CS 559: Algorithmic Aspects of Computer Networks **Spring 2010**

Lecture 14 — March 4

Lecturer: John Byers *BOSTON UNIVERSITY* *Scribe: Gonca Gursun*

In today's lecture, we talked about a game-theoretic approache to modeling network creation and formation.

Consider a network with n nodes and e edges. If there are costs associated with building nodes and edges, it is desirable to build the network in a way that the cost is minimized to achieve a desired objective. In a classical optimization framework, problems such as these can be formulated as a combinatorial optimization problem solved by a central authority. Often such problems are NP-hard. Moreover, in the case of Internet, there is no central authority, therefore the goal of minimizing the cost is challenging.

Below are some facts about Internet:

- No central authority.
- Competing objectives/goals across ISPs, i.e. the objective of ISPs is not achieving social optimality.
- Self-interest is the primary goal for ISPs.

From the view of economics, Internet is a computational artifact with the following properties:

- multiple agents
- self-interested agents
- individual, not global objectives
- computations are distributed
- solutions must be computationally tractable

Fabrikant et al proposes a game-theoretic model of network creation [1]. In this game, there are n nodes (players) and their strategy choices create an undirected graph. The goal is creating a *good* network. A good network is associated with keeping routing costs and construction costs low. They use the following routing cost:

$$\sum_{i,j \wedge i \neq j} d(i,j), \tag{14.1}$$

where $d(i,j)$ is the hop count on the shortest path between node i and j. A network is also associated with an edge cost, α ($\alpha > 0$, and the game changes by different values of α). The cost of building the network (or graph G), $C(G)$, is given by the following equation:

$$C(G) = \alpha \times |E| + \sum_{i,j \wedge i \neq j} d(i,j), \tag{14.2}$$

where $|E|$ is the total number of edges in the network.

In this game, individuals elect to build edges that are adjacent to themselves and they can not influence the rest. An edge between a node pair (i,j) is paid by either node i or node j (there are also games in which i and j share the cost of the edge). Nodes play their *strategies* (the action of building/removing edges) repeatedly. S_i denotes the strategy of the node i, s.t. $S_i = [0110....010]$ is an array of length n, where each entry j denotes if i is willing to build an edge with node j. The total cost of an individual node i after it plays its strategy S_i is given by the following equation:

$$C_i(G_{S_i}) = \alpha \times |E_{i,S_i}| + \sum_{i,j \wedge i \neq j} d(i,j), \tag{14.3}$$

where $|E_i|$ is the total number of edges i is willing to pay for in its strategy S_i.

Each player has 2^{n-1} choices of strategies. Node i with strategy S_i is at a local optimum (equilibrium) if $\forall \hat{S}_i \neq S_i : C_i(G_{\hat{S}_i}) \geq C_i(G_{S_i})$.

If all players are simultaneously in equilibrium, then a Nash Equilibrium (NE) is achieved, i.e. no player would unilaterally prefer to employ a different strategy.

Questions:

1. Is there a NE? Yes.

2. Is the NE unique? No, not generally. It is unique for small and large values of α.

3. If there is a NE, can we compute it? Yes, because there are finite number of strategies, $O(2^n \times 2^n \times ... \times 2^n)$

4. How about in polynomial time? No, it is NP-complete. However, NE can be achieved deterministically for this game by Iterative Best Response. Players repeatedly (e.g. Round Robin) update their strategy to minimize the cost. If it converges to a state where none of the players wants to update its strategy, then NE is reached. Iterative Best Response may cause cycles in some games but it doesn't for this game.

5. What is the gap between socially optimal (global objective) and game theoretic/selfish outcomes (as initially motivated by [2])?

$$\text{Price of Anarchy} = \frac{cost(G_{NE})}{cost(G_{SO})}, \tag{14.4}$$

where G_{NE} is the graph/network in which the worst NE (highest cost) is reached, and G_{SO} is the graph/network in which best (socially optimum) NE is reached. For the network formation game, $PoA \leq \frac{4}{3}$ [1].

Bibliography

[1] A. Fabrikant, A. Luthra, E. Maneva, C. Papadimitriou, and S. Shenker. On a network creation game. *PODC '03: Proceedings of the twenty-second annual symposium on Principles of distributed computing*, 2003.

[2] E. Koutsoupias and C. Papadimitriou. Worst-case equilibria. In *in Proceedings of the 16th Annual Symposium on Theoretical Aspects of Computer Science*, pages 404–413, 1999.