Neural Network Training:
Understanding Backpropagation requires an Understanding of Multivariate Functions, Derivates, and the General Chain Rule

Single Variate Derivative

$f(x)=6x^2$

$f'(x)=12x$ Lagrange notation

$df/dx = 12x$ Leibnitz notation

Single Variate Derivative

$f(x)=6x^2 = 3(2x^2) = 3\ g(x)$ nested function

Chain rule:

$g'(x)=4x$

$f'(x) = 3\ g'(x) = 3\ (\ 4\ x) = 12\ x$

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$df(g)/dx = df/dg * dg/dx$

Example of a 3D function and its partial derivatives:

$f(x,y) = 3x+y$

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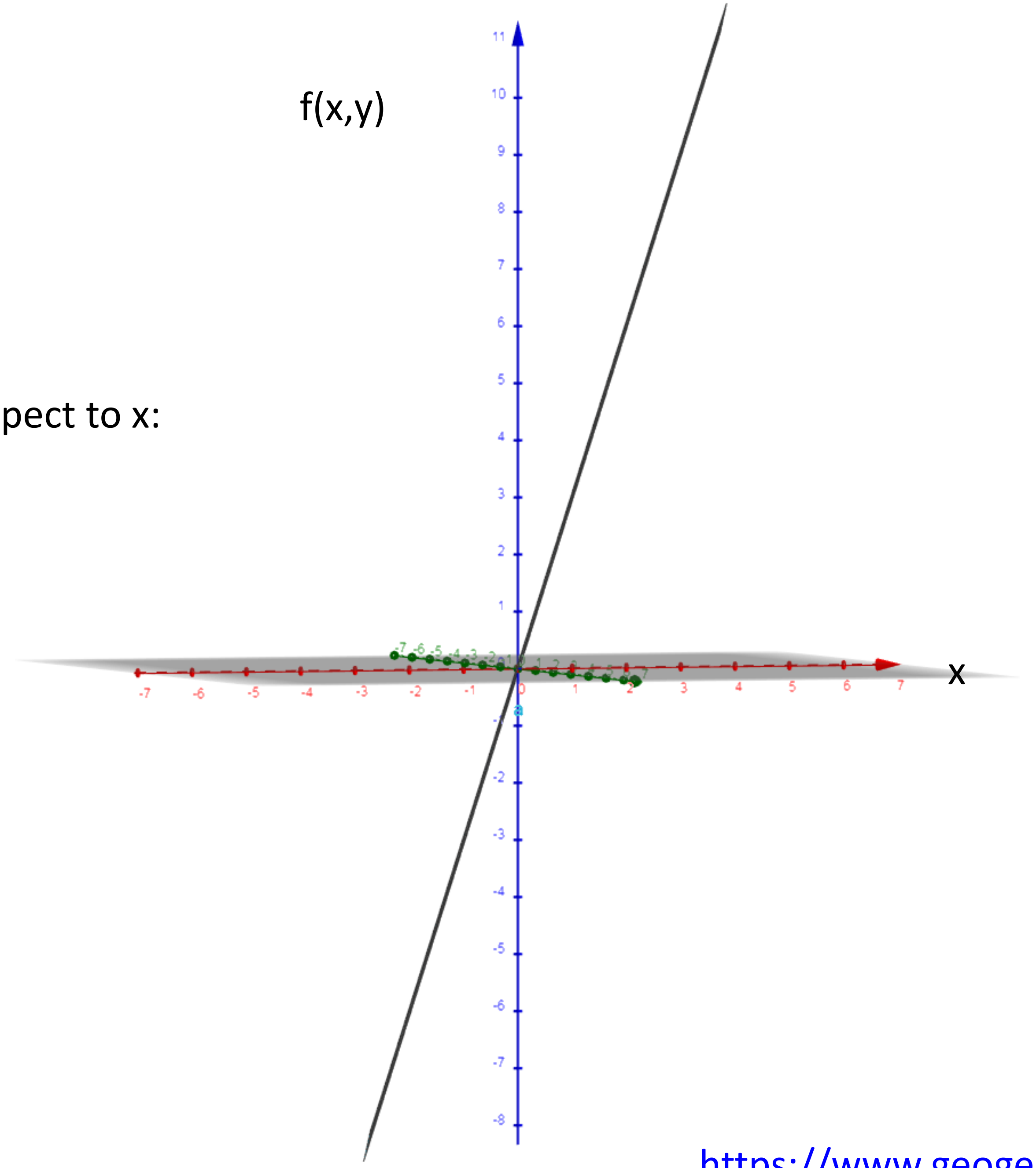
Slope in x:

Derivative with respect to x:

