This homework will be graded. A couple of these problems are taken from J. Finkelstein.

1. Prove by induction that $n! > 2^n$ for all $n \geq 4$.
   Your proof must specify the variable on which the induction occurs, the base case, and the inductive hypothesis.

2. Let $A$ be any fixed finite set of 4 or more elements. Prove that the number of subsets of elements in $A$ is less than the number of permutations of elements of $A$.

3. Let $A = \{0, 1, 2\}$, and let language $L$ be defined by $L = \{waw^Rbw|w \in A^*\}$.
   (i). What is $|A^3|$ (that is, the number of elements in $|A^3|$) ?
   What is $|A^n|$ ? What is $|A^*|$?

   (ii). Thinking of $L$ as a language, what is its alphabet ?
   How many elements are there in $L$ which have length 8 ?
   Give two examples of strings that are in the language $L$ and two examples of strings that are not in the language. Give an English language description of the elements of $L$.

4. (i). How many functions are there with domain $\{0, 1, 2\}$ and range $\{4, 5\}$

   (ii). How many 1-1 functions are there with domain $\{0, 1, 2\}$ and range $\{4, 5\}$
   I don’t expect a proof here but I do expect an answer and some explanation of why your answer is right.

5. Rank the order of growth of the following functions from smallest to largest.
   $6n^2$, $n(\sqrt{n})$, $n^2 \log \log n$, $5n$, $3^n$, $n^2 \log 6n$, $n \log n$, $n^2 \log n$

6. Consider the following 2 step procedure $P$ and explain why it is not a legitimate algorithm of the kind we discussed in class.
   The input is three natural numbers $a,b,c$.

   1. Try all possible assignments of natural numbers to $a,b,c$ and for each of these possible settings test if $a^3 + b^3 = c^3$

   2. If the test you make of the equation in 1 is ever true, then $P$ outputs true, if it all of the tests you make in 1 are false then $P$ outputs false.