1. A standard Turing Machine (TM) has an input alphabet $\Sigma$ and takes input strings $x$ which is a finite strings of symbols from $\Sigma$.

$M(x) =$ the result of TM M on input $x =$ either "accept" or "reject" or "loop". (Here loop means "never halt" and the TM M may either actually loop on a finite part of its tape or run forever moving farther and farther along on its tape.)

2. The TM M recognizes a language L defined by, $L = \{x | x$ is an input string of M and $M(x)$ accepts $\}$. Given a TM M, the language L is a subset of $\Sigma^*$ and is unique.

3. The language L is decidable if there is some TM M which recognizes L and which halts on every legal input string $x$ for M. In this case we say M accepts $x$ if $x \in L$ (i.e. ", x is in L"), and otherwise it rejects L.

4. A language is undecidable if there is no TM which decides it. A language is unrecognizable if there is no TM which recognizes it.

5. Recall that a TM that decides a language L also recognizes L. It follows from this that if L is unrecognizable then L is undecidable.

6. The language $A_{TM} = \{ < M, w > | M$ is a TM that accepts the input string $w \}$. $A_{TM}$ is is a language which is recognizable but $A_{TM}$ is not decidable. In fact, $A_{TM}$ is recognized by UTM, the universal Turing Machine.

From this it follows that the complement of $A_{TM} = \{ < M, w > | M$ is a TM and $w$ is a legal input string for M and M does not accept the input string $w \}$ in not even recognizable.