$\partial f/\partial x \quad = 3$

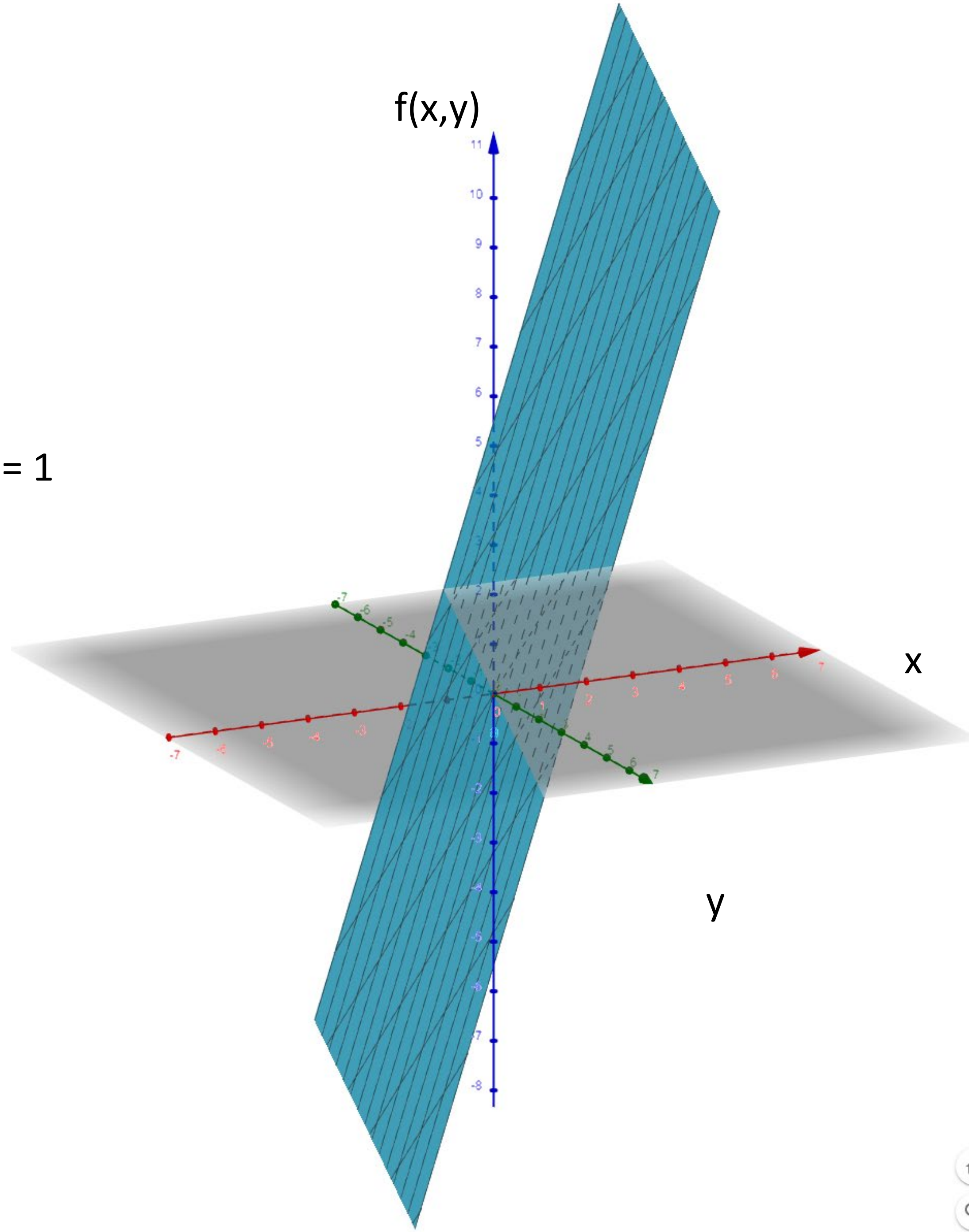


https://www.geogebra.org

f(x,y)=3x+y

Slope in y:

$\partial f/\partial y$ = 1



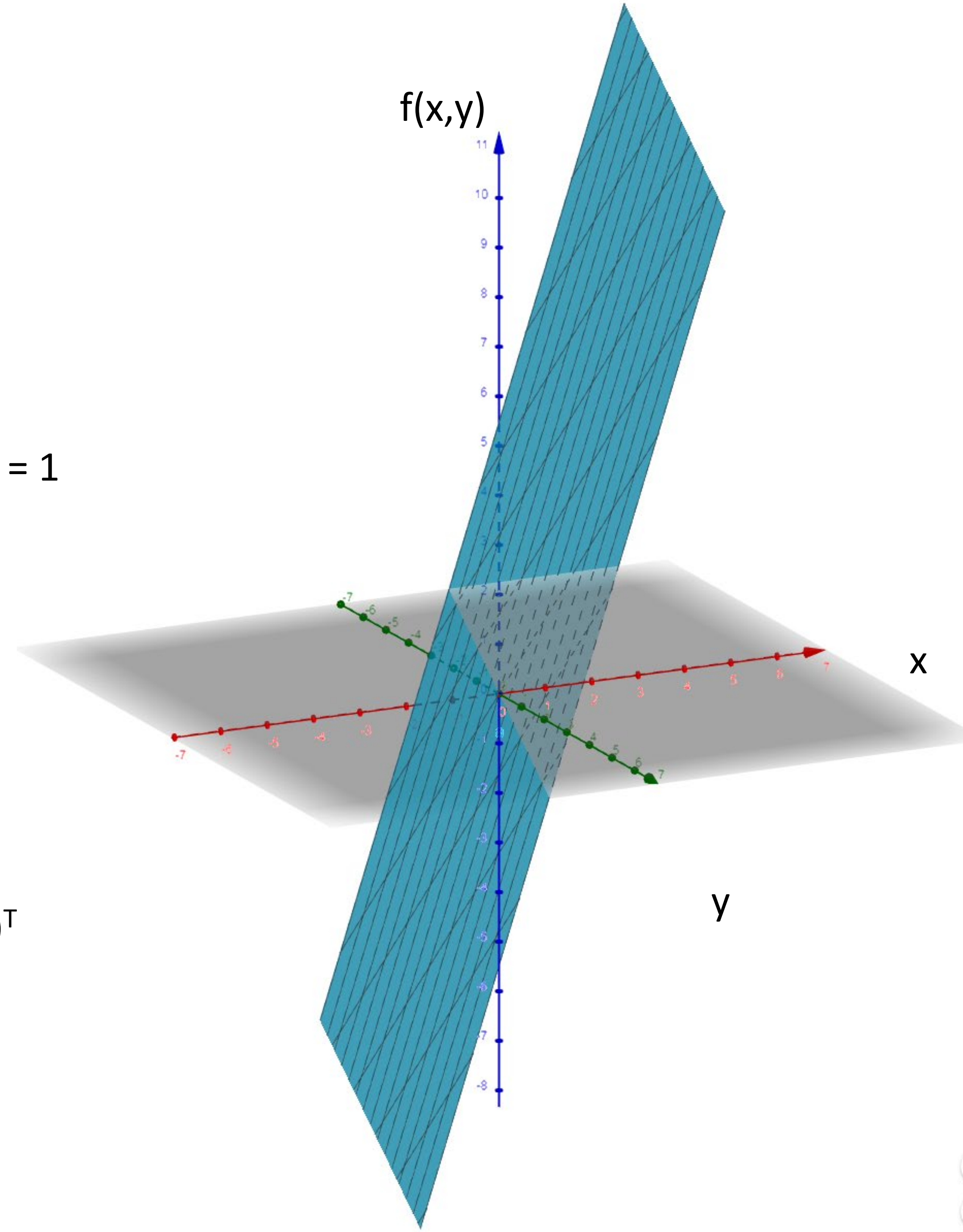
$f(x,y)=3x+y$

Slope in y:

