1. The vertex cover problem (VC) is defined below.

Show that the VC problem is in NP.

Vertex Cover Problem:
A vertex cover C of a graph G is a set of vertices of G where every edge of G contains at least
one vertex from C. The size of C is just the number of vertices in the set C.

VC = \{(G,k) \mid G is a graph and k is an integer and G has a vertex cover of size at most k\}

First state what you need to do to show VC is in NP. Then show it.

2. Show that NP is closed under intersection.

3. This problem concerns the question of whether NP is closed under complement, meaning
that if L is in NP is it also true that \bar{L} = the complement of L is in NP.

In class we showed that P is closed under complement. This was done by noting that Time
(n^k) is closed under complement for any integer k. And this followed from the fact that if we
simple reverse accept and reject for the output of a Time(n^k) algorithm we get a Time(n^k) for \bar{L} = complement of L.

Here is the same attempt to show that NP is closed under complement:

We show that NTime (n^k) is closed under complement for any integer k. This follows from the
fact that if we simple reverse accept and reject for the output of a NTime(n^k) algorithm we get a
NTime(n^k) for the complement of L.

This argument, which works for P and Time(n^k), does not work for NTime (n^k). Explain why.

4. Prove that \leq_m^P is transitive.
This means: Prove that if A \leq_m^P B and B \leq_m^P C then A \leq_m^P C.

5. (Question of whether NP is closed under the relation \leq_m^P)
Assume that A \leq_m^P B and B is in NP. Is it true that then A is in NP ?
If true, give a proof, if false, say why it is false.

6. Show that the problem max-cut, as defined in problem 7.25 on page 296 is in NP.
First state what you need to do to show this. Then show it.