CS 332 - Homework 1

Due: Wednesday, February 8.

Reading: Chapter 3, pages 137-148

Problems:

1. Prove by induction that $2^n < n!$ for all $n \geq 4$.
   Your proof must specify the variable on which the induction occurs, the base case, and the inductive hypothesis.

2. Rank the order of growth of the following 6 functions from smallest to largest. By order of growth I mean the Big-O notation.
   $4n^3$, $n \sqrt{n}$, $5n$, $3^n$, $n^3 \log 5n$, $n \log n$, $n^3 \log n \log n$

3. Consider the following 2 step procedure P and explain why it is not a legitimate algorithm of the kind we discussed in class. The input to P is three natural numbers a,b,c.
   i. Try all possible assignments of natural numbers to a,b,c and for each of these possible settings test if $a^3 + b^3 = c^3$
   ii. If the test you make of the equation in i. is ever true, then P outputs true, if it all of the tests you make in 1 are false then P outputs false.

4. Describe a TM which accepts the language $L = \{w2w \mid w \text{ is a string in } \{0,1\}^* \}$.
   Here I mean informally describe how the TM works. You need not give the full program or diagram.
   We are looking for something like the description of the TM in examples 3.11 or 3.12 of the book on pages 146 and 147.

5. Write a TM program which decides the set of binary strings which when interpreted as decimal integers have the values $5+8n$, where $n=0,1,2,3,...$. (So these values are 5, 13, 21, 29, 37, 45, ...)