$\partial f/\partial y = 1$

Gradient:

$\nabla f = (3, 1)^T$



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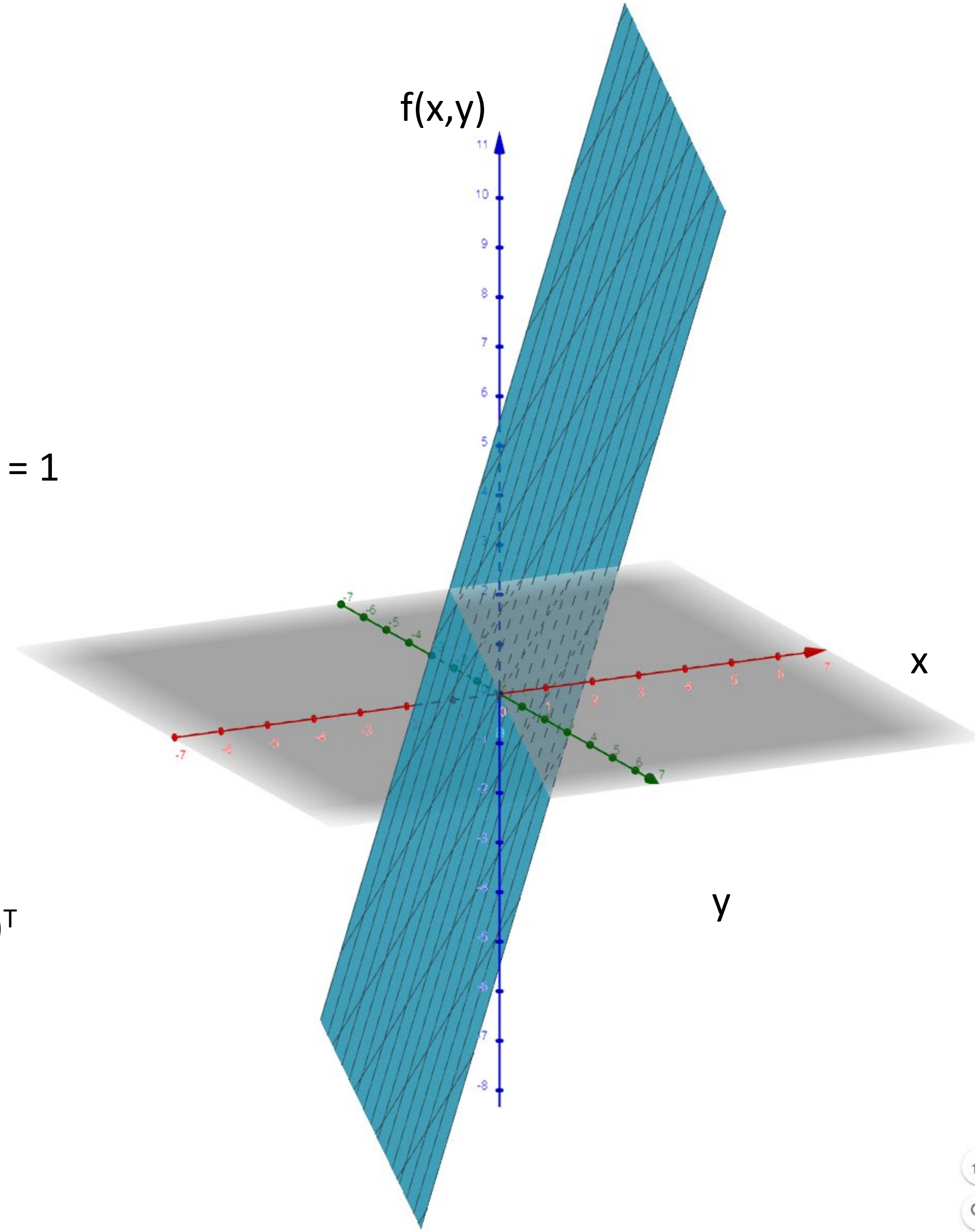

