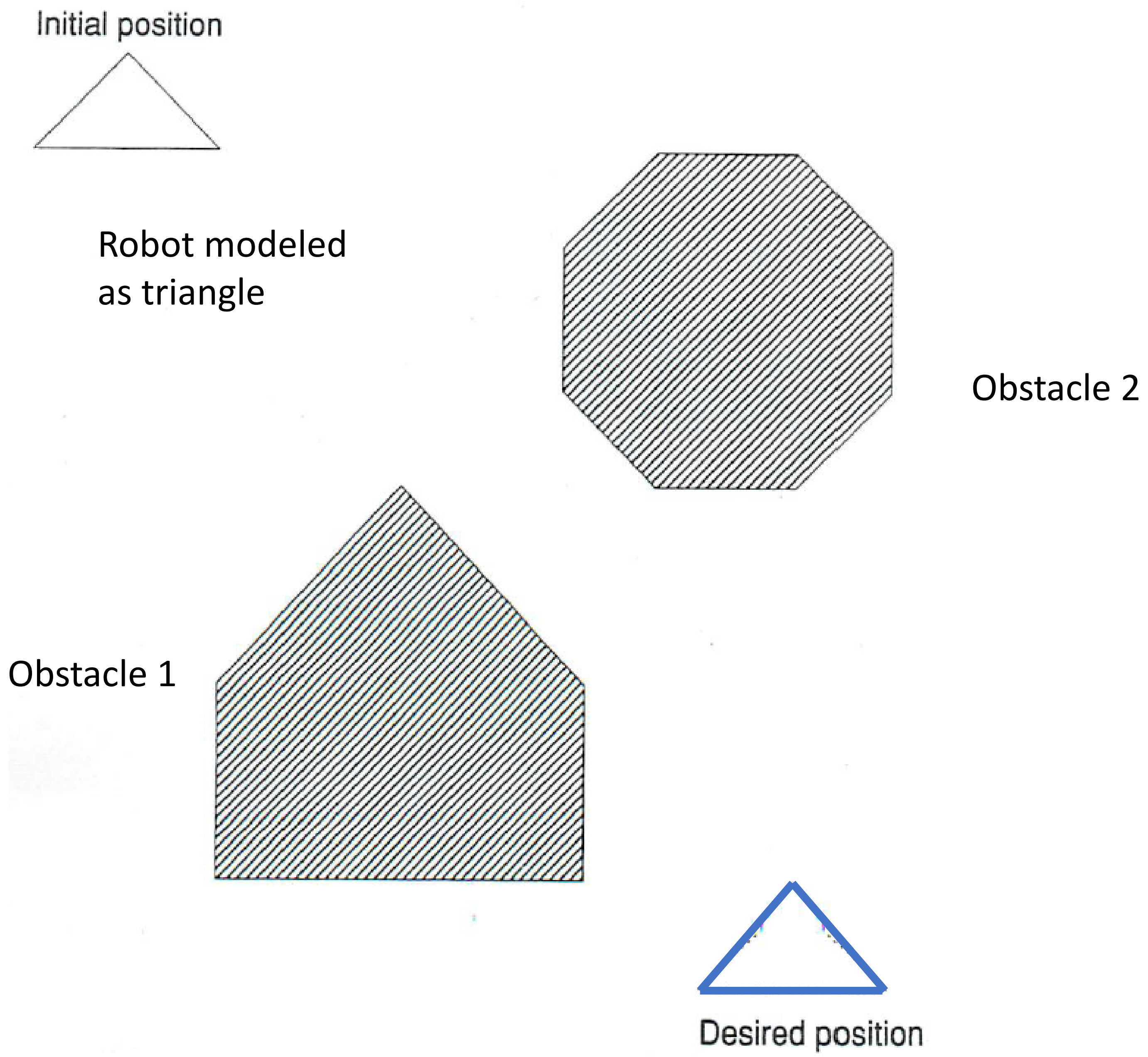
- Run A* on Visibility Graph in

  "Free Space" F = C - O



AI System Task:

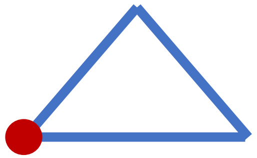
Find shortest path for robot to move from initial to goal position without robot hitting the obstacles



Create a fence around each obstacle:

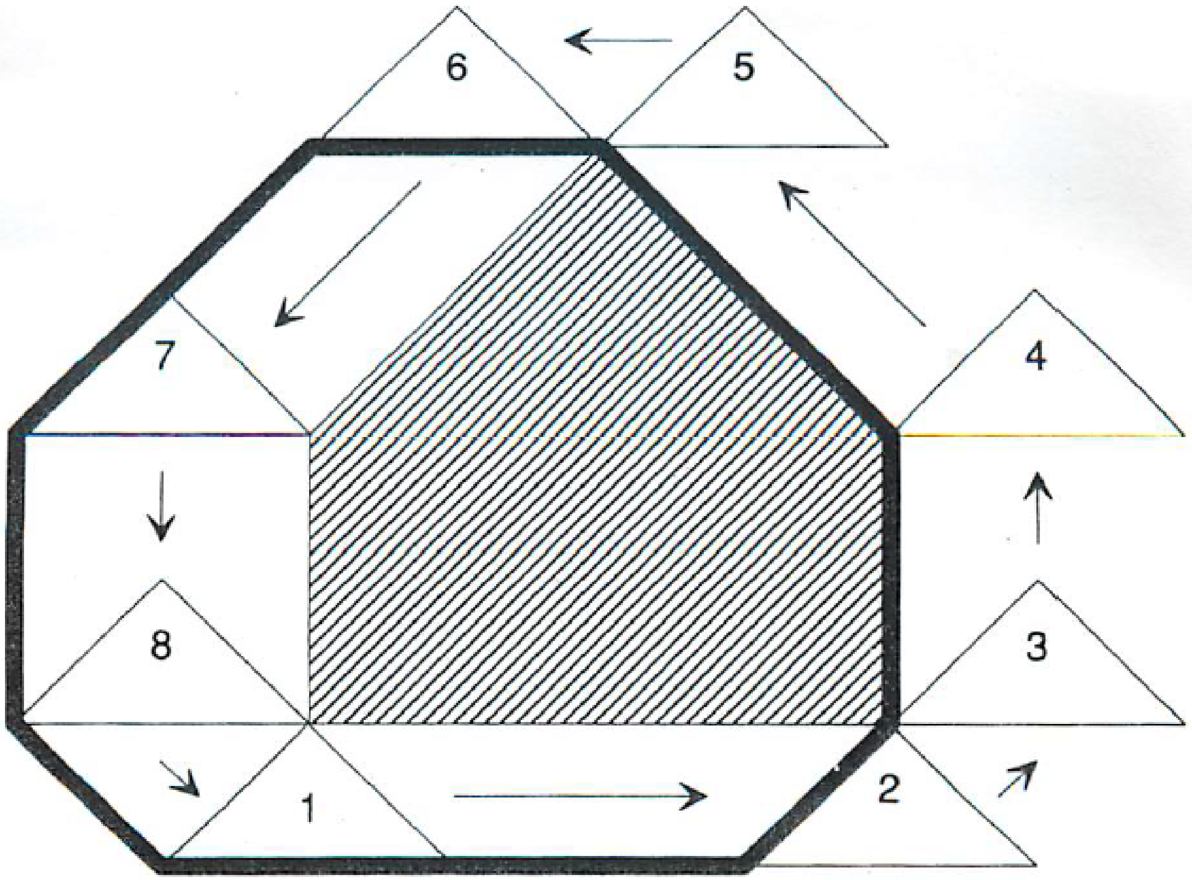
1. Select robot reference point
2. Slide robot shape around obstacle
3. Mark locations of reference point as fence

