Pricing Differentiated Brokered Internet Services

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ABSTRACT

Price war, as an important factor in undercutting competitors and attracting customers, has spurred considerable work that analyzes such conflict situation. However, in most of these studies, quality of service (QoS), as an important decision-making criterion, has been neglected. Furthermore, with the rise of service-oriented architectures, where players may offer different levels of QoS for different prices, more studies are needed to examine the interaction among players within the service hierarchy. In this paper, we present a new approach to modeling price competition in service-oriented architectures, where there are multiple service levels. In our model, brokers, as the intermediaries between end-users and service providers, offer different QoS by adapting the service that they obtain from lower-level providers so as to match the demands of their clients to the services of providers. To maximize profit, players at each level, compete in a Bertrand game, while they offer different QoS. To maintain an oligopoly market, we then describe underlying dynamics which lead to a Bertrand game with price constraints at the providers' level. Numerical examples demonstrate the behavior of brokers and providers and the effect of price competition on their market shares.

Keywords: Service-oriented architecture, Quality of Service (QoS), oligopolistic competition, service differentiation, Bertrand competition, price constraints.

I. INTRODUCTION

In today's highly competitive internet service market, service providers, in order to survive, should offer their customers more flexibility in their quality-of-service (QoS) / price offerings to meet a variety of customer needs. Clearly, any successful solution for a service provider to stay in the market, not only depends on supporting new and updated technologies, but also involves economic aspects. However, pricing the services of the network, even without considering quality differentiation, is a challenging problem that involves different issues. There have been many studies that attempted to address these issues with or without considering differentiated QoS. Pricing approaches include Paris Metro pricing [12], congestion pricing [2, 4], rate-reliability pricing [7], and fairness pricing [6]. On the other hand, with the rise of service-oriented architectures, such as computational clouds and recursive networks [14], there is a need for more advanced solutions that manage the interactions among service providers at multiple levels. Brokers, as the intermediaries between clients and lower-level providers, play a key role in improving the efficiency of service-oriented structures by matching the demands of clients to the services of providers. They can downgrade or upgrade a service by sharing it among customers or by combining several services to satisfy customers' demand.

In this study we propose a multi-layer network market in which service brokers and service providers compete at different levels in an oligopoly to maximize their profit. In our setting, brokers can pay a cost to upgrade or downgrade the service that they buy from (lower-level) providers so as to offer a new service to the market (customers). The broker incurs costs when adapting a lower-level service as it expends resources to either enhance the service extended to its customers (*e.g.*, by employing delay jitter reduction or capacity allocation techniques over a best-effort service) or degrade it (*e.g.*, by multiplexing client demands over a guaranteed service). We consider the competition among providers and among brokers separately, while brokers impose some preference constraints on providers. We also consider conditions that may lead to a monopoly market and study how players act under such conditions. Our numerical results show that more service differentiation yields more profit for all players, while affecting only their market share. On the other hand, in the case of positive quality-price user utility, if profit increases for one player, profit decreases for the other player.

The rest of this paper is organized as follows. In the next section, we review related work. Then in Section III, we develop a game-theoretic model by characterizing the competitive behavior of players with one another at each level of the service hierarchy. Section IV presents our analysis and numerical results and section V concludes the paper.

II. RELATED WORK

Network economics has been an active research area in which pricing and regulating the market have been studied widely. However the exponential growth of internet services in hierarchical (*i.e.*, multi-layer) markets requires a deeper study of new market features that will become available.

Our work is inspired by Zhang $et\ al.$ [15] and Nagurney and Wolf [10]. They propose an economic model for the interaction and competition among service providers, network providers, and users. Both studies develop a two-stage game, where service providers compete in a Cournot game and network providers compete in a Bertrand game. In our framework, providers at all levels compete in Bertrand games (*i.e.*, competition on price). [15] studies a market with two service providers and two network providers offering the same level of service quality. However, [10] considers a market with more than two providers for service and network, in which network providers offer different levels of service quality. In their model, the market is managed through the demand-price functions, which depend on both quality and quantity. In the model we present here, we consider a market where at the providers' level, players can offer different qualities of service, while at the brokers' level, players can upgrade or downgrade the service to optimize their profit.

Semret $et\ al.$ [13] also consider a retail market where they have three kinds of players: one service provider, one broker and end users for each network. However, they develop a decentralized auction-based bandwidth pricing for differentiated internet services. They show that Progressive Second Price (PSP) provides stable pricing in the market, in which service providers receive most of the profits and the brokers' profit margin is small.