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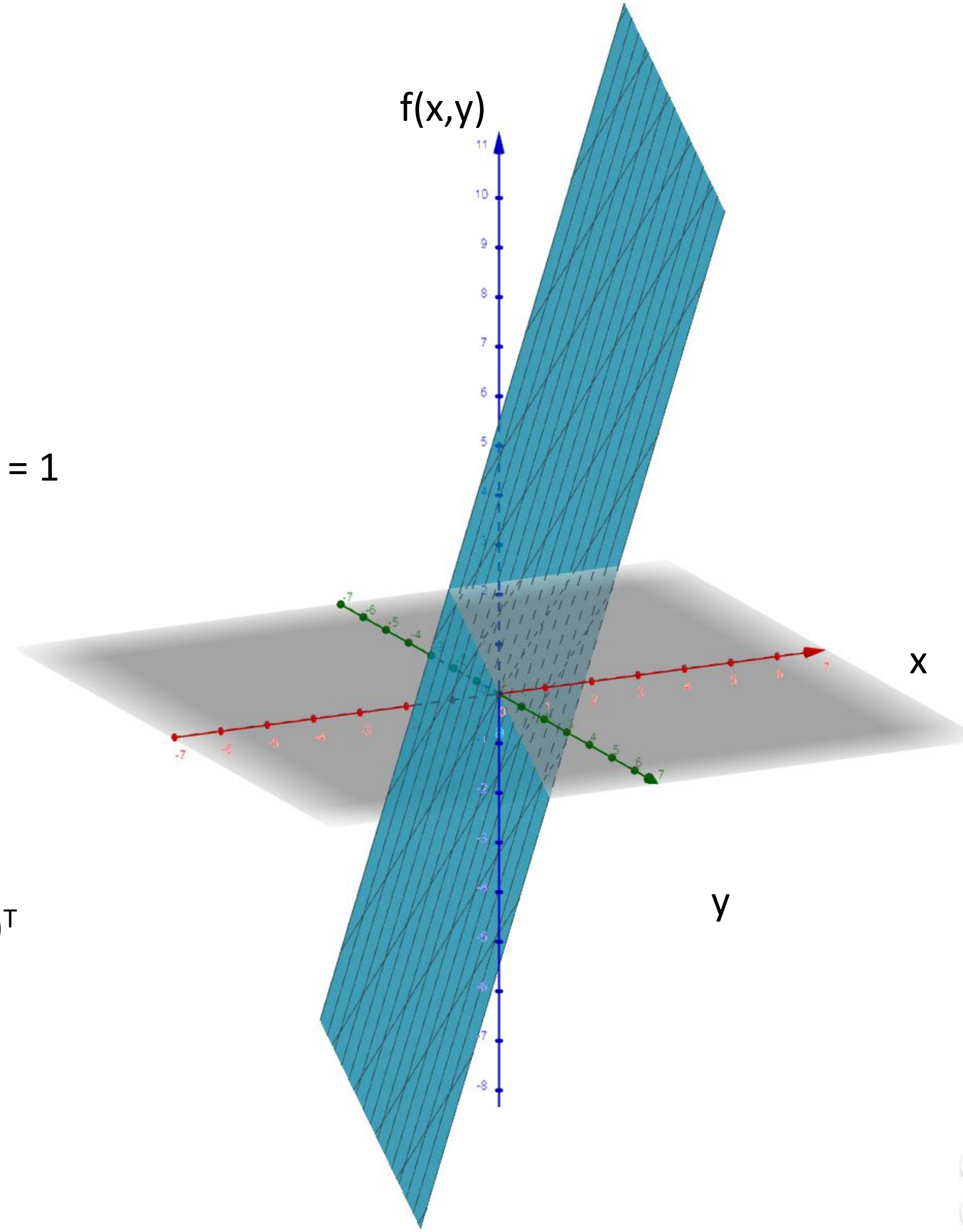

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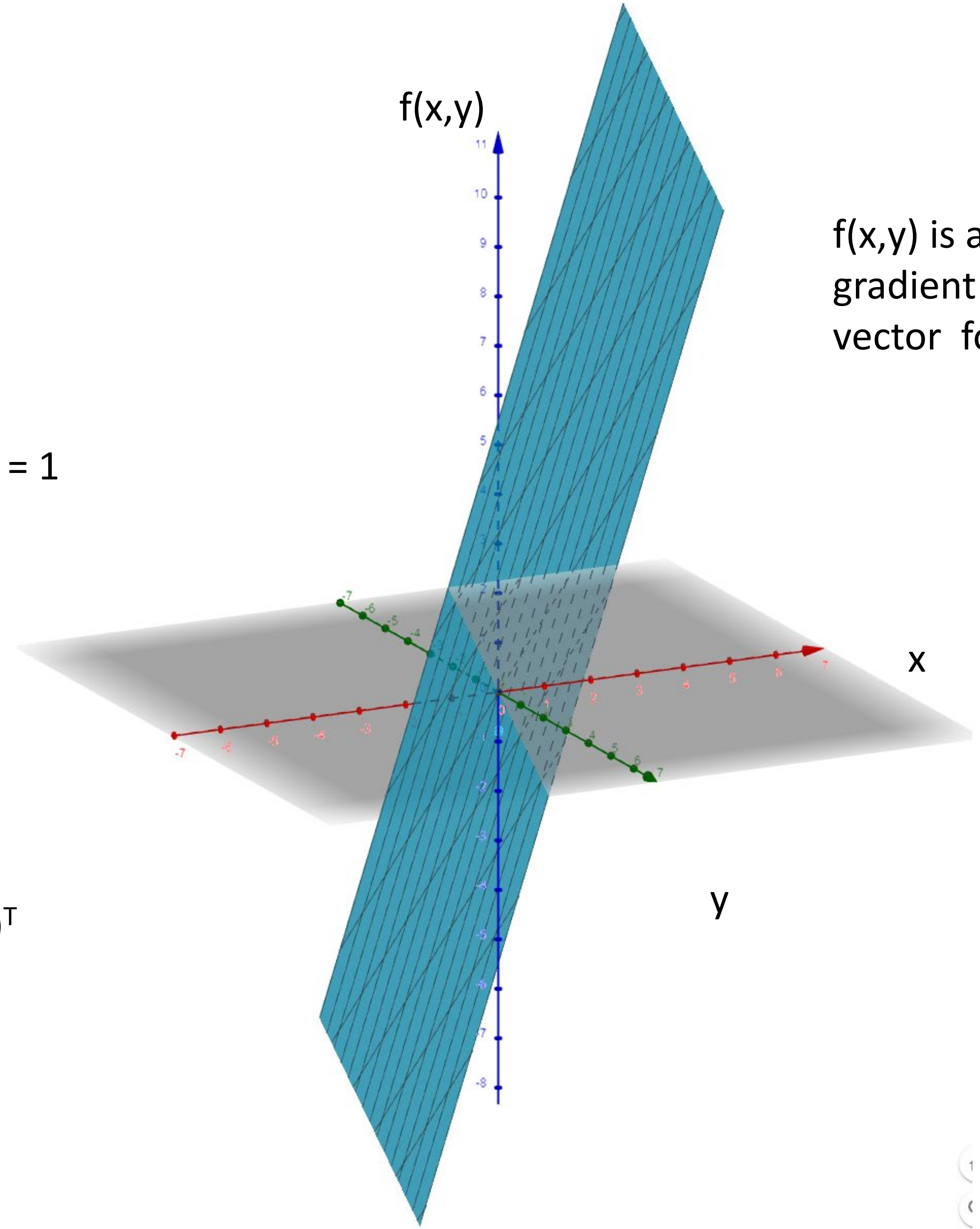

f(x,y) is a plane, so the gradient is the same vector for all points

Another example of a 3D function and its partial derivatives:

$$f(x,y) = -0.5x+0.25y^2$$

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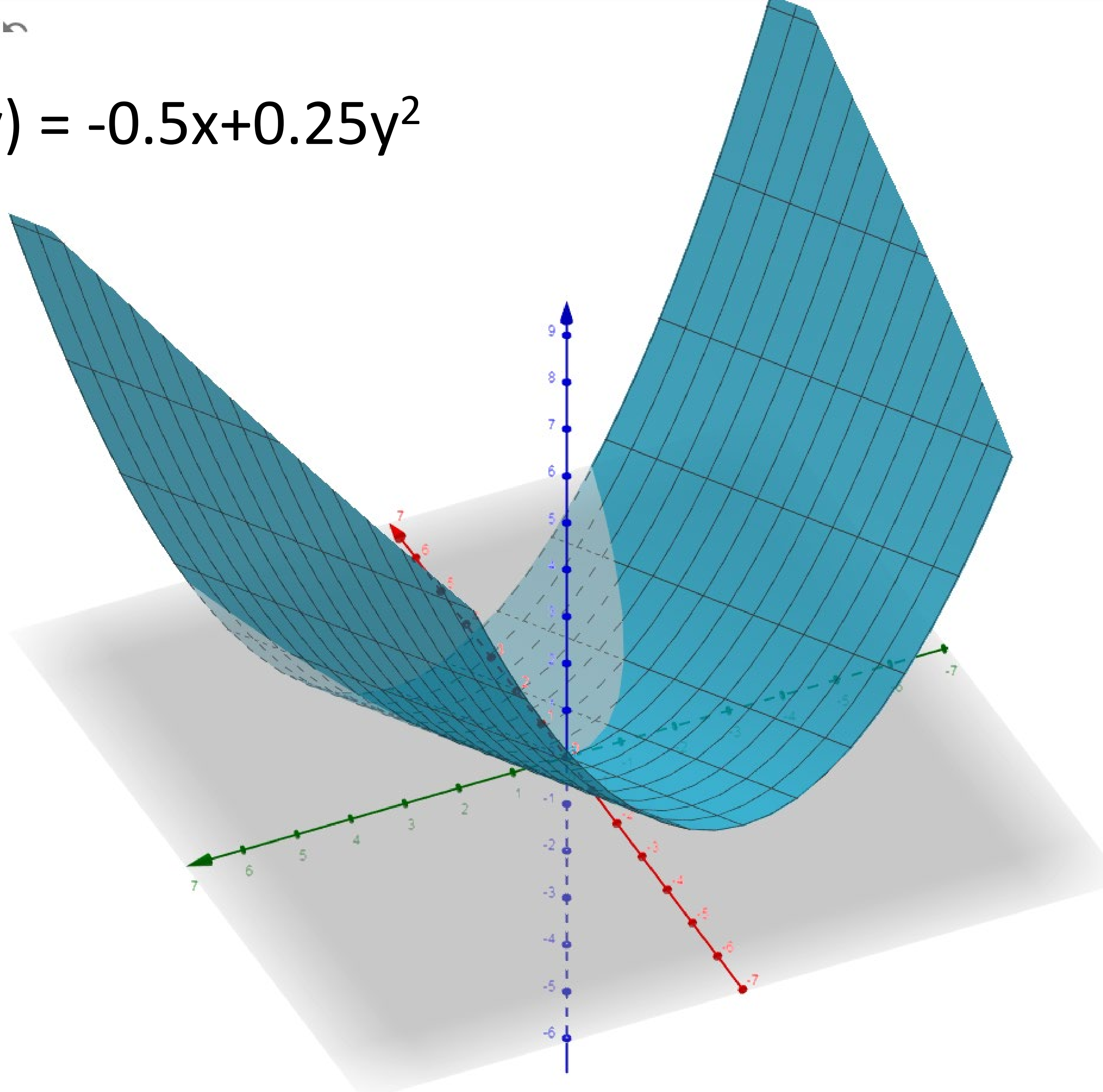
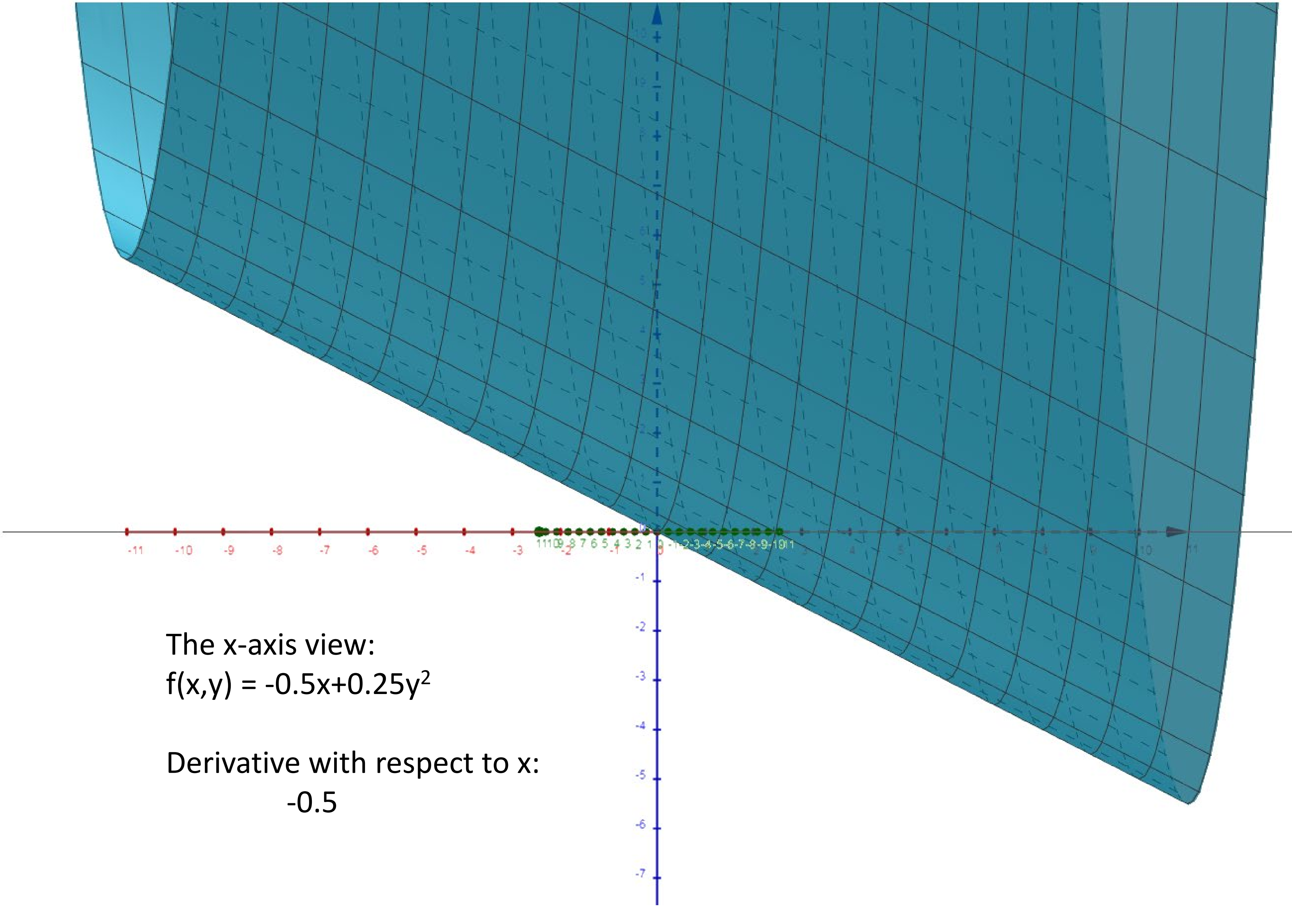




