1. Write out the full program of a Turing machine which decides the language $L$ of all binary strings which contain an equal number of 0's and 1's. 5 points

2. Describe a TM which accepts the language $L = \{wCy \mid w \text{ is a binary positive integer, } C \text{ is just the letter } C, \text{ and } y \text{ is the binary integer whose value is } 2w\}$. 5 points

Here I mean informally describe how the TM works. You need not give the full program or diagram.

We are looking for something like the description of the TM in examples 3.11 or 3.12 of the book on pages 146 and 147.

Note: A binary positive integer is a binary string whose first bit is a 1.

3. Explain why the collection of all decidable languages is closed under intersection. 5 points

Use the TM definition of decidable here. So you need to show the if $L$ and $J$ are two decidable languages then $L \cap J$ is also decidable.

4. Write a TM which decides the set of binary strings which when interpreted as binary integers are evenly divisible by 8. 5 points

Give the full TM program or draw a state diagram. Hint: This is easier than you might think.

5. Show that a set of natural numbers is decidable if and only if it can be enumerated in increasing order. 10 points

6. Show that a set $S$ of natural numbers is recognizable and if its complement $N-S$ is recognizable then $S$ is decidable. 5 points