Pricing for single-level games has been studied widely. He and Walrand [5] consider a self-regulated service model, where market demand determines the service quality. In their model, there is one Internet Service Provider (ISP) offering two classes of service, with different prices to manage congestion. They show that if the price does not match the service quality, the system may end up in an equilibrium similar to the Prisoner's Dilemma game.

Li $et\ al.$ [8] and Fulp and Reeves [3] provide a traffic-sensitive pricing scheme for differentiated network services. While the focus of [3] is to maximize the profit of the service provider who buys a differentiated service connection from domain brokers and sells it to users, the goal of [8] is to provide economic incentives to the users so as to maintain a certain level of traffic load.

Ma, Baslam $et\ al.$ and Nagurney $et\ al.$ [1, 9, 11] present different game-theoretic models of a differentiated service market of users and service providers. [11] proposes a game-theoretic model where service providers compete with duration-based contracts for differentiated service. [1] considers a joint price-quality market with a Stackelberg game where providers are leaders and users are followers. In their model, providers consider the migration of users when setting their price and quality. [9] studies a congestion-prone market with usage-based pricing. They propose a model for users' preference over their value and sensitivity to congestion, and based on the model, characterize the market share and optimal price for providers. Our framework considers multi-layer differentiated service games where service obtained from the lower level can be upgraded or downgraded, and sold to the higher level. Our analysis also applies price constraints when players' optimal price would lead to losing their market share, and we devise how players should then update their price.

III. MODEL AND SOLUTION

In this section, we present our model and analysis of a two-level game configuration and focus on the competition among providers and brokers and what emerges as pricing of their services. Figure 1 illustrates the game-theoretic model: At the lower level, we have two service providers, while at the higher level, we have two service sellers or brokers that deal directly with users. We start by presenting our notation and some basic settings, then we discuss some analytical and numerical results.

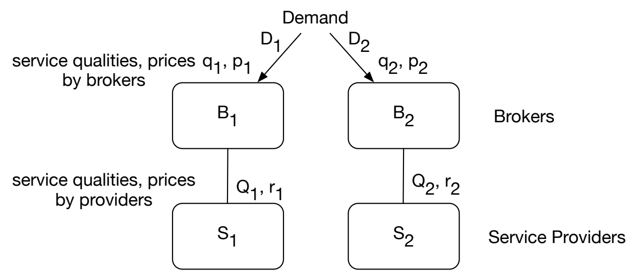

Fig. 1: Game-theoretic Model of a Two-level Competition.

A. Model Description

We consider a system with N customers, two service sellers (brokers), denoted by $B_i,\ i = 1, 2$, and two service providers, $S_i,\ i = 1, 2$. Customers have different preference for quality (utility) described by:

$$\theta q - p$$

where θ is the customer's marginal willingness to pay for quality q, and p is the price of service. There is a distribution of θ among customers. For simplicity, we assume that θ is uniformly distributed on an interval $\theta \in [\theta_{min}, \theta_{max}]$ and $\theta_{max} > 2\ \theta_{min}$. Customers are looking for a broker that maximizes their utility.

Both brokers and service providers can offer services with different qualities. The service quality offered by brokers is denoted by q_i and is in an interval $q \in [q_{min}, q_{max}]$, and the quality offered by service providers is denoted by Q_i. Also, we assume that brokers and service providers compete in an imperfectly competitive market. Furthermore, there is no supply constraint. We also assume that each broker buys just from one of the service providers that is more economical for the broker. Without loss of generality, we suppose that S_i is more economical for B_i (unless as we note in Section III-F, the market does not support that).

B. Demand Distribution

Brokers first choose the quality of service that they will provide to customers, then they compete on prices. If the brokers choose the same quality, then the customers decide only based on the price and this leads to a Bertrand competition with identical goods, whose prices should be set equal to costs, and no one makes profit. Thus the brokers should choose to offer different service qualities to make profits. We assume that $q_2 > q_1$, $p_2 > p_1$, and also $Q_2 > Q_1$. Therefore, customers with a high willingness to pay for quality will buy from B_2, while customers with a low willingness will buy from B_1. We can characterize the demand for each broker by identifying the customers who are indifferent between the two differentiated qualities. The indifferent customers, denoted by θ^*, satisfy:

$$\theta^* q_1 - p_1 = \theta^* q_2 - p_2 \Leftrightarrow \theta^* = \frac{p_2 - p_1}{q_2 - q_1} \tag{1}$$