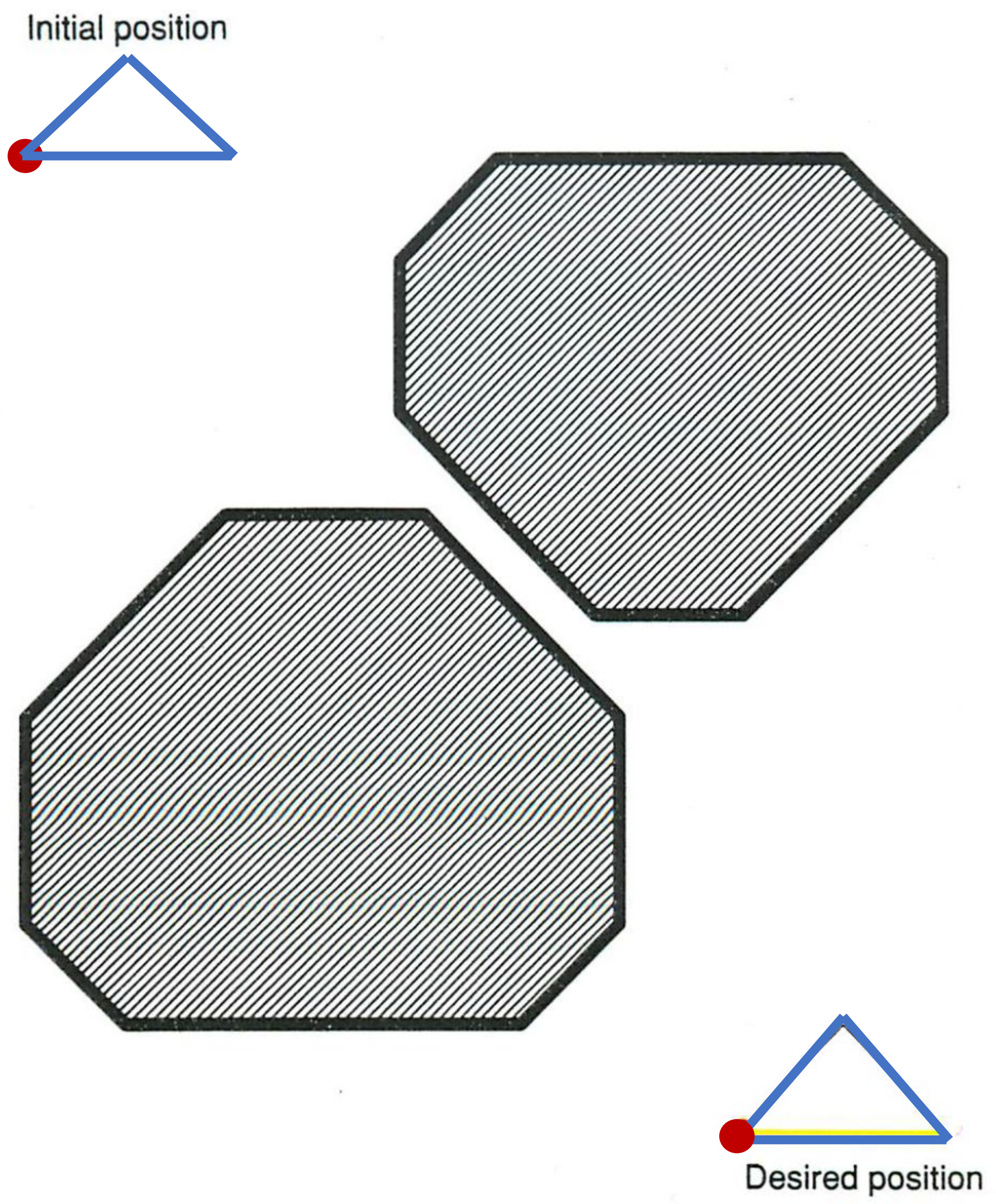
HERE:

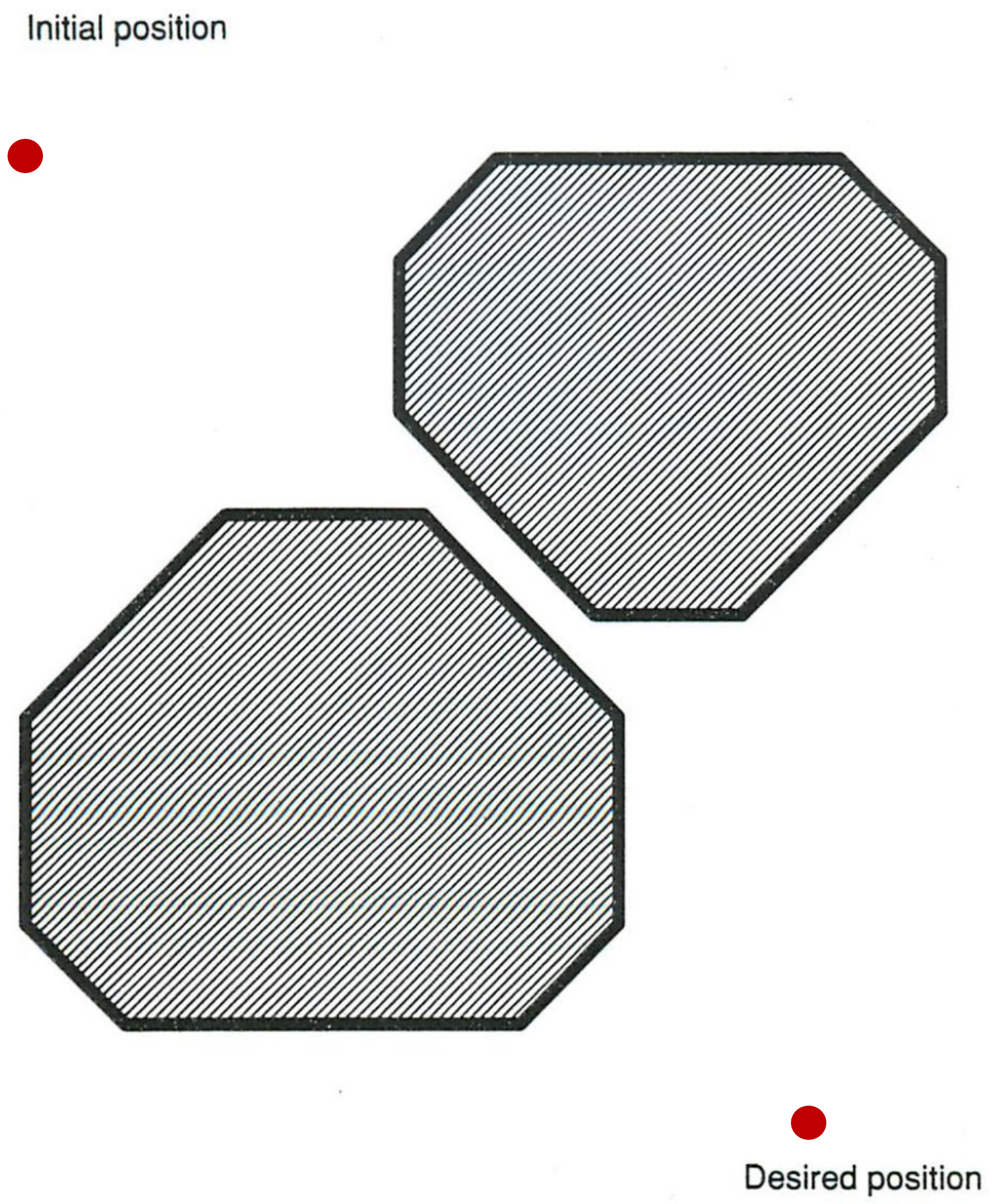
1. Reference point: 

