CAS CS 113. Problem Set 2

Due by 12:10pm Thursday, September 28, 2006, in the CS 113 drop box on the first floor of the CS department, or 12:30pm in class.

Problem 1. Prove by induction the sum of the geometric series formula:

$$\sum_{i=0}^{n} b^i = \frac{b^{n+1}-1}{b-1}$$

Problem 2. Prove by induction that the sum of the first n odd integers is n^2:

$$\sum_{i=1}^{n}(2i-1) = n^2$$

Problem 3. Prove by induction the sum of the first n squares formula:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Problem 4. Prove by (strong) induction that any postage greater than 4 cents can be posted exactly using only 2-cent and 5-cent stamps.

Problem 5. Consider a product of $n+1$ numbers: $x_0 x_1 \dots x_n$, and assume that parenthesis are inserted into the above expression in some arbitrary order. Prove by (strong) induction that computing the product requires n multiplications no matter how the parenthesis were inserted.

Problem 6. Consider the following simple game (similar to Nim) for two players: on the table there are two piles of matches, n matches in each. Each player in turn can remove any positive (> 0) number of matches from *one* pile of her choice. The player taking the last match from the table wins. Show that the player who moves second always wins.

Extra credit: Who wins if the rules are change so that the player taking the last match **looses**? Prove your answer.

Problem 7. Odometer principle: To find a successor to a particular odometer setting (representing a natural number) and using base b (that is assuming each odometer digit has b possible values — from 0 to $b-1$) Do the following
Start by considering the rightmost digit.

1. If the current digit is not $b-1$ (i.e., it is $< b-1$), then replace it by the next digit in order and stop;
2. If the current position is an empty (blank) space (i.e., to the left of all the digits), then write in it 1 and stop;
3. If it is neither (i.e., the current digit is $b-1$) then replace this current digit with the digit 0, move one place to the left, and return to 1.

To prove (by induction) that the Odometer Principle works and indeed gives you the representation $x_{n-1} \dots x_1 x_0$ for the natural number

$$X = x_{n-1} b^{n-1} + \ldots + x_1 b + x_0 = \sum_{i=0}^{n-1} x_i b^i$$

do the following:

(a) Prove the above for binary: $b = 2$

(b) Generalize the proof for any integer $b \geq 2$