Having uniformly distributed θ, the demand for each broker, B_1 and B_2, is given by:

$$\begin{aligned} D_1(p_1, p_2) &= \frac{\theta^* - \theta_{min}}{\Delta\theta} = \frac{1}{\Delta\theta}\left(\frac{p_2 - p_1}{q_2 - q_1} - \theta_{min}\right) \\ D_2(p_1, p_2) &= \frac{\theta_{max} - \theta^*}{\Delta\theta} = \frac{1}{\Delta\theta}\left(\theta_{max} - \frac{p_2 - p_1}{q_2 - q_1}\right) \end{aligned} \tag{2}$$

where $\Delta\theta \equiv \theta_{max} - \theta_{min}$.

C. Brokers' Profits

Now that we have the demand distribution, we can calculate broker i's profit, assuming that converting Q_i to q_i (whether to upgrade or downgrade the service) has a marginal cost c_i:

$$\Pi_i = p_i D_i - \frac{q_i D_i}{Q_i} r_i - c_i D_i (Q_i - q_i)^2 \tag{3}$$

where r_i is the price of service that broker B_i pays to service provider S_i and $\frac{q_i D_i}{Q_i}$ is the amount of service that B_i needs to buy to supply its own market. We assume here that the cost to the broker, c_i, for converting the service quality it is getting Q_i to that it offers its customers q_i, is proportional to the square of the difference in quality, $(Q_i - q_i)$. Intuitively, the cost increases more rapidly as the service quality increases, or alternatively, there is a diminishing return in service quality as more resources are allocated and cost increases. We henceforth, for simplicity, assume that $c_1 = c_2 = c$.

As we mentioned earlier, we assume that B_i buys service from S_i, *i.e.*, the broker with lower quality (B_1) buys from the lower quality provider (S_1) and the higher quality broker (B_2) buys from the higher quality provider (S_2). This is the only valid assumption to have an oligopoly market at the level of service providers. Otherwise, if B_i prefers to buy from S_j, then the cost of buying from S_j must be less than the cost of buying from S_i:

$$\frac{q_i}{Q_j} r_j + c(q_i - Q_j)^2 < \frac{q_i}{Q_i} r_i + c(q_i - Q_i)^2$$

Given $(q_i - Q_j)^2 > (q_i - Q_i)^2$, we have $\frac{r_j}{Q_j} < \frac{r_i}{Q_i}$. Thus, given $\frac{r_j}{Q_j} < \frac{r_i}{Q_i}$, and also $(q_j - Q_j)^2 < (q_j - Q_i)^2$, we deduce that it is less costly for broker B_j to buy from S_j as well, which results in a monopoly market at the level of service providers.

In the first stage, given the service prices r_i, and service qualities Q_i, the brokers compete in a Bertrand game with differentiated goods. Plugging Equation (2) into Equation (3), and solving $\partial \Pi_i / \partial p_i = 0$ for achieving Nash equilibrium, leads to:

$$\begin{aligned} p_1 = \frac{1}{3}\Big(& (q_2 - q_1)(\theta_{max} - 2\theta_{min}) + \frac{2 q_1 r_1}{Q_1} + \\ & \frac{q_2 r_2}{Q_2} + 2c(q_1 - Q_1)^2 + c(q_2 - Q_2)^2 \Big) \end{aligned} \tag{4}$$

$$\begin{aligned} p_2 = \frac{1}{3}\Big(& (q_2 - q_1)(2\theta_{max} - \theta_{min}) + \frac{q_1 r_1}{Q_1} + \\ & \frac{2 q_2 r_2}{Q_2} + c(q_1 - Q_1)^2 + 2c(q_2 - Q_2)^2 \Big) \end{aligned} \tag{5}$$

Now we have the brokers' prices, p_1 and p_2, as a function of the brokers' and providers' service qualities, and providers' prices r_i's. The next step is to plug them into D_i's to obtain:

$$\begin{aligned} D_1 = & \frac{1}{3\Delta\theta}(\theta_{max} - 2\theta_{min}) + \\ & \frac{\frac{q_2 r_2}{Q_2} - \frac{q_1 r_1}{Q_1} - c(q_1 - Q_1)^2 + c(q_2 - Q_2)^2}{3\Delta\theta(q_2 - q_1)} \end{aligned} \tag{6}$$

$$\begin{aligned} D_2 = & \frac{1}{3\Delta\theta}(2\theta_{max} - \theta_{min}) + \\ & \frac{\frac{q_1 r_1}{Q_1} - \frac{q_2 r_2}{Q_2} + c(q_1 - Q_1)^2 - c(q_2 - Q_2)^2}{3\Delta\theta(q_2 - q_1)} \end{aligned} \tag{7}$$

Now, D_1 and D_2 are dependent on service providers' prices r_i's. The next step is to find the optimal r_i's.