2. Eight unique shape positions

3. Fence: Thick black line




Initial position
Desired position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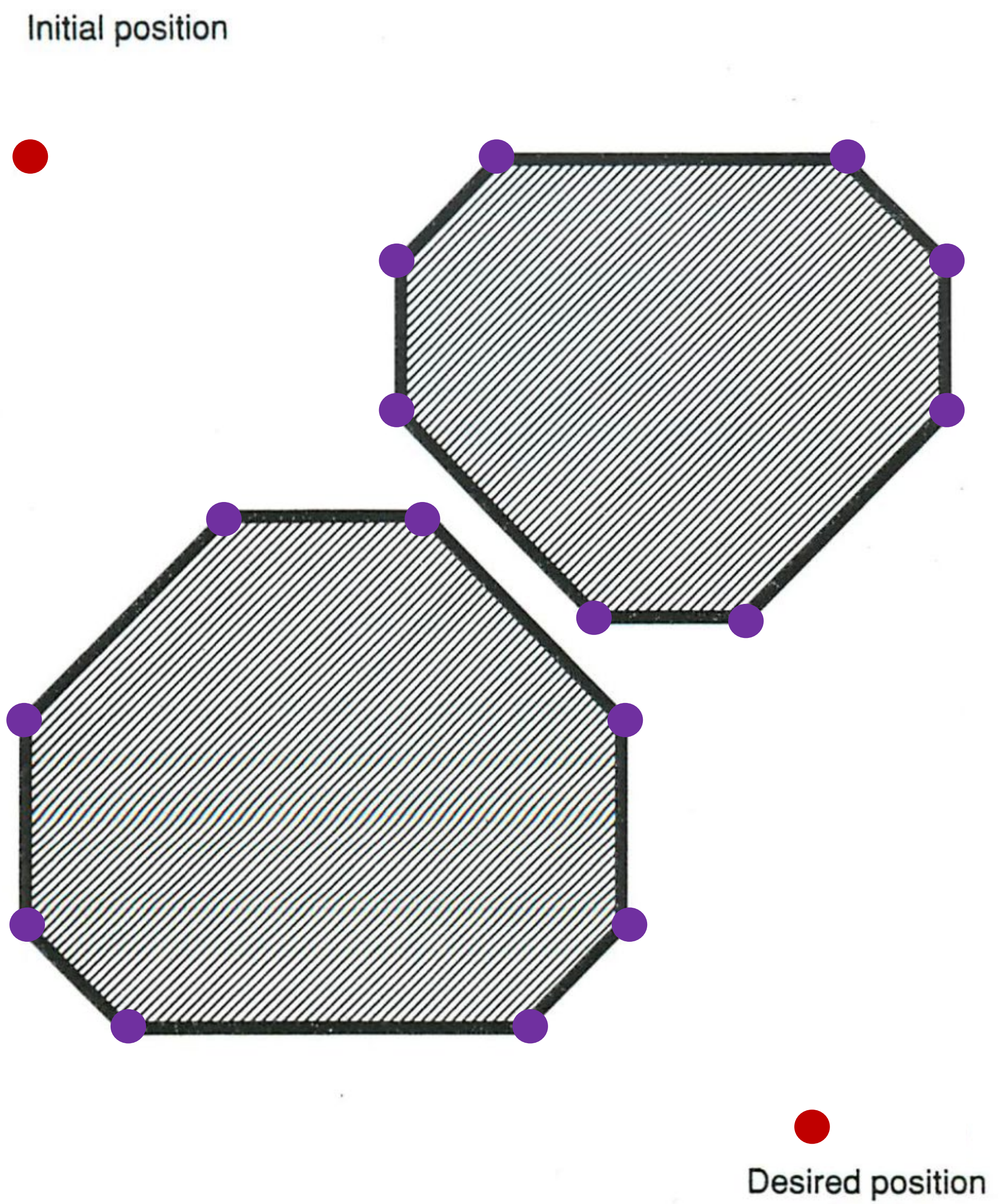
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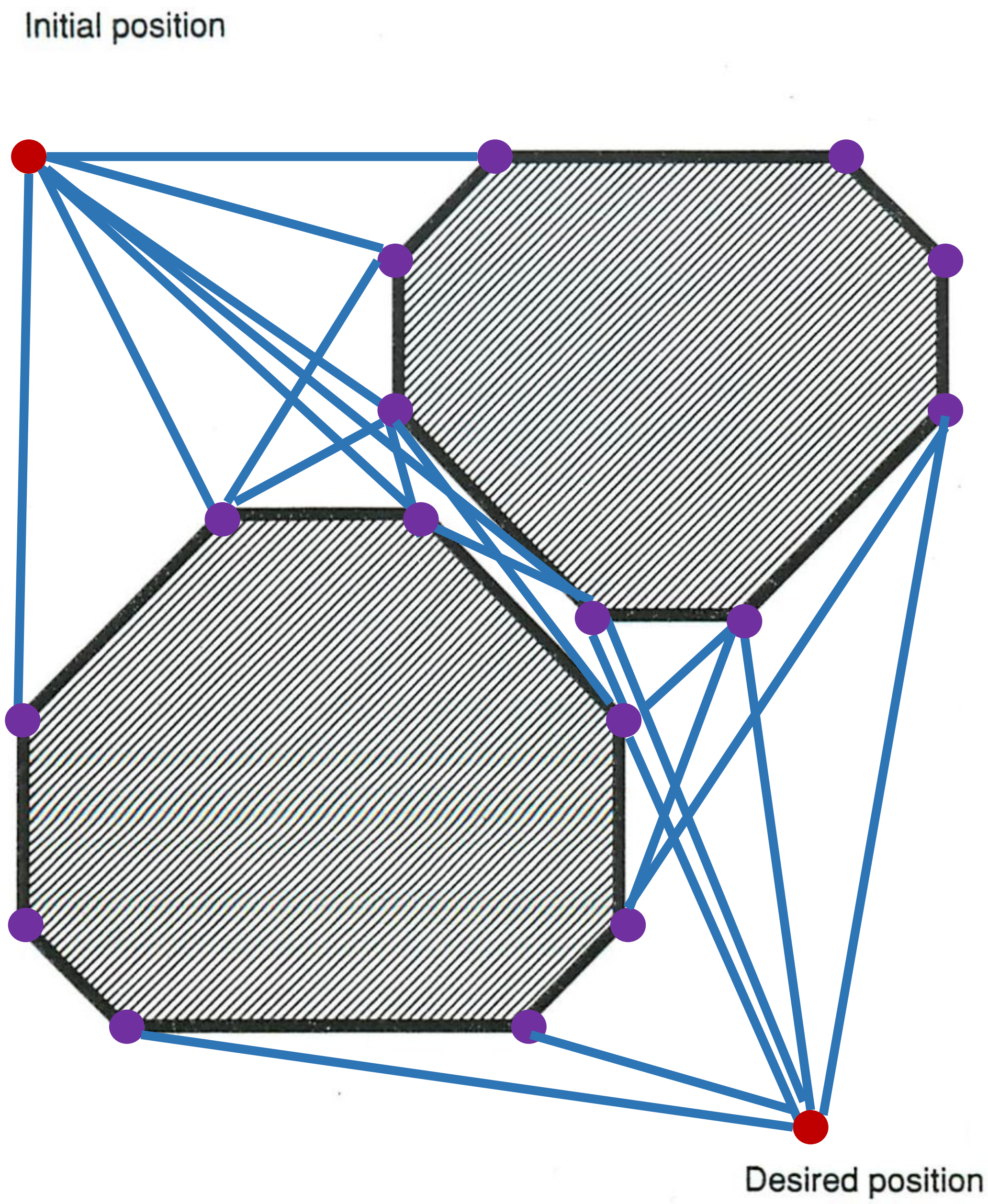
# Visibility Graph