The x-axis view:

$f(x,y) = -0.5x+0.25y^2$

Derivative with respect to x:

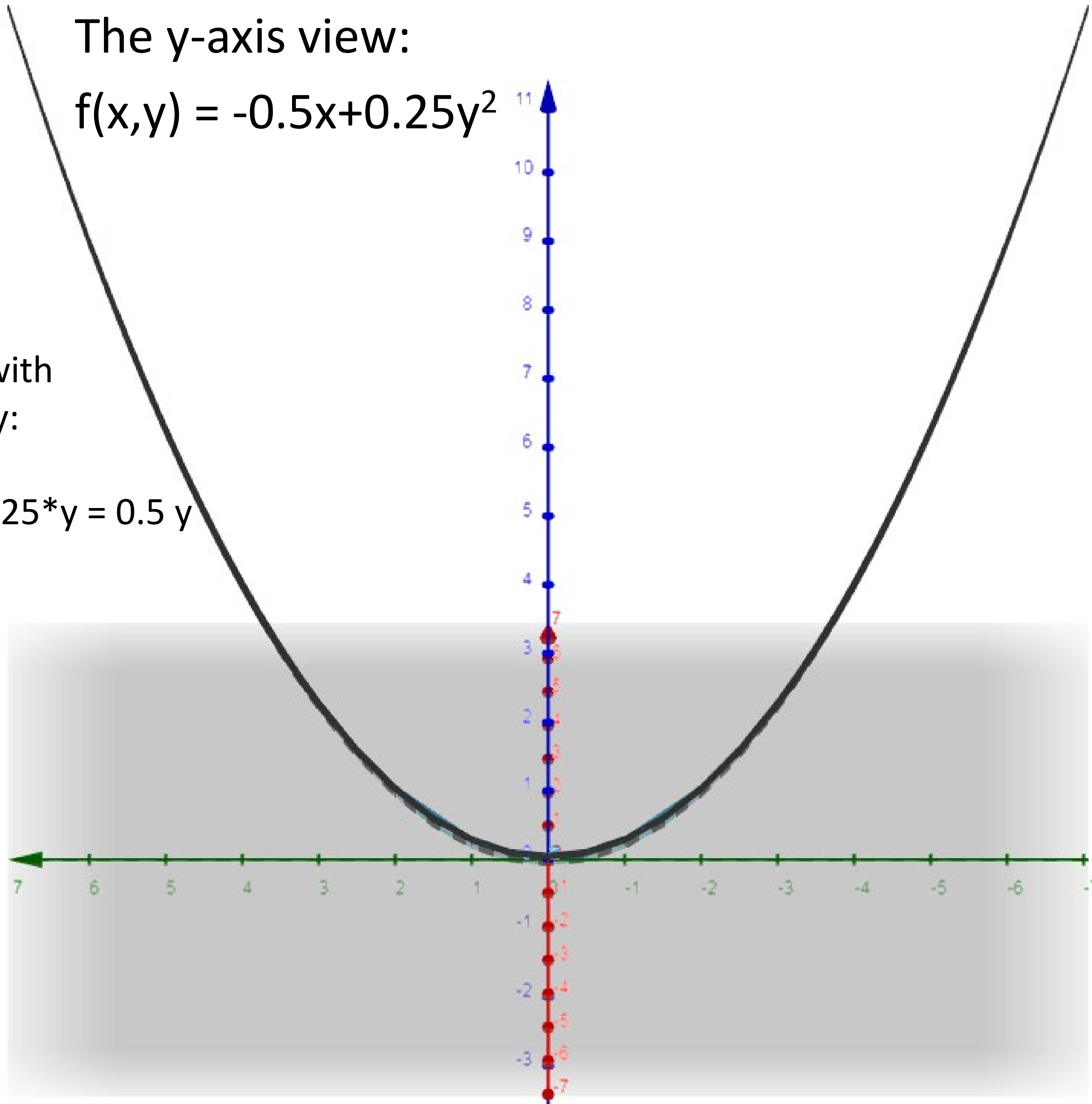
-0.5

The y-axis view:

$f(x,y) = -0.5x+0.25y^2$

Derivative with respect to y:

