CS 332 - Homework 2

Due: Wednesday, February 22.

Reading: Chapter 3, pages 148-159 and Chapter 4, pages 165-169

Problems:

1. Let A be any fixed finite set of 4 or more elements. Prove that the number of subsets A is less than the number of permutations of elements of A.

2. Suppose you have a Turing machine T that computes the addition function for binary integers. So T computes f(n,m) = n+m.

   Describe a TM M that computes the function g(n) = n^2.

   Hint: Given n, you can directly use T to compute f(n,n) = n+n = 2n Also we know that n^2 = n + n + n + n + ... + n, n times. So now use T repeatedly to get a TM M as described above.

3. For this problem you may assume that the sets L and J are both subsets of N.

   i. Show that any infinite decidable language L has an infinite decidable subset.
   Note: This is a trick question.

   ii. Show that any infinite decidable language L has an infinite decidable subset J with the property that L − J is also infinite.

   iii. Does the statement in part i of this problem still true if L is only recognizable. (That is, is it true that any infinite recognizable language L has an infinite decidable subset ?) If it is true, say why. If not true, find a counter example.

4. Assume that your TM’s M are allowed to have the read head stay (S) where it is instead of only moving L or R at any transition. So a line of the TM program species either an L, S, or R tape head move. Show that any such M can be simulated by a standard TM T which only moves L or R as it computes.

   To do this you should explicitly say how, when given a machine M as above, you can change its program into a standard program for T which accepts and rejects the same strings. You are free to add some new symbols or new states to T if that helps.

5. Page 161 (Page 189), problem 3.15 parts d. and e.

6. Let A = {0, 1, 2}, and let language L be defined by L = \{wawbw\mid w \in A^*\}.
(i). What is $|A^4|$ (that is, the number of elements in $A^4$) ?
(Note: Here I use $A^4$ to mean the concatenation of $A$ with itself 4 times = AAAAA = \{wxyz|w, x, y, z \in A\}. (See page 14 of the textbook.)

What is $|A^n|$ ? What is $|A^*|$ 

(ii). Thinking of the language L, what is its alphabet ?
How many elements are there in L which have length 8 ?

PART 2: A few problems which are just exercises and should not be turned in.


2. Page 159 (Page 187), problem 3.2 c.

3. Page 160 (Page 188), problem 3.6

4. Page 161 (Page 189), problem 3.16 part d. and b.