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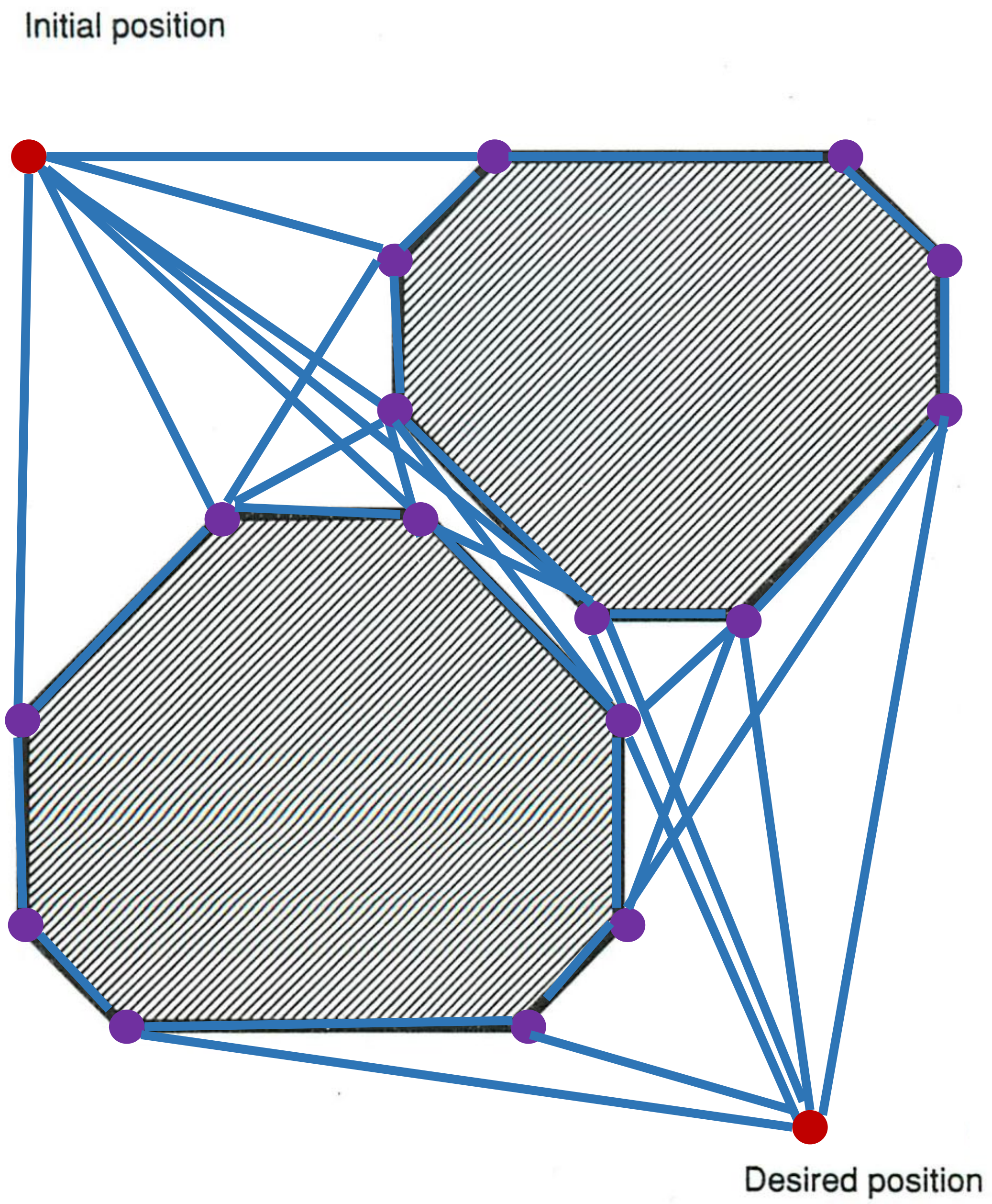
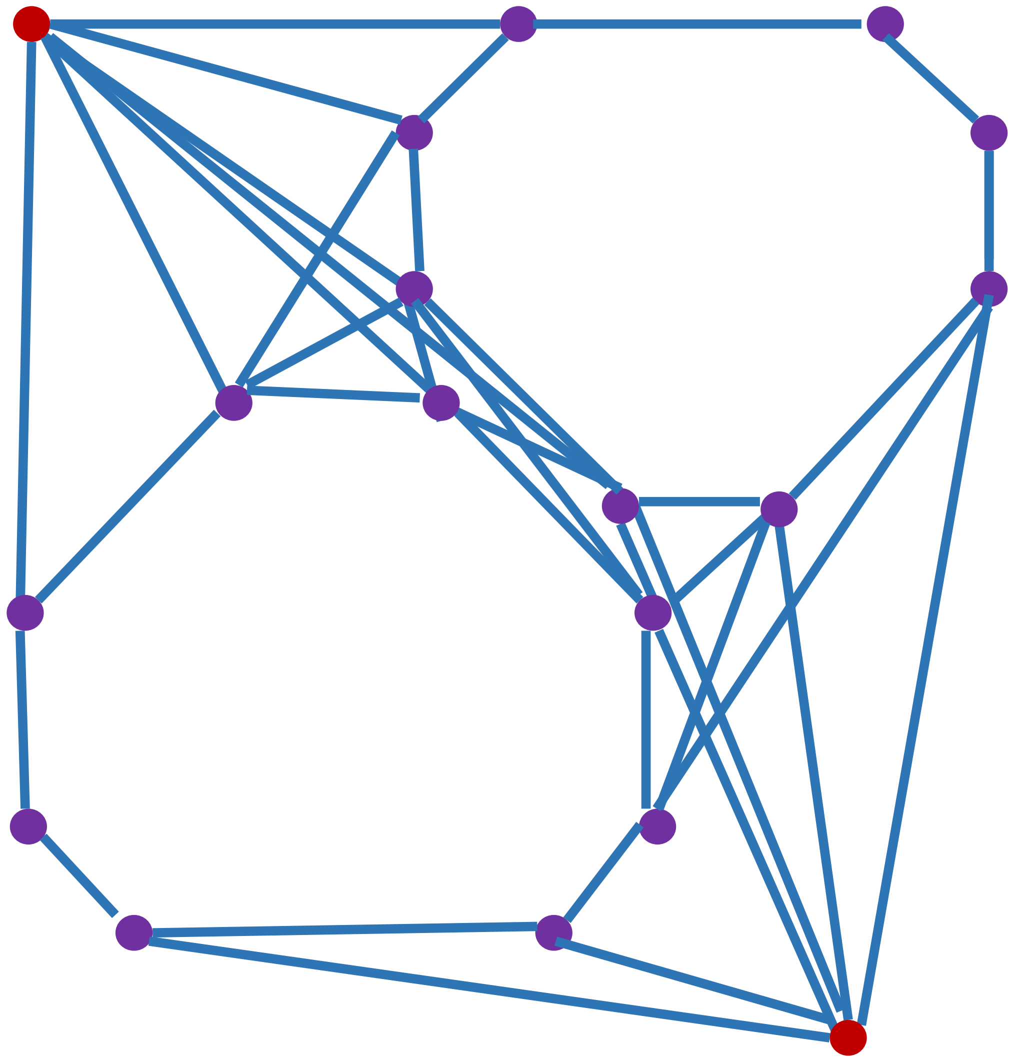
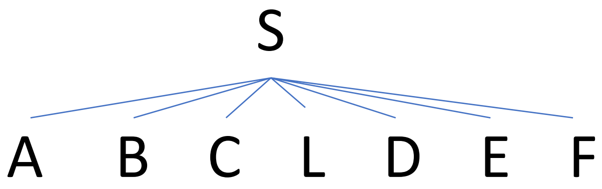


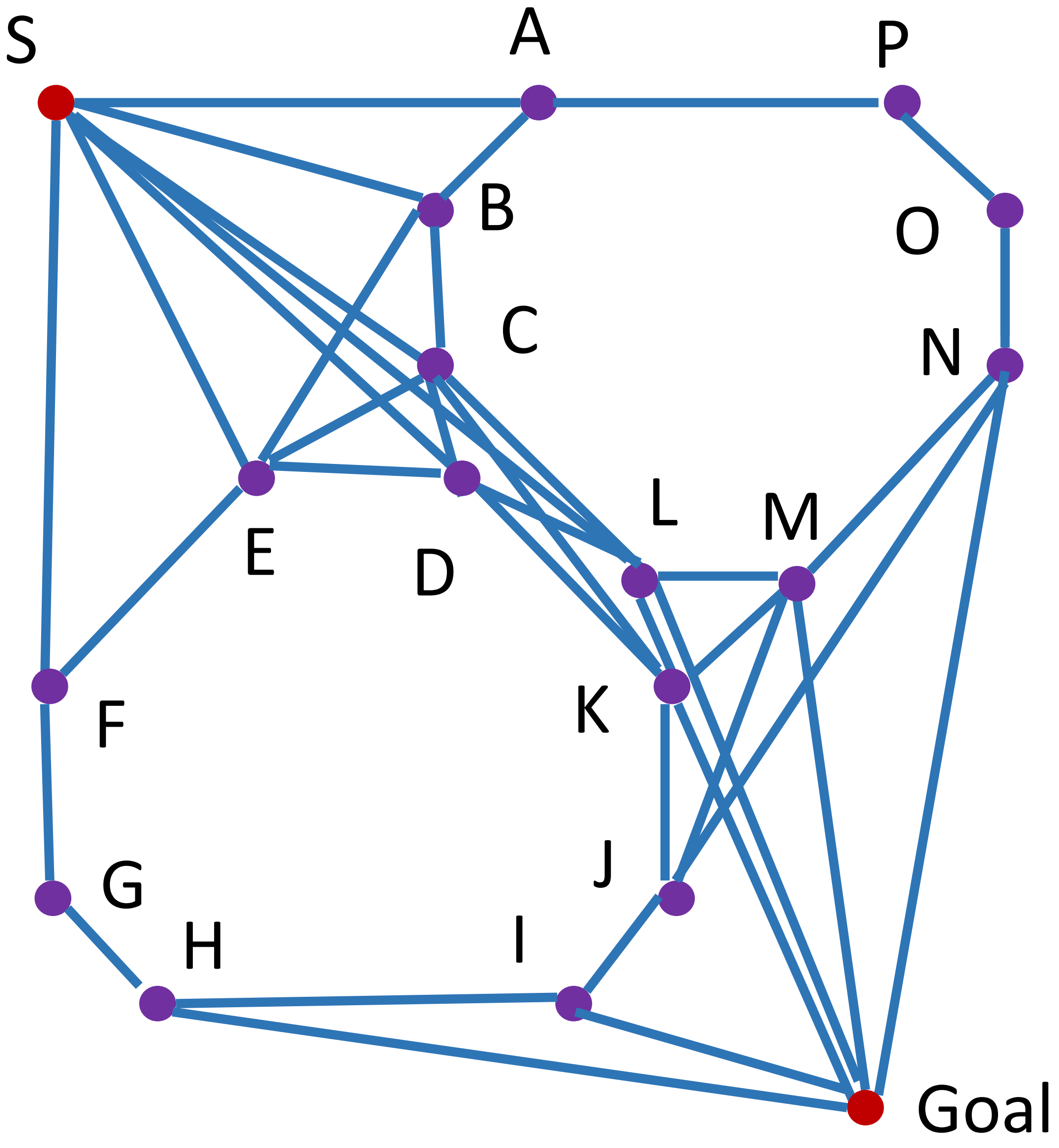
## Visibility Graph



## Run A* Algorithm on Visibility Graph:




Node-to-node distance
+
Underestimate to Goal



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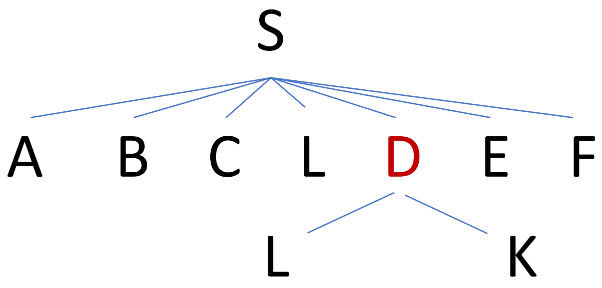


