1. Let $L$ be a language which is infinite and recognizable. Show that there is a subset $K$ of $L$ which is infinite and decidable.

2. True or false: Any finite set $F$ of Boolean strings is decidable. Explain your reasoning.

3. Give a 1-1 and onto correspondence between $N$ and $J$. ($J$ is the set of all integers, positive, negative and 0.)

4. True or false: The set of all Turing machines (as defined in the textbook) is countable. Explain your reasoning.

5. Show that the set of all finite sets of integers is a countable set.

6. Show that the recognizable languages are closed under concatenation. That is, if $J$ and $L$ are two languages in that are recognizable, then so it the language $JL$. ($JL$ is the concatenation of $J$ and $L$.)

7. Consider the following nondeterministic Turing machine $N$ where $N$ has states $q_i$ for $i = 1, 2, 3, 4, a, b$. $N$’s input alphabet is $\{0,1\}$, $N$’s tape alphabet is $\{0,1,B\}$, $N$’s has the following $\delta$ function as it’s program. Note: If the definition of $\delta$ is not specified then you may assume the program simply halts and is in the $q_r$ state when it does. So for example if at some point the TM is in state state $q_2$ and reads a 0 then it goes to state $q_r$ and halts.

So here is $\delta$: 
$\delta(q_0, 0) = \{q_0, 0, R\}$
$\delta(q_0, 1) = \{(q_0, 1, R), (q_1, 1, R)\}$
$\delta(q_1, 0) = \{(q_0, 0, R)\}$
$\delta(q_1, 1) = \{(q_2, 1, R), (q_3, 1, R)\}$
$\delta(q_2, 1) = \{(q_a, 1, R)\}$
$\delta(q_3, 0) = \{(q_4, 0, R)\}$
$\delta(q_4, 1) = \{(q_a, 1, R)\}$
i. Run the TM on the input strings 001101 and on 1001. Does N accept either of these strings? Explain why.

ii. Describe which strings are in the language $L(N)$, where $L(N) = \{ w \mid \text{the machine N accepts w} \}$?