D. Providers' Profits

At this stage, we have the total demand served by (service sold by) each broker. To have an imperfectly competitive market at the level of service providers, the combination of their price and quality should be such that each broker prefers a different service provider. Assuming B_1 prefers S_1 and B_2 prefers S_2, the following inequalities should hold for B_1 and B_2, respectively:

$$\begin{aligned} \frac{q_1}{Q_1} r_1 + c(q_1 - Q_1)^2 &< \frac{q_1}{Q_2} r_2 + c(q_1 - Q_2)^2 \\ \frac{q_2}{Q_2} r_2 + c(q_2 - Q_2)^2 &< \frac{q_2}{Q_1} r_1 + c(q_2 - Q_1)^2 \end{aligned} \tag{8}$$

These constraints ensure that broker B_i chooses provider S_i as the cost is lower than that of getting service from the other provider S_j. We will discuss later the situation when one of these constraints is violated.

In this stage of the game, service providers compete in another Bertrand game. The profit of each provider is defined as:

$$U_i = \frac{D_i q_i}{Q_i}(r_i - k_i) - e Q_i^2$$

where eQ_i^2 is the cost of providing quality Q_i, r_i is the service price and k_i represents some general cost (fee). After plugging Equations (6) and (7) into the providers' profit, we obtain quadratic equations in r_i. To obtain the optimal solution (Nash equilibrium), we solve $\partial U_i / \partial r_i = 0$ which yields:

$$\begin{aligned} r_1 = & \frac{2k_1}{3} + \frac{k_2 q_2 Q_1}{3 q_1 Q_2} + \frac{Q_1}{3 q_1} \times \\ & \left[c(q_2 - Q_2)^2 - c(q_1 - Q_1)^2 - (q_1 - q_2)(4\theta_{max} - 5\theta_{min})\right] \end{aligned}$$

$$r_2 = \frac{2k_2}{3} + \frac{k_1 q_1 Q_2}{3q_2 Q_1} + \frac{Q_2}{3q_2}\times \big[c(q_1 - Q_1)^2 - c(q_2 - Q_2)^2 - (q_1 - q_2)(5\theta_{max} - 4\theta_{min})\big]$$

By substituting r_i's in Equations (4) and (5), we get the final values for p_i's only as functions of user preferences and service qualities:

$$p_1 = \frac{1}{9}\left(5c(q_1 - Q_1)^2 + 4c(q_2 - Q_2)^2\right) + \frac{4k_2 q_2 Q_1 + 5k_1 q_1 Q_2}{9Q_1 Q_2} + \frac{1}{9}(q_2 - q_1)(16\theta_{max} - 20\theta_{min})$$

$$p_2 = \frac{1}{9}\left(4c(q_1 - Q_1)^2 + 5c(q_2 - Q_2)^2\right) + \frac{5k_2 q_2 Q_1 + 4k_1 q_1 Q_2}{9Q_1 Q_2} + \frac{1}{9}(q_2 - q_1)(20\theta_{max} - 16\theta_{min})$$

We obtain the final values for D_i's from Equations (6) and (7):

$$D_1 = \frac{1}{9\Delta\theta}(4\theta_{max} - 5\theta_{min}) + \frac{c(q_1 - Q_1)^2 - c(q_2 - Q_2)^2}{9\Delta\theta(q_1 - q_2)} + \frac{-k_2 q_2 Q_1 + Q_2 k_1 q_1}{9\Delta\theta(q_1 - q_2)Q_1 Q_2}$$

$$D_2 = \frac{1}{9\Delta\theta}(5\theta_{max} - 4\theta_{min}) + \frac{c(q_2 - Q_2)^2 - c(q_1 - Q_1)^2}{9\Delta\theta(q_1 - q_2)} + \frac{k_2 q_2 Q_1 - Q_2 k_1 q_1}{9\Delta\theta(q_1 - q_2)Q_1 Q_2}$$