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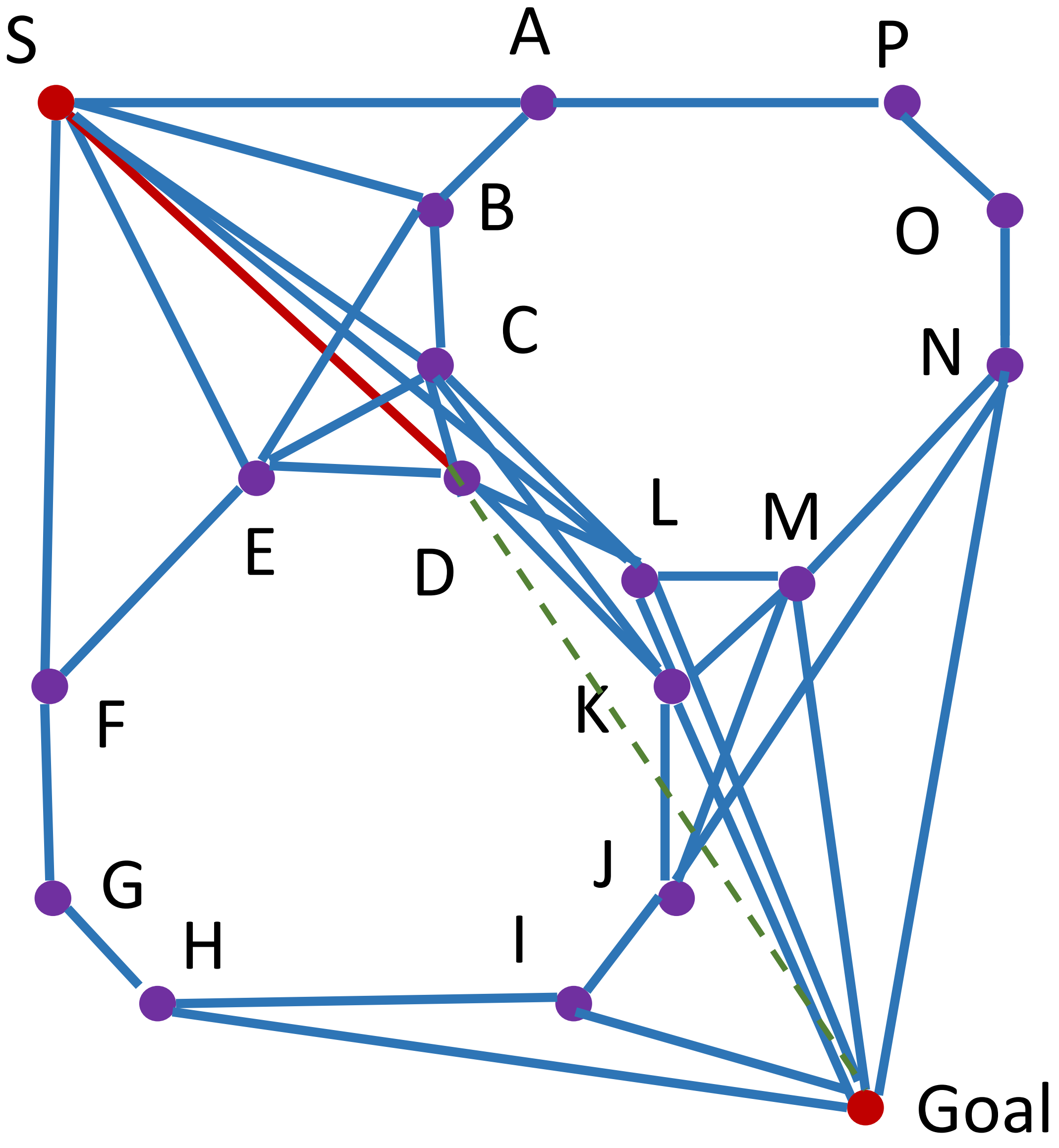

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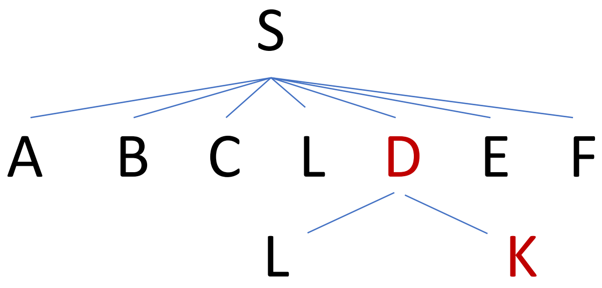


