CS 559: Algorithmic Aspects of Computer Networks		Spring 2010
	Lecture 19 — April 1	
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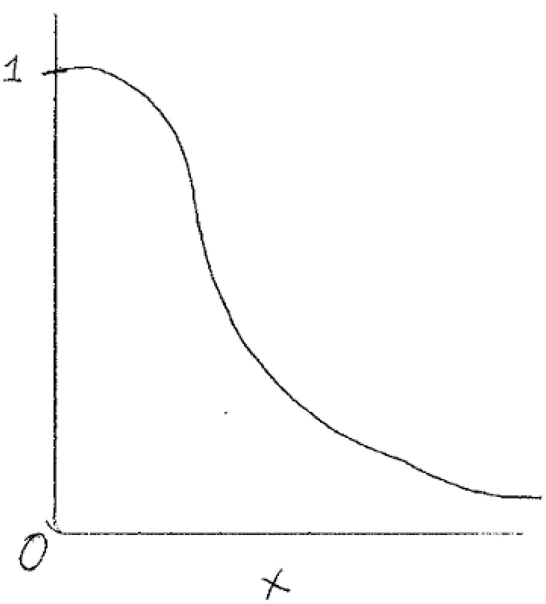
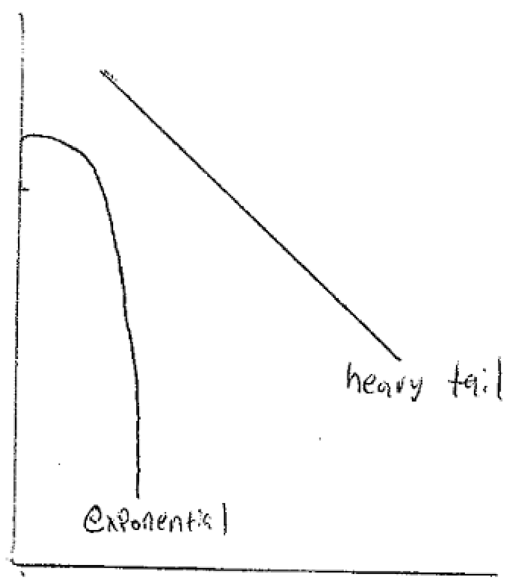
In today's lecture, we learned about the **Barabasi-Albert** model for *scaling random networks*. This model aims to improve average path length as well as degree distribution. Most likely, it will improve the clustering coefficient, but this is not one of its goals.

19.1 Heavy Tailed Degree Distributions

$\Pr[X \leq x] \approx cx^{-\alpha}$
$Pr\left[X \leq x\right] \approx e^{-cx}$
$log(Pr\left[X \leq x\right]) \approx log(c) - \alpha * log(x)$
$log(Pr\left[X \leq x\right]) \approx cx * log(e)$

19.2 Complimentary Cummulative Distribution Functions





19.3 Proposed Model – Preferential Attachment

This model works on growing graphs, something previous models did not consider. At time t=0, m initial nodes arise and form a clique At t=i, one new node arrives and connects to m other nodes but not at random - preferentially. Probabilistically attach to node j with probability proportional to degree of j. Normalize probability over all degrees in network

$$\frac{d_j}{\sum_k d_k}$$

Example:

$$\sum d_k s = 5, d_j = 6, d_x = 2.$$

When adding a node, attach it to d_j with probability $\frac{6}{50}$ and to d_x with probability $\frac{2}{50}$. Hence, nodes which are already connected get more connected. This leads to a heavy tailed degree distribution.

We also discussed some historical footnotes for this model; it was published in *Science* in 1999 which was a big deal. But almost immediately, economists recongnized it as a model they had since 1920 known as "The Simon Model".

19.4 Analysis via differential equations:

First, X_j is the number of nodes with incoming-degree j. In order to find the amount function grows or shrinks with respect to time t, take derivative with respect to t:

$\frac{dX_j}{dt} = \alpha$ - $\frac{x_i - 1}{t}$ + $(1-\alpha)$ $\frac{(x_i-1)^2}{\sum_k x_k}$

Assume edges are directed, so new nodes point to old nodes. Instead of completely random or completely preferential, use a hybrid method:

- α is probability we'll attach at random
- $1 - \alpha$ probability to attach preferentially.

Consider m=1: Number of nodes with in-degree j could grow (nodes which previously have j-1 get pointed to by new node) or could shrink (nodes currently with j get pointed to by new node).
Ideally, the fraction of nodes in each class is constant (fixed in the limit).
The average degree is fixed because of adding m edges each time. $m * t$=edges in graph. We want $\frac{dX_j}{dt} \approx \frac{1}{t}$

19.5 Monkey Typing Randomly

After looking at through this analysis, we looked at the case of random generation with the following conditions:

- Monkey hits the spacebar with probability c.
- Mokey hits all other keys uniformly at random.
- A "word" is whitespace delimited.

The result? A power law.

19.6 Conclusion

Out of the Barabasi-Albert model we get something hierarchical or tree-like. But in a logical network (such as the internet), nodes with high degree are at the edges of the graph where ISPs provide service to users. The core of the network is high speed but low degree. High degree is essentially aggregation points. What other models might better represent this? We'll look at the Power Law Random Graph (PLRG) next lecture.