E. Positive Utility

In the previous setting we assumed that customers buy service either from B_1 or B_2 even if their utility is negative. Here we solve a game with only positive utility customers, *i.e.*, customers whose value of $\theta q - p$ is positive. Therefore, customers with zero utility provide a lower bound on θ which can be found by solving $\theta q_1 - p_1 = 0$. Thus θ_{min} is replaced by $\frac{p_1}{q_1}$:

$$\begin{aligned} D_1(p_1, p_2) &= \theta^* - \frac{p_1}{q_1} = \frac{p_2 - p_1}{q_2 - q_1} - \frac{p_1}{q_1} \\ D_2(p_1, p_2) &= \theta_{max} - \theta^* = \theta_{max} - \frac{p_2 - p_1}{q_2 - q_1} \end{aligned} \tag{9}$$

Like the previous setting, this is a two-stage Bertrand game, and the Nash equilibrium for each game is found by replacing the D_i's into the profit functions and solving $\partial\Pi_i/\partial p_i = 0$ and $\partial U_i/\partial r_i = 0$. We discuss the difference between this positive utility game and the previous (unconstrained utility) game later in Section IV.

F. Game with Constraints

At the lower level of service providers, the constraints (8) are not considered while the equilibrium points are calculated. Therefore, in some situations, one of the constraints might be violated. Let us assume that after finding r_i's, the constraint for B_i is violated, *i.e.*, $\frac{q_i}{Q_i} r_i + c(q_i - Q_i)^2 \geq \frac{q_i}{Q_j} r_j + c(q_i - Q_j)^2$. This means that, under this condition, for broker B_i, it incurs more cost to buy service from provider S_i than provider S_j; so if provider S_i does not change its price, B_i will get service from S_j, and this situation leads to a monopoly market at the providers' level.

To find an optimal point that also meets the constraints (8), provider S_i should set its price such that $r_i < \frac{Q_i}{q_i}(\frac{q_i}{Q_j} r_j + c(q_i - Q_j)^2 - c(q_i - Q_i)^2)$. In response, provider S_j updates its price by plugging r_i into $\partial U_j/\partial r_j = 0$ which leads to $r_j = f(r_i)$, *i.e.*, r_j as a function of r_i. Thus, S_i can replace the r_j with $f(r_i)$ in its inequality to calculate an optimal price that satisfies the constraint:

$$r_i = \frac{Q_i}{q_i}\left(\frac{q_i}{Q_j} f(r_i) + c(q_i - Q_j)^2 - c(q_i - Q_i)^2\right) - \epsilon \qquad \epsilon > 0$$

In this stage of the game, S_i should find a positive value for ϵ that maximizes its profit. By substituting r_i and r_j as functions of ϵ, U_i is a decreasing quadratic function of ϵ. Solving $\partial U_i/\partial\epsilon = 0$ results in optimal ϵ. If $\epsilon < 0$, it can be replaced with a small positive number close to zero. Since U_i is decreasing with respect to ϵ, any other positive value larger than the chosen ϵ leads to less profit. Clearly, the new set of prices for the service providers is an equilibrium point for the game, since it maximizes the revenue of both providers while meeting the constraints so each service provider does not lose its market (*i.e.*, one of the two brokers stays as its customer); therefore neither of the service providers has an incentive to change its price independently.

IV. Numerical Analysis

We present here some numerical results to illustrate the effect of choosing different qualities of service by both brokers and providers.

We consider a setting where $\theta_{max} = 1.5$, $\theta_{min} = 0.2$, $c = 0.1$, and $k_i = .01265 \times Q_i^{1.5}$. The service quality of the providers are set to $Q_1 = 20$ and $Q_2 = 45$. For the brokers, q_2 varies between 30 and 60, and we set q_1 to different values so it is less than, equal to, or larger than Q_1 to see how the market changes under different conditions. Examining a first set of plots shown in Figures 2 and 3, we note that the total demand constitutes the whole market. So, when the demand for one broker/provider side decreases, the demand for the other side increases and vice versa. But this is not the case for prices and profits – they increase or decrease together. Furthermore, observing the behavior for different values of q_1, we see from the brokers' and providers' price plots, when broker B_1 downgrades the lower-level service obtained from its provider S_1 (*i.e.*, $q_1 < Q_1$), all brokers and providers can offer their service at higher prices and make more profit. Similarly, by comparing the behavior for higher values of q_2, where $q_2 > Q_2$, with that for lower values where $q_2 < Q_2$, we observe that a better strategy for broker B_2 is to upgrade the lower-level service that it obtains from S_2 (*i.e.*, $q_2 > Q_2$). This happens because upgrading q_2 or downgrading q_1 leads to a larger gap between q_1 and q_2, therefore the two sets of broker and provider can offer more differentiated services at higher prices.

