# CS-113. Problem Set 7

*Due Tuesday, Dec.5.*

## Graphs

1) How many different

   a) directed
   b) undirected

   graphs are possible for a given set $V$ of $\boldsymbol{n}$ nodes?

   for example, for $V=\{a,b,c\}$, $E_1=\{ab, bc\}$ and $E_2=\{ab, ca\}$, consider $G_1=(V,E_1)$ and $G_2=(V,E_2)$ as different graphs.

2) How many different simple paths of edge-length $\boldsymbol{i}$ are there between given nodes $s$ and $t$ in a complete graph $\boldsymbol{K_n}$ with $\boldsymbol{n}$ nodes for a given $\boldsymbol{i}$?

3) How many different simple cycles of length $\boldsymbol{i}$ are there in a complete graph $\boldsymbol{K_n}$ with $\boldsymbol{n}$ nodes for a given $\boldsymbol{i}$?

4) How many different simple paths are there between given nodes $s$ and $t$ in a tree with $\boldsymbol{n}$ nodes?

5) How many different ways are there to 2-color a given tree of $\boldsymbol{n}$ nodes?

An undirected graph $G=(V,E)$ is **bipartite** iff $\exists V_1 \subseteq V \;\; \forall vw \in E \,.\, v \in V_1 \Leftrightarrow w \notin V_1$. Then we write $G=(V_1, V_2, E \subseteq V_1 \times V_2)$, where $V=V_1 \cup V_2$ and $V_1 \cap V_2 = \varnothing$.
If $|V_1|=\boldsymbol{n}$ and $|V_2|=\boldsymbol{m}$ and $E= V_1 \times V_2$ then we say the bipartite graph is complete and denote it as $\boldsymbol{K_{n,m}}$ .

6) How many different bipartite graphs $G=(V_1, V_2, E \subseteq V_1 \times V_2)$ are there for given $V_1$, $V_2$ ?

7) How many different simple paths of edge-length $\boldsymbol{i}$ are there between given nodes $s$ and $t$ in a complete bipartite graph $\boldsymbol{K_{n,m}}$ for a given $\boldsymbol{i}$?

8) How many different ways are there to 2-color a bipartite graph $G=(V_1, V_2, E \subseteq V_1 \times V_2)$?

9) How many different ways are there to 2-color a connected bipartite graph $G=(V_1, V_2, E \subseteq V_1 \times V_2)$?

10) Give a simple and efficient algorithm to 2-color a graph (or determine that it is not 2-colorable)? What is the complexity of your algorithm (in terms of big-Theta)? Explain why it does not seem easy to find a similar efficient algorithm for 3-coloring.