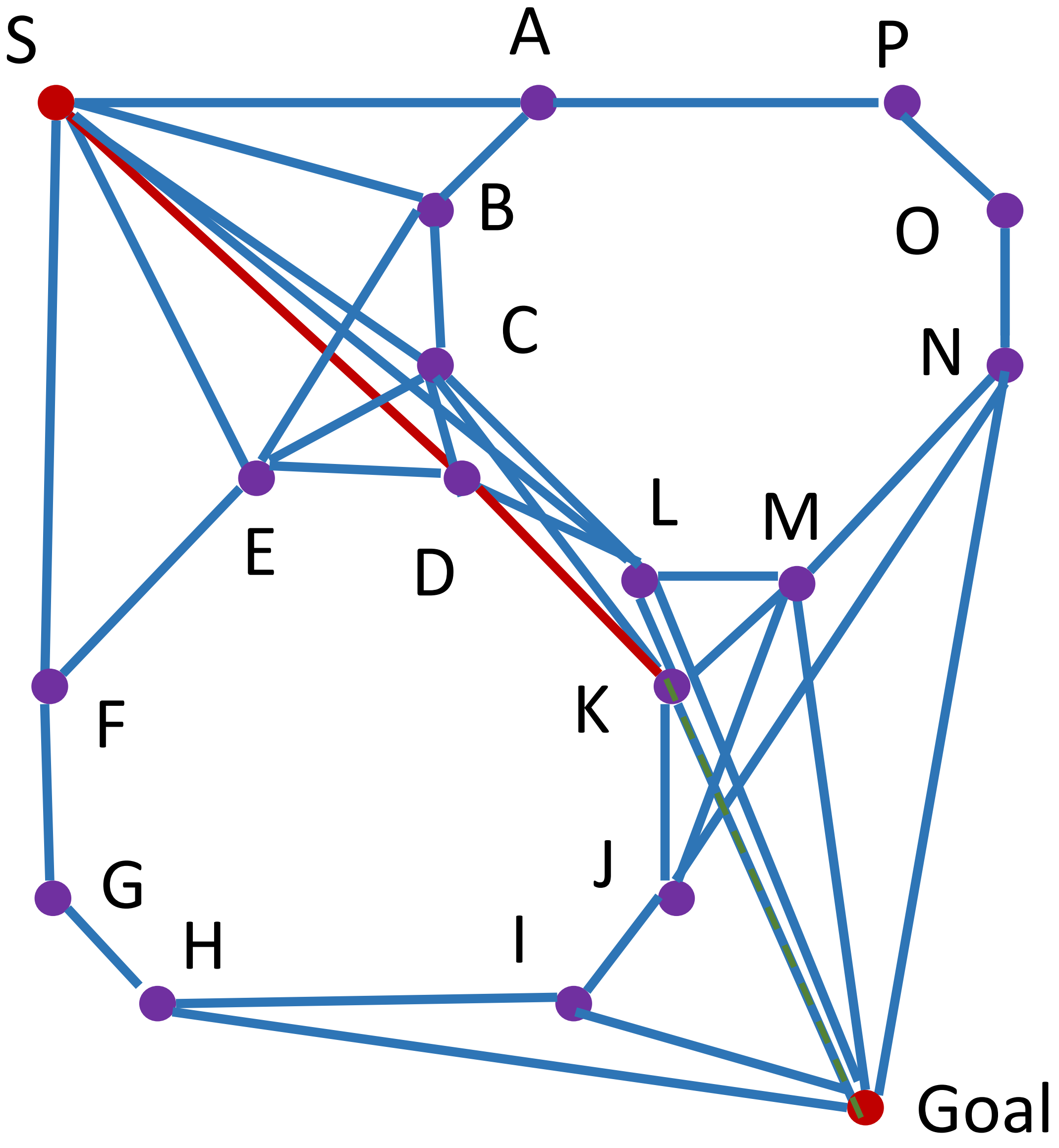


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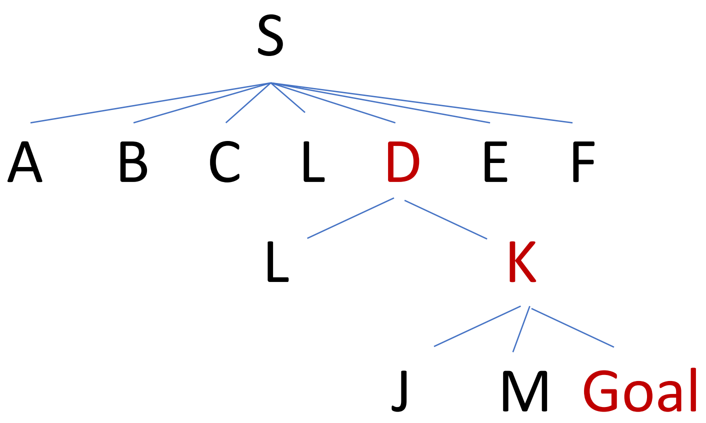
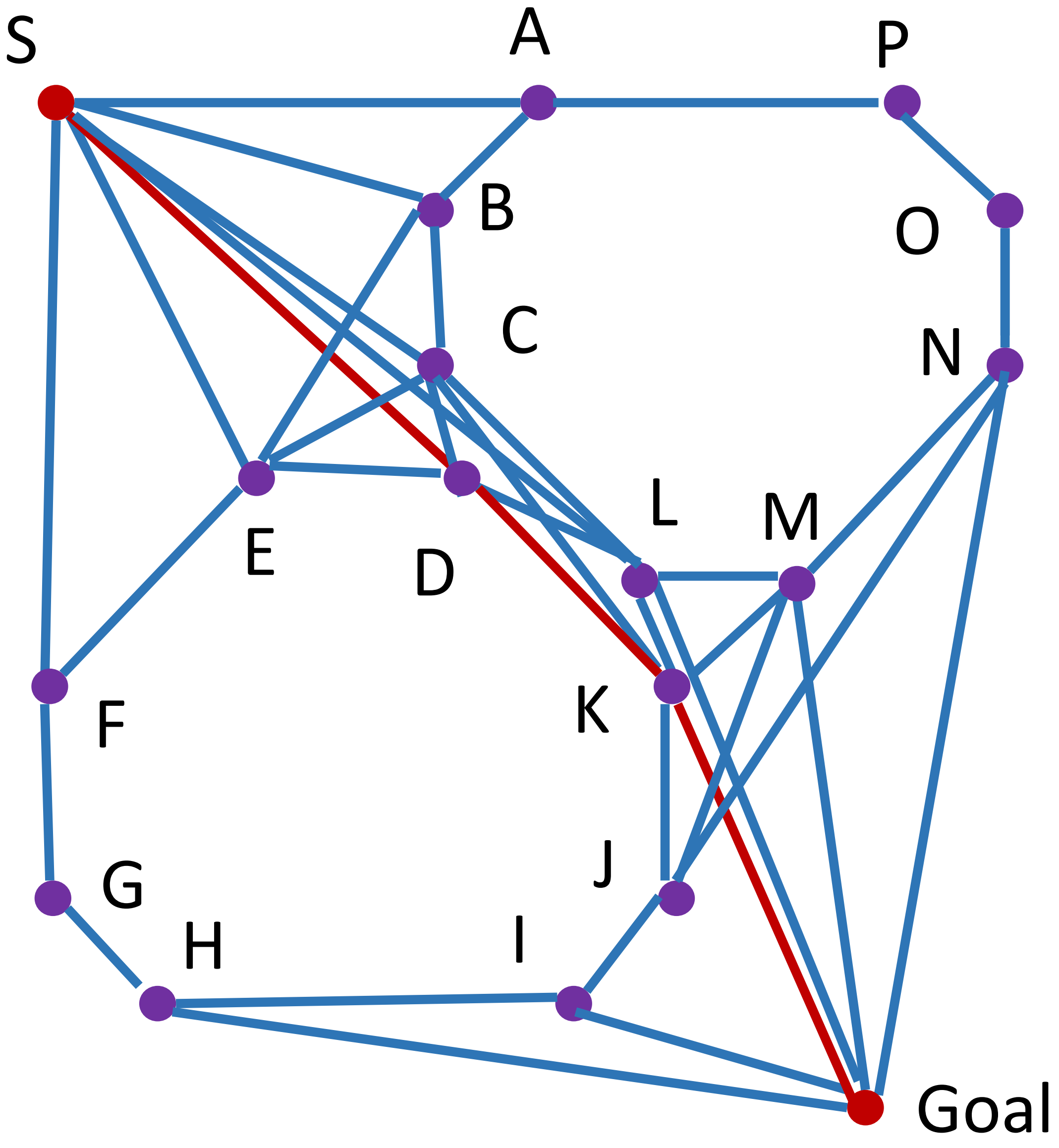
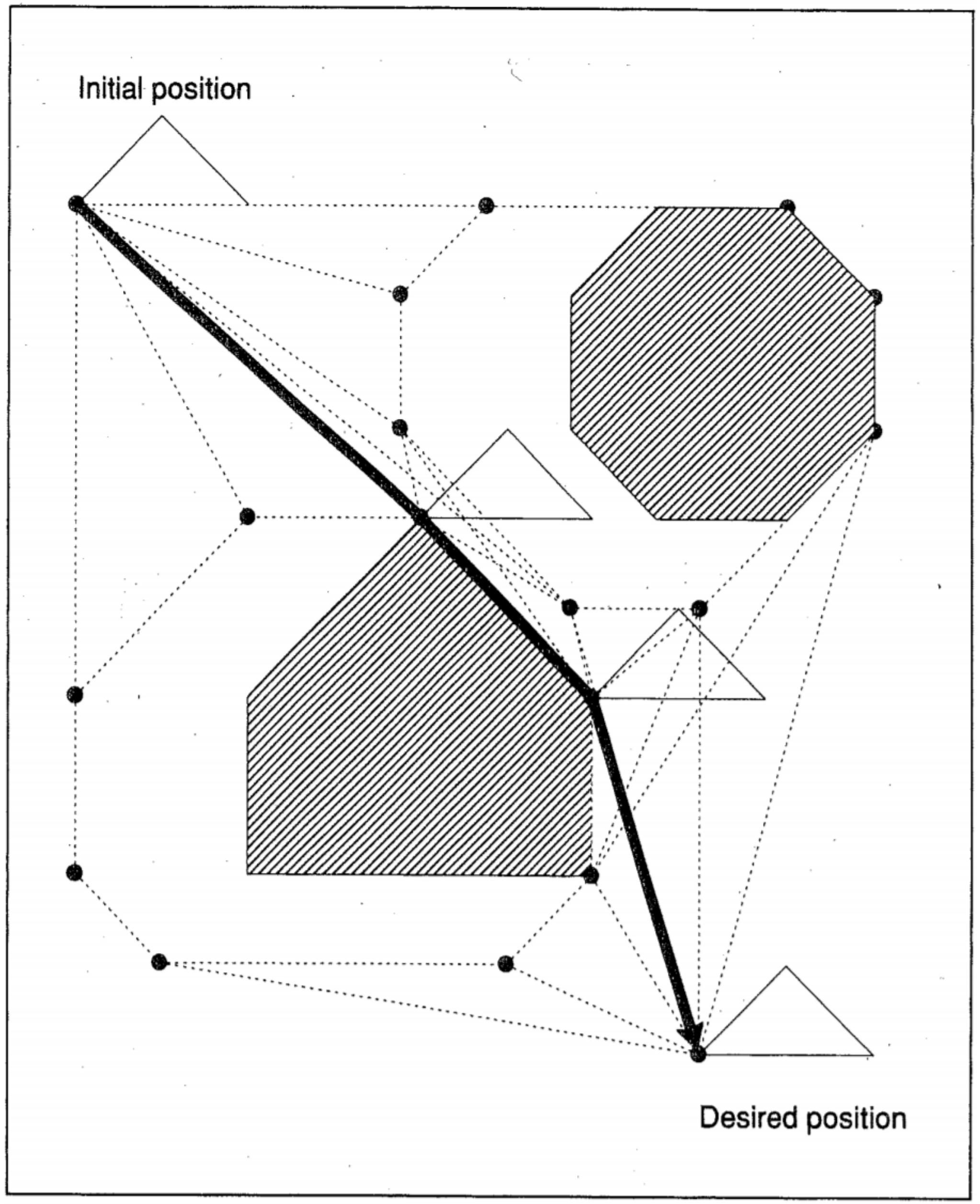


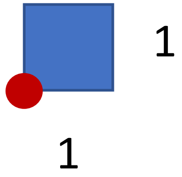



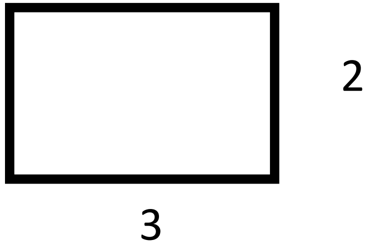

Node-to-node distance
+
Underestimate to Goal

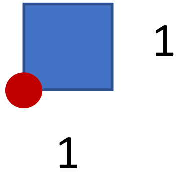
# Shortest path  computed by A*

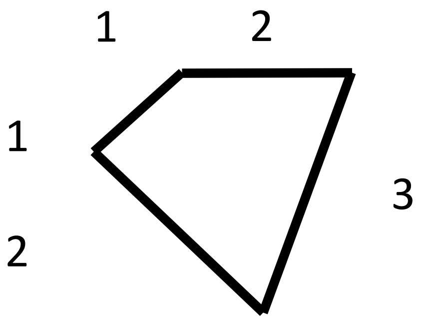


# Your task:
# Define Fence for the following robots & obstacles

1. Robot: 


Obstacle: 