$= 2*0.25*y = 0.5\ y$

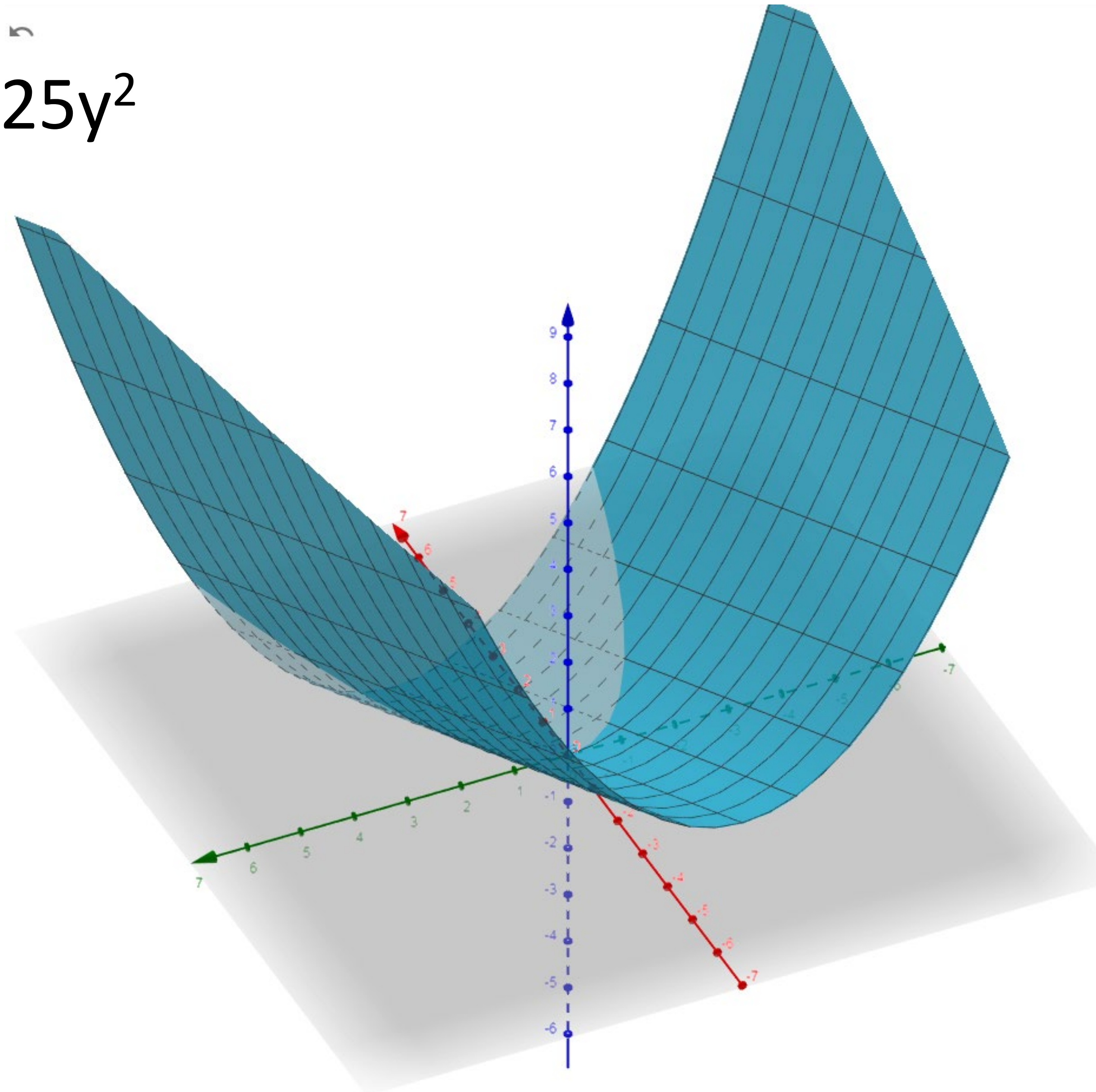


$f(x,y) = -0.5x+0.25y^2$

Gradient

∇f

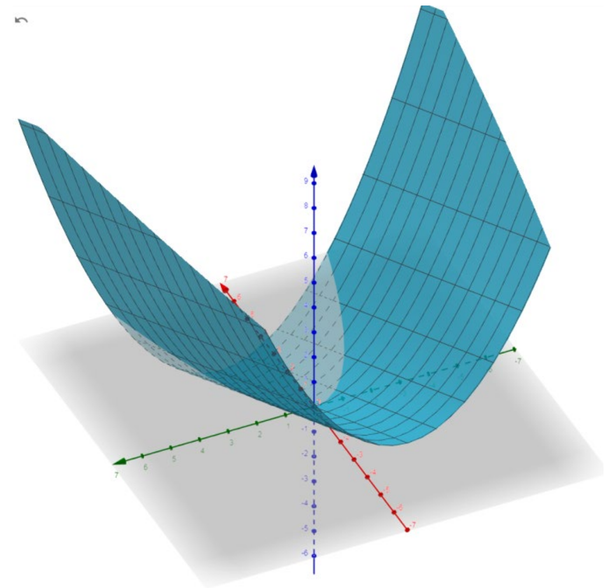
$= (-0.5,\ 0.5y)^T$



$f(x,y) = -0.5x+0.25y^2$

$= 0.5\,(-x+0.5y^2)$

$p(o(\boldsymbol{x})) = 0.5\, o(\boldsymbol{x})$, where $\boldsymbol{x}=(x,y)$



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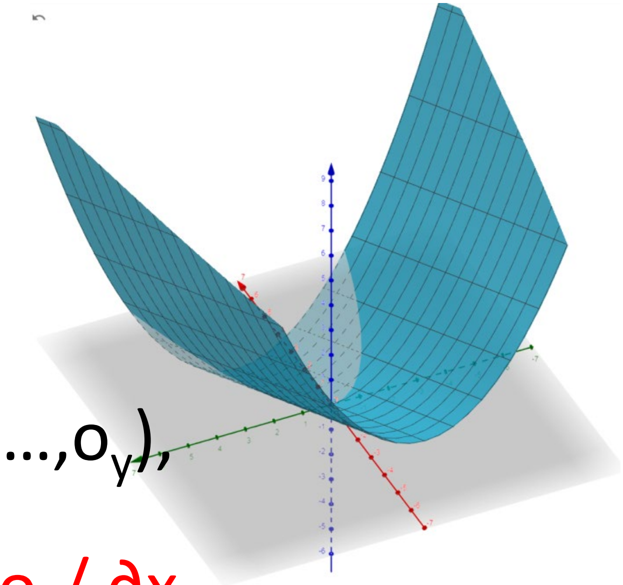

