Question 1 (Multicast Address Allocation):

(a) Among the numerous problems with this approach are that a centralized solution quickly becomes a bottleneck as the size of the request pool grows large. Also, if such a central authority ever fails, nobody has the capability to lease out multicast addresses.

(b) This question can be interpreted two ways: 1) to compute the probability that there is at least one collision in the first $N$ allocations, or 2) to compute the probability that the first collision is the $N$’th. Either interpretation was fine, and here we give the solutions to the first case.

Suppose we have allocated $i < N$ addresses without a collision. The probability that our random allocation of the $i+1$st address does not cause a collision is $1 - \frac{i}{65536}$. In general, we can chain these probabilities together to derive the probability that $N$ consecutive allocations were collision-free. Letting $C_i$ denote the event that the first $i$ collisions were collision-free, we can write:

\[
Pr[C_i] = Pr[C_i|C_{i-1}] \cdot Pr[C_{i-1}]
\]

\[
= \left(1 - \frac{i}{65536}\right) \cdot Pr[C_{i-1}]
\]

Noting that at the base case, $Pr[C_1] = 1$, we have that:

\[
Pr[C_i] = \left(1 - \frac{1}{65536}\right) \left(1 - \frac{2}{65536}\right) \cdots \left(1 - \frac{N-1}{65536}\right)
\]

\[
= \prod_{i=1}^{N-1} \left(1 - \frac{i}{65536}\right)
\]

Now you would like to solve for the smallest $i$ such that $Pr[C_i] \leq 0.99$. This is possible analytically, but is easier to solve in simulation. It turns out that the value of $i$ is only 37, which means that you have only exhausted one-twentieth of one percent of your address space before you see a collision with one percent probability.

A famous, and closely related problem arises in the Birthday Paradox: How many individuals must there be in a room before there is a good chance that two of them were born on the same day of the year? It turns out that you should bet on it (> 50% chance of winning) even when there are only 23 people in the room.
Question 2 (ALOHA analysis):

(a) The efficiency of the protocol is \( \frac{k}{k+x} \), where \( x \) is the expected number of consecutive wasted slots. Now we determine \( x \). Let \( Y \) be a random variable denoting the number of slots until a success, we have \( P(Y = y) = a(1 - a)^{y-1} \) where \( a \) is the probability of a success, which is \( Np(1 - p)^{N-1} \). So \( x = E(Y) - 1 = \frac{1-a}{a} \).

Putting them together, efficiency = \( \frac{k}{k + \frac{1-Np(1-p)^{N-1}}{Np(1-p)^{N-1}}} \).

(b) The \( p \) that maximizes the efficiency is the one that minimizes \( x \), which in turn maximizes \( a \), so it is \( \frac{1}{N} \).

(c) Substitute \( p \) with \( \frac{1}{N} \) in the formula above, we get \( \lim_{n \to \infty} \text{efficiency} = \frac{k}{k + e - 1} \).

(d) Apparently, when \( k \) approaches \( \infty \), efficiency approaches 1.

Question 3 (Ethernet capture effect)

(a) A can choose \( k_A = 0 \) or 1; B can choose \( k_B = 0, 1, 2, 3 \). A wins outright if \((k_A, k_B)\) is among \((0, 1), (0, 2), (0, 3), (1, 2), (1, 3)\); there is a 5/8 chance of this.

(b) Now we have \( k_B \) among 0...7. If \( k_A = 0 \), there are 7 choices for \( k_B \) that have A win; if \( k_A = 1 \) then there are 6 choices. All told the probability of A’s winning outright is 13/16.

(c) \( P(\text{winning race 2}) = 5/8 \) and \( P(\text{winning race 3}) = 13/16 \); generalizing, we assume the odds of A winning the \( i \)th race exceed \( 1 - \frac{1}{2^{i-1}} \). We now have that \( P(\text{A wins every race given that it wins races 1-3}) \geq (1 - 1/8)(1 - 1/16)(1 - 1/32)... \approx 3/4 \).

(d) B gives up on it, and starts over with \( B_2 \)

(e) Let us consider the same two stations \( A \) and \( B \) from the previous question. So if \( A \) just transmitted then it will wait for one or two time slots, during which \( B \) can try and successfully send data.
Question 4 (Throughput of wireless vs. wireline):

(a) Messages from $C$ to $A$ need to be relayed by $B$ because $A$ cannot hear $C$. So the answer is 1 message per 2 slots.

(b) The transmissions do not interfere with each other, so the answer is 2 messages per slot.

(c) Because $B$ can hear both $A$ and $C$, at one slot only one of them can transmit, so the answer is 1 message per slot.

(d) Let us assume that a wired network means point to point connection. Then there is no concern of conflict, so the answers to the above questions are: 1 message per 2 slots, 2 messages per slot and 2 messages per slot, respectively.

(e) (a) It takes 2 slots to get the message and 2 slots to return the ACK, so the answer is 1 message per 4 slots.

(b) The transmissions do not interfere, but the ACKs will interfere. So in addition to the 1 slot to send the original message, ACK from $B$ to $A$ and ACK from $C$ to $D$ will each take 1 slot. So the answer is 2 messages per 3 slots.

(c) The transmission of the second message can be concurrent with the ACK of the first message as $D \rightarrow C$ and $A \rightarrow B$ do not interfere. So we need 1 slot to transmit the first message, 1 slot to transmit the second message as well as ACK to the first message, and 1 additional slot to transmit ACK to the second message. So the answer is 2 messages per 3 slots.