2. Robot: 


Obstacle: 

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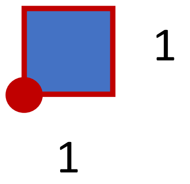
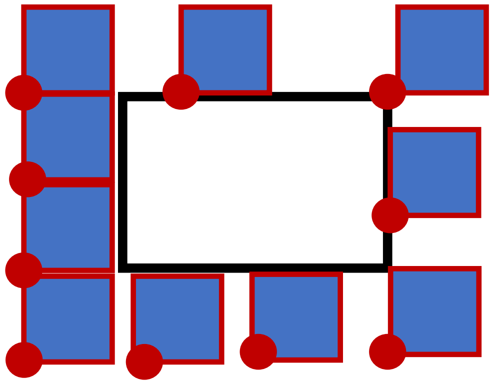



Obstacle:



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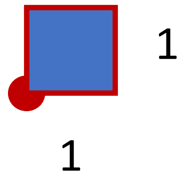
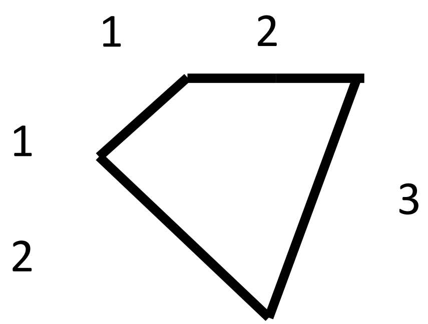



Obstacle:



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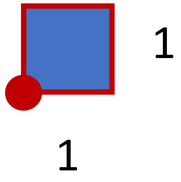
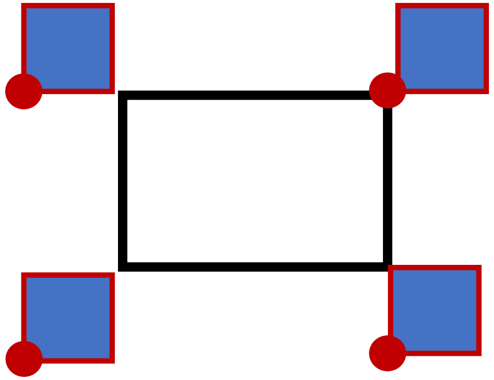
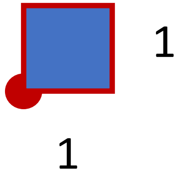
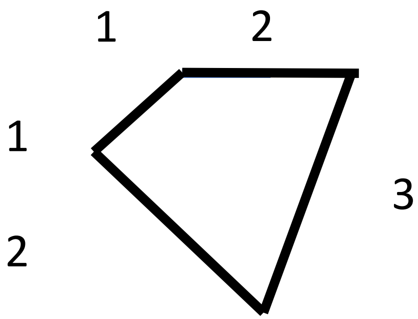
1. Robot: 

Obstacle: 

2. Robot: 

Obstacle: 

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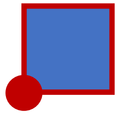
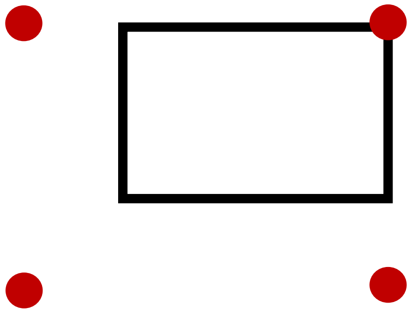
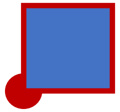
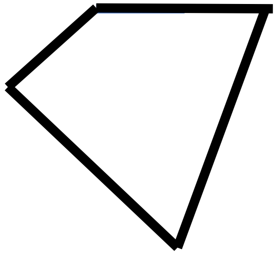
1. Robot: 

Obstacle: 

2. Robot: 

Obstacle: 

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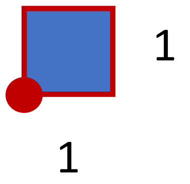
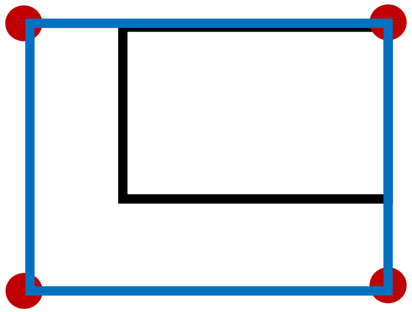
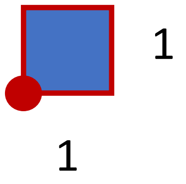
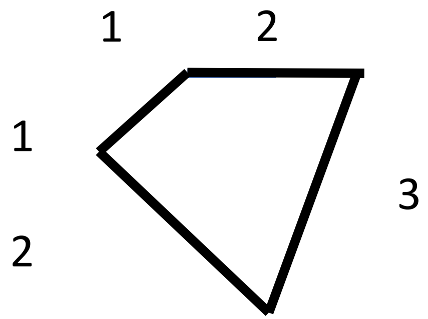
1. Robot: 


   Obstacle: 

2. Robot: 


   Obstacle: 

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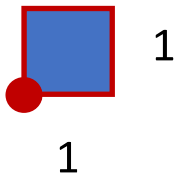
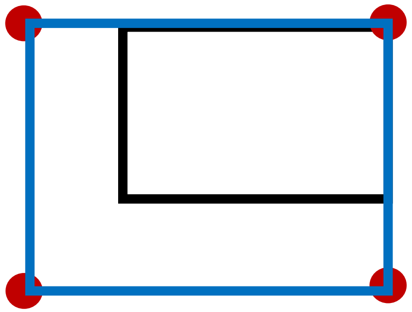
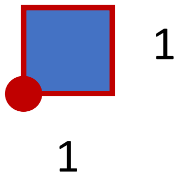
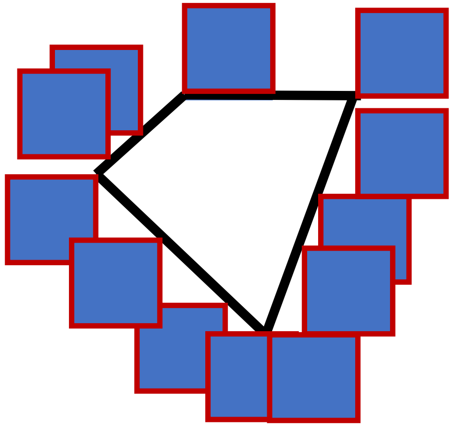
1. Robot: 


Obstacle: 

2. Robot: 


Obstacle: 

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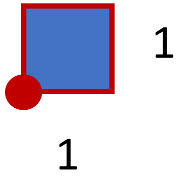
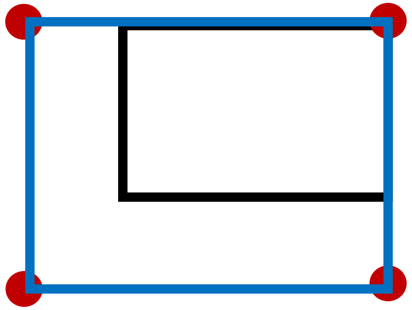
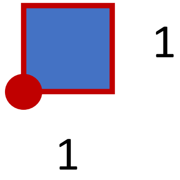
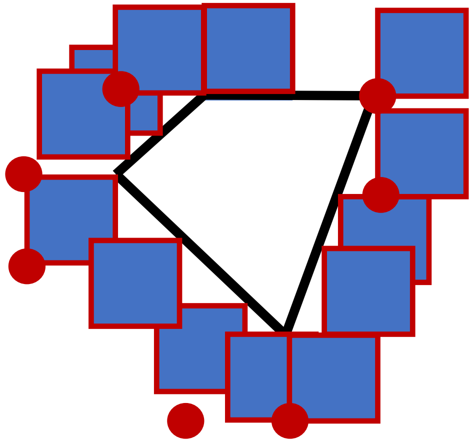
1. Robot: 


   Obstacle: 

2. Robot: 


   Obstacle: 

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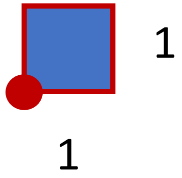
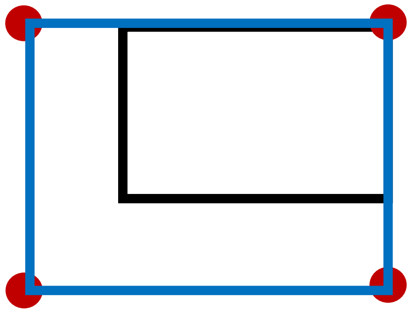
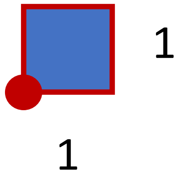
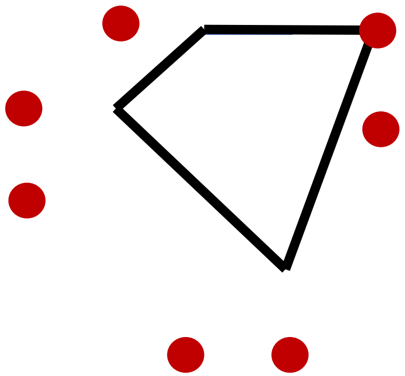
1. Robot: 