For $q_1 = 29$, the market exhibits abnormal behavior when the gap between q_1 and q_2 is small, while the gap between

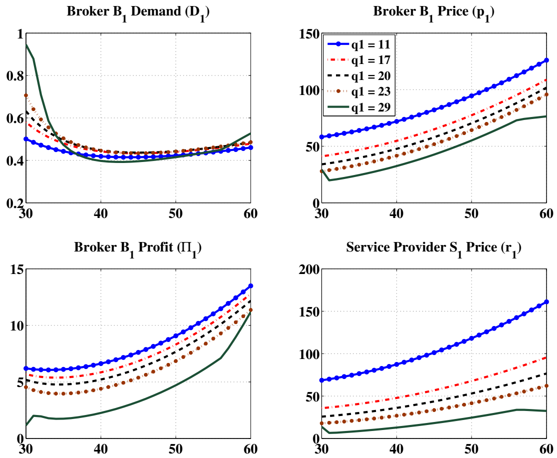

Fig. 2: Price, profit and demand distribution for broker B_1, $Q_1 = 20,\ Q_2 = 45,\ 30 \le q_2 \le 60$.

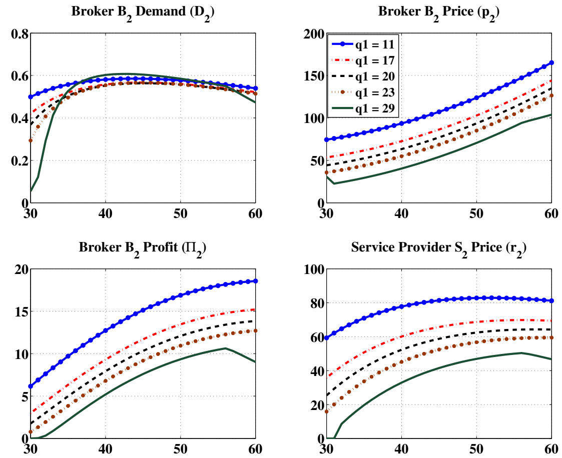

Fig. 3: Price, profit and demand distribution for broker B_2, $Q_1 = 20,\ Q_2 = 45,\ 30 \le q_2 \le 60$.

providers' qualities and brokers' qualities is large. Specifically, the market approaches a monopoly where S_2 and B_2 have a small market share when broker B_2 is downgrading the service obtained from its service provider S_2, *i.e.*, $q_2 < Q_2$. Observing the results when the values of q_2 are close to 30, we note that here, although the providers' game is a monopoly at some points where S_2's price $r_2 = 0$, the brokers' game is not, and B_2 can have a small share of the market D_2 while it gets service from provider S_1. This is because when the gap between q_1 and q_2 is not significant, most of the customers prefer the cheaper service provided by broker B_1. When the market is a monopoly, the provider or broker who remains in the market can increase its price to a value that the other competitor cannot enter the market even if it lowers its price to equal its cost, thus there is no way for the competitor to make profit and is prevented from entering the market.

On the other hand, for $q_1 = 29$, when broker B_2 is upgrading the service quality obtained from S_2, *i.e.*, $q_2 > Q_2$, as the gap between q_2 and Q_2 gets larger, S_2 starts to decrease its price to cover the cost of the quality upgrade for B_2 so as not to lose its market share. Since the value of q_1 is somewhere between Q_1 and Q_2, it is more economical for B_1 to buy service from S_2 rather than S_1 at the optimal prices, *i.e.*, the optimal price of S_1 violates constraints (8) and it should update its price r_1 as we explained in Section III-F. Consequently, S_2 should also update its price. Since there is a substantial gap between q_1 and q_2, both providers can compete in the market.

