Section I. Examples: For each of the following problems give examples of the languages $J$, $K$, $L$ that are described in the problems. In each case briefly say why the example is a correct one for the stated problem. Each problem is worth 5 points

1. Give examples of two languages $J$ and $K$ both of which are NOT decidable but whose union is decidable.

2. Let $I = \{x \mid |x| \text{ is odd}\}$. (So $I$ is decidable.) Give an example of a language $K \subseteq I$ with $K$ undecidable.

3. Give examples of two languages $J$ and $L$ both of which are NOT decidable but where $J \cap L$ is infinite and decidable.

4. Give an example of an infinite decidable set $L$ where $L \subseteq \{M \mid M$ is a Turing machine that does not accept any string\}

Section II: For each of the following languages $J$, $K$, $L$ use a reduction to prove that the language is undecidable.

In each case you must,

- State precisely what problem you are reducing to the problem $J$, $K$, or $L$ and also state precisely what you need to show to establish the reduction, (2 points each) and then
- Give the reduction (4 points each).

5. Show that it is not decidable to determine if a TM $M$ on input $w$ ever goes into the specific state $q_9$ during the computation of $M(w)$. That is, show $J = \{(M, w) \mid \text{the TM } M \text{ on input } w \text{ goes into state } q_9 \text{ during its computation}\}$ is undecidable.

6. Let $K = \{M \mid M \text{ is a Turing machine with input alphabet } \Sigma \text{ and } M \text{ accepts every string in } \Sigma^*\}$. Prove $K$ is undecidable.

7. Let $L = \{M \mid M \text{ is a Turing machine with input alphabet } \{0, 1\} \text{ and } M \text{ accepts the string 001}\}$. Prove $L$ is undecidable.