Obstacle: 

2. Robot: 

Obstacle: 

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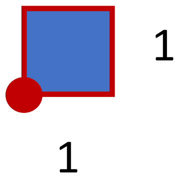
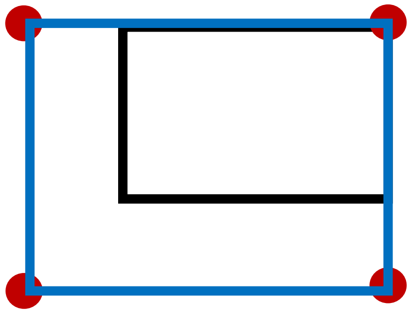
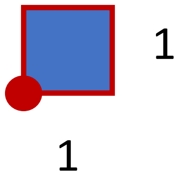
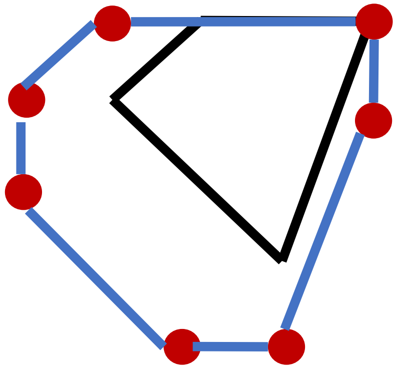
1. Robot: 

Obstacle: 

2. Robot: 

Obstacle: 

# General Problem of Robot Motion Planning

- Robot with k degrees of freedom:
  State or configuration of robot: $(q_1, q_2, \ldots, q_k)$

- So far $(q_1, q_2)$ for two-dimensional position

- PUMA robot: 6 joint angles: $(q_1, q_2, \ldots, q_6)$
  6D configuration space

# General Problem of Robot Motion Planning

Given initial point $c_1$ and destination point $c_2$, in configuration space C:

Robot can safely move between corresponding points in physical space

if and only if

There exists a continuous path between $c_1$ and $c_2$ that lies entirely in the free space.

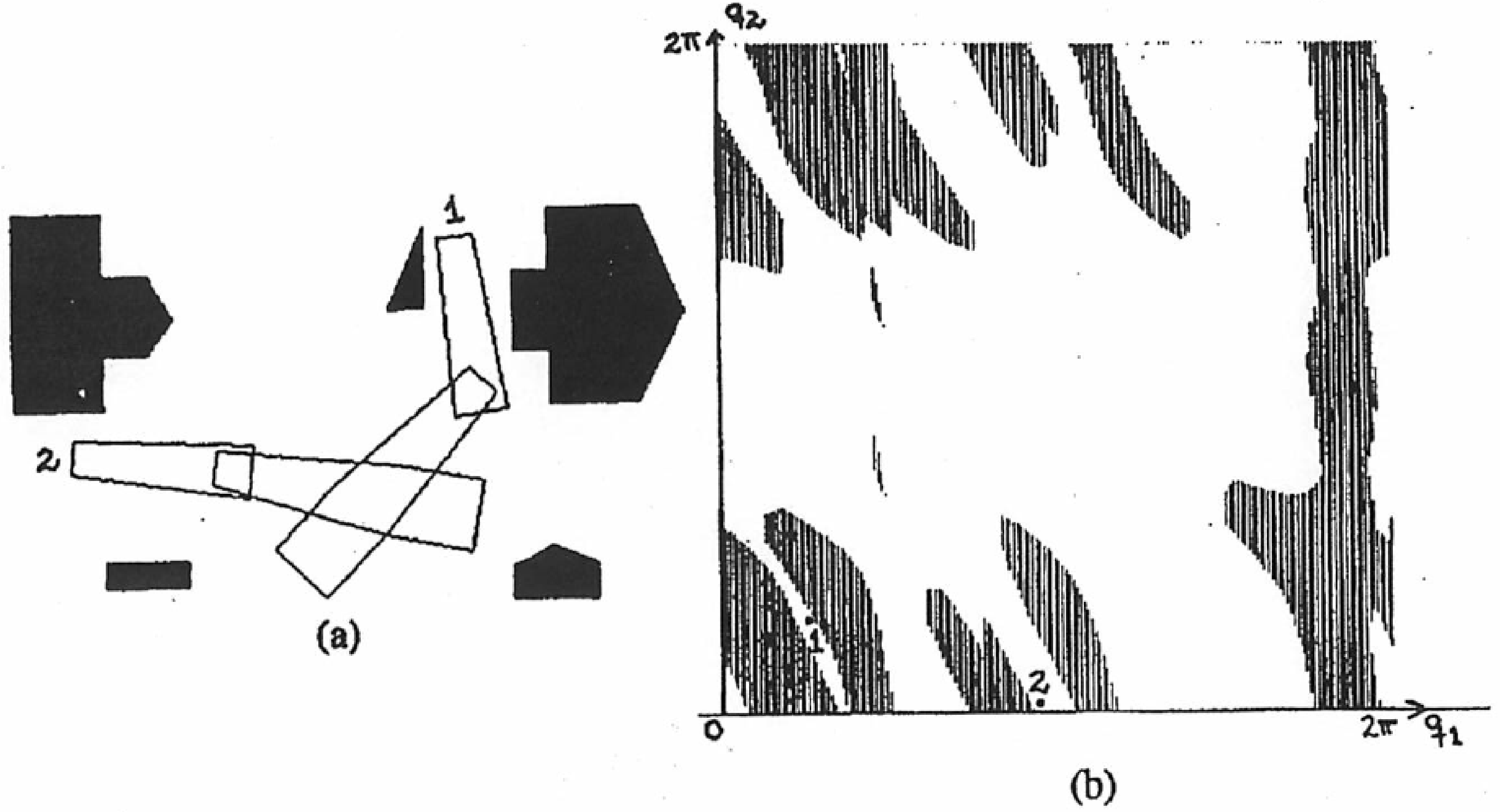
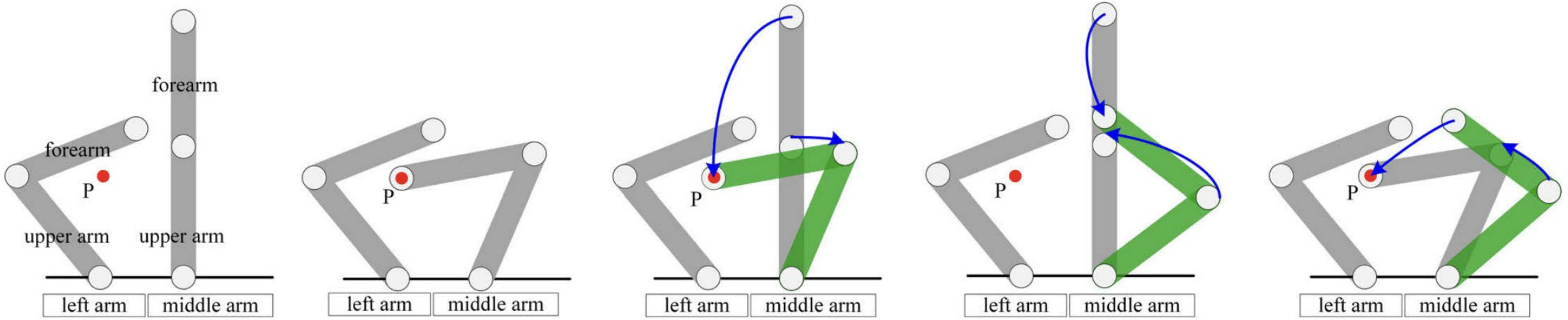



Fig. 4. (a) Two-link revolute manipulator and obstacles. (b) Two-dimensional C space with obstacles approximated by list of one-dimensional slice projections (shown dark). Initial and final position of manipulator are shown in input space and C space.

Lozano-Perez, 1987

# Robot Obstacle Avoidance




(a) zero position (b) Target location (c) interference occurs (d) obstacle avoidance path (e) achieve the target point

Zhao et al., 2020

# Learning Outcomes of this Lecture

- Understand how search algorithms are evaluated
- Understand the unique properties of AI searching tasks (versus general search algorithms)
- Can explain 11 path-based search algorithms and run them on an example
- Can explain the dynamic programming principle
- Know what an admissible and a monotone heuristic function is for the A* algorithm
- Can design a configuration space from a 2D obstacle map & a translating robot
- Can design a visibility graph in free space
- Can run A* on a visibility graph for robot path planning
- Understand configuration spaces of robot arms