Problems:

1. Prove that there is a subset of \{1\}^* which is undecidable. (5 points)

2. Assume that we have two languages S and T and that S is mapping reducible to T.
   (As defined in Sipser, S mapping reducible to T means that there is a (total) computable
   function f such that for any string x, x \in A if and only if f(x) \in B.)
   i. Prove that if T is decidable then S is decidable. (5 points)
   ii. Prove that if T is recognizable then S is recognizable. (5 points)
   iii. Use ii. to show that the accepting language \text{A}_{TM} is NOT mapping reducible to \text{E}_{TM},
       where \text{E}_{TM} = \{ M \mid M \text{ is a TM and } L(M) = \emptyset \}
       (\text{E}_{TM} \text{ is defined in section 5.1 of the textbook.}) (5 points)

3. i. Given a graph with n vertices, how many 3 cliques can it have (at most) ? (3 points)
   ii. Show that L = \{ G \mid G \text{ is a graph containing a 3-clique } \} is in P. (3 points)
   iii. How about J = \{ G \mid G \text{ has n vertices and contains an n/2-clique} \} ? Can you use the
       same reasoning to show J is in P ? Why or why not ? (3 points)

4. Let COMP = the set of natural numbers that are not prime = \{ n \mid n \in \mathbb{N} \text{ and } n = ab \text{ where } a \text{ and } b \text{ are both natural number which are greater than 1 } \}
   Show that COMP is in NP. (5 points)

5. Assume that you know that the clique problem is NP-complete. Use this to prove that the
   independent set problem is NP-complete, where IS = the independent set problem = \{ G,k \mid G \text{ has an independent set of size } k \}
   Note: An independent set, is a set of vertices no two
   of which have an edge connecting them.
   Please do this carefully and with details. That is, first prove that the IS problem is in NP.
   Next state exactly which problem you are reducing to Which and why this show the IS
   problem is NP-complete. Finally give the reduction and justify (briefly) that it works.
   (10 points)