General Chain Rule: $\boldsymbol{x}=(x_1,\ldots,x_n),\boldsymbol{o}=(o_1,\ldots,o_y),$

$$\partial p(\boldsymbol{o}(\boldsymbol{x}))/\,\partial x_i = \sum_{j=1,..,J} \partial p(\boldsymbol{o})/\,\partial o_j \;\; \partial o_j\,/\,\partial x_i$$

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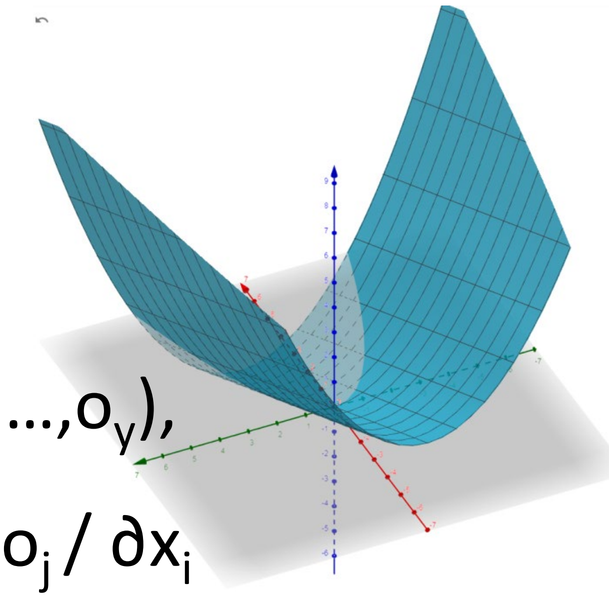

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Here: J=1, f(x,y) is a scalar, n=2 => 2 partial derivatives

$\partial p/\partial x =$

$\partial p/\partial y =$

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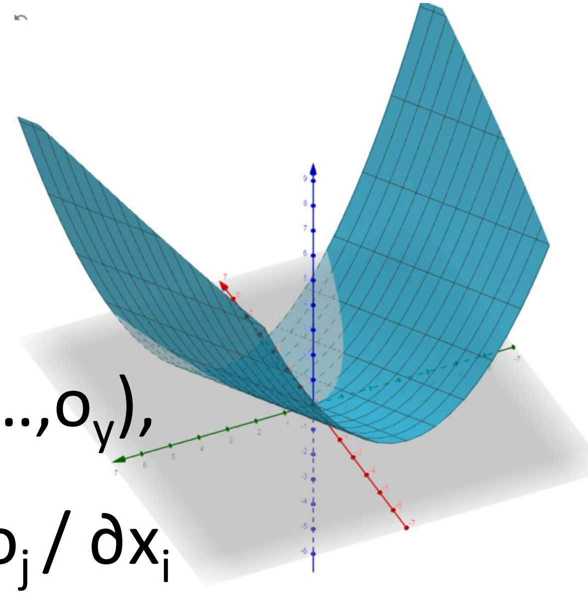

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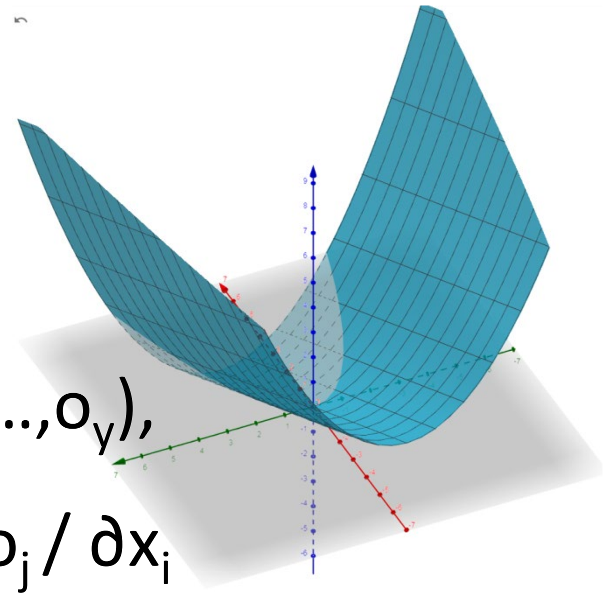

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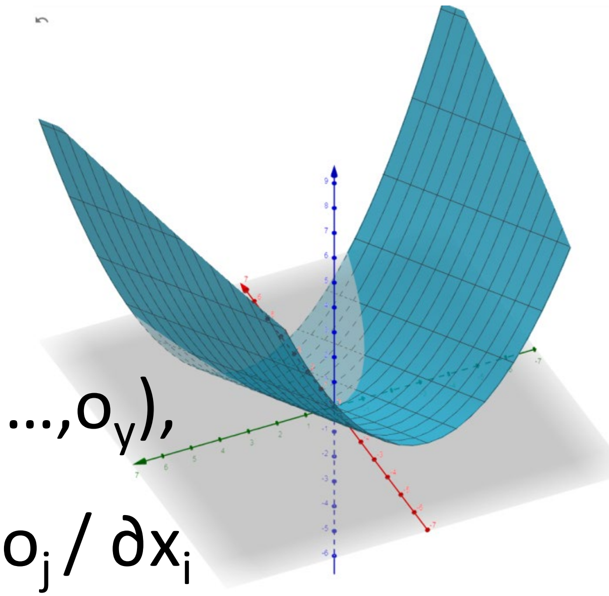
$$\partial p(o(\boldsymbol{x}))/\partial x_i = \quad \partial p(o)/\partial o \quad \partial o / \partial x_i$$

$\partial p/\partial x =$ 0.5 -1 = -0.5

$\partial p/\partial y =$ 0.5 2 (0.5)y = 0.5 y



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$\partial p/\partial x = -0.5$

$\partial p/\partial y = = 0.5\, y$

$\nabla f = (-0.5, 0.5y)^T$

as computed before !