A. Positive Utility Results

We now consider the case of positive utility competition. Intuitively, we expect to see some restriction on the prices for all brokers and providers, otherwise they lose part of the market for which the utility $(\theta q - p)$ is negative. Thus it is a compromise between price and demand. The numerical results confirm this intuition. Comparing the prices of brokers and providers under positive utility and unconstrained utility, for the same conditions, shows that the highest prices under positive utility are below half of the prices in the latter case, while the demands are less as well; compare Figures 4 and 5 with Figures 2 and 3.

Also in this positive utility game, whether brokers upgrade or downgrade the service obtained from their providers, the behavior is different from that in the unconstrained utility game. Specifically, since the positive utility market is more sensitive to prices, a smaller gap between the service quality offered by the broker and the quality it gets from its provider yields more profit. Furthermore, while for both brokers, upgrading the service obtained from lower-level providers (and in turn, selling a higher quality service to customers) is generally more profitable, B_2 makes more profit when B_1 downgrades the obtained (lower-level) service, and B_1 makes slightly more profit when B_2 upgrades the obtained (lower-level) service.

Unlike the unconstrained utility game, if profit increases for one player, profit decreases for the other player. Another interesting observation from these plots is when the market is a monopoly: while there are conditions under which broker B_1 or B_2 can lose their market share, service provider S_1 can manage to stay in the market under all conditions.

V. Conclusion

In this paper, we developed a game-theoretic model that captures the interaction among players in a multi-level market. In our model, brokers, as the intermediaries between users and service providers, adapt the quality of the service that they get from lower-level providers so as to attract more customers and maximize their profit. The game consists of two service providers, two brokers, and users. Numerical results show that the more differentiation between the quality of service offered by brokers, the more profit they can make. An interesting result is that although players compete for more profit, the competition only affects their market share; the profit increases for one player if it increases for the other one.

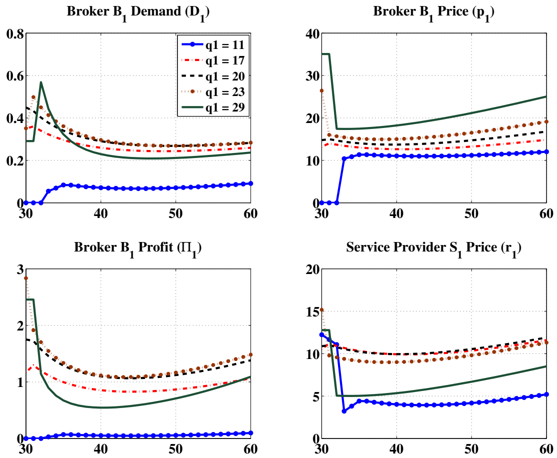

Fig. 4: Price, profit and demand distribution for the positive utility game, $Q_1 = 20,\ Q_2 = 45,\ 30 \le q_2 \le 60$.

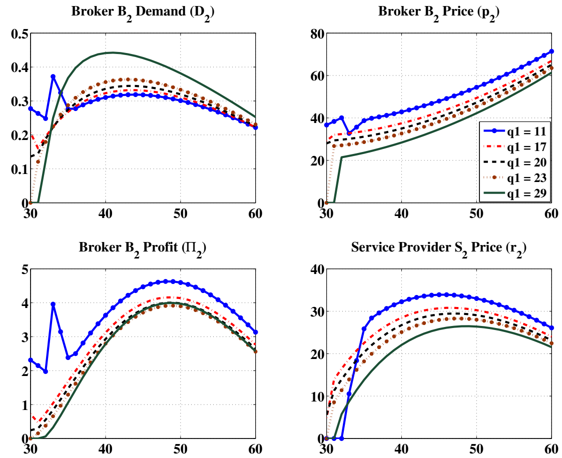

Fig. 5: Price, profit and demand distribution for the positive utility game, $Q_1 = 20,\ Q_2 = 45,\ 30 \le q_2 \le 60$.

We also studied situations where both brokers prefer to buy service from just one of the lower-level providers, *i.e.,* the providers' market is about to become a monopoly. We developed a Bertrand game with price constraints to keep the market as an oligopoly if possible. Moreover, we explored the case where customers buy the service only if the combination of price-quality has positive utility for them. In this situation, players try to offer the service cheaper to attract more customers. Unlike the unconstrained utility game, if profit increases for one player, profit decreases for the other player.

We plan to further extend our model to more brokers and providers, capture cooperation among some players (i.e., to pool resources to meet user demand) and study the resulting dynamics. We are also interested in considering more complex distribution of customer behavior and modeling a market where players maximize their profit by adjusting both price and offered service quality.

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