The Internet is ...

**HUGE**

- So HUGE that no one really knows how big
- But, rough estimates:
  - Users ~ 1.8B in 2009 (source: eTForecasts)
  - Web sites > 182M active in Oct 2008 (source: netcraft)
  - Web pages ~ 150B (source: Internet archive)

The Internet is ...

**DYNAMIC**

- Users log in and out
- New services get added
- Routing policies change
- Denial-of-Service (DoS) attacks
Motivation

- How to manage such a huge and highly dynamic structure like the Internet?
- How can we build Future networks?
  - Can’t build and hope they work
  - Understand the steady-state and dynamics of what we are building
- Need methodologies
  - Optimization Theory
  - Control Theory

Focus

- Congestion Control
- Adopt techniques from
  - Optimization Theory
  - Control Theory
- With emphasis on “Modeling”
- Prices
  - Congestion Prices
  - Exogenous Prices
    - non-load related, e.g. random wireless losses

An Optimization Theoretical Framework
Life ... involves daily decisions
Gas Prices are affecting these decisions

- Drivers will observe prices, decide
  - Drive
  - Walk
  - Bike
  - Stay home
  - Take the subway
  
Can still go to the movies 😊
If it is raining
Utility
- How much driving means to me compared to other things in life?
- Unknown to the gas stations
- Each driver has his/her own utility

Drivers, observe the gas price and drive the total demand
Market (OPEC + Government + Oil companies), based on demand, sets the prices

- System is in equilibrium if demand is balanced with supply

Users drive the demand on the network
Have different Utilities
- Download music, play games, make phone calls, deny service,...

Network, observes the demand, sets prices
Price as real money
- Smart Market [MV95], Paris metro [O97]

Price as a congestion measure
- Queuing Delay, packet loss or marking, additional resources to be allocated

What is the goal of Network Design? [S95]
Make users happy
Maximize the sum of Utilities for all users
Network users’ Utilities

- Users have different utilities, however
  - Higher the rate, the better
  - Decreasing marginal utility

- Formally: Elastic traffic [S95]
  - User $r$ has utility $U(x_r)$ when allocated $x_r > 0$ rate
  - $U(x_r)$ is an increasing function, strictly concave function of $x_r$
  - $U'(x_r)$ goes to $\infty$ as $x_r$ goes to 0
  - $U''(x_r)$ goes to 0 as $x_r$ goes to $\infty$

Network Model

- Consider a network of $J$ resources
- Consider $R$ the set of all possible routes
- Associate a route $r$ with each user
- Define a 0-1 routing matrix $A$ s.t.
  - $a_{jr} = 1$ if resource $j$ is on route $r$
  - $a_{jr} = 0$ otherwise
An Optimization Problem [K97]

\[
\begin{align*}
\text{SYSTEM}(U, A, C): \\
&\max \sum_{i=1}^{n} U_i(x_i) \\
\text{subject to} & \quad Ax < C \\
\text{over} & \quad x > 0.
\end{align*}
\]

- A (unique) solution exists
- However, utilities are unknown to the network

Introducing prices ...

- Break the problem into:
  - \( R \) different problems, a problem for each user
  - 1 Network problem

- Prices act as a mediator between the network and the users
  - Prices can be used to measure utilities
  - Users choose an amount to pay for the service
  - Network, based on the load, charges a price

User Maximization Problem

- Let user \( r \) pays \( w_r \) per unit time, to receive \( x_r \) proportional to \( w_r \):

\[
\lambda_r = \frac{w_r}{x_r} \left( \frac{\$/t}{b/t} = \$/b \right)
\]

- \( \lambda_r \) is the charge per unit flow

\[
\text{USER}_r(U_r; \lambda_r) : \\
\max U_r \left( \frac{w_r}{x_r} \right) - w_r \\
\text{over} & \quad w_r \geq 0.
\]
Let the network knows the vector $W$
Then the Network Maximization problem:

$$\text{NETWORK}(A, C; w):$$

$$\max x \sum_{r \in R} f(x_r, w_r)$$

subject to

$$Ax \leq C$$

over

$$x \geq 0.$$ 

A Greedy network choice

Indeed, for $w=1$, maximizes overall throughput
But, lacks traditional fairness concepts
Here is a simple example:

Fairness criterion depends on the function that
the network is optimizing for

Max-Min Fairness

Fair
- all sources get an equal share on every link
  provided they can use it

Efficient
- each link is utilized to the maximum load possible

150
150
150
(50, 50, 50, 100)
Fairness criterion (1/3)

- **Max-min Fairness**
  - No rate can increase, no matter how large, while decreasing another rate that is less than it, no matter how small
  - Absolute priority to small-rate users
- **X is proportionally fair if [K97]:**
  - Feasible $x \geq 0$ and $Ax \leq C$
  - For any other feasible vector $x^*$, the aggregate of proportional changes is zero or negative:
    $$\sum_{r \in R} \frac{x_r^* - x_r}{x_r} \leq 0.$$ 

Fairness criterion (2/3)

- **X is weighted proportional fair if**
  $$\sum_{r \in R} w_r \frac{x_r^* - x_r}{x_r} \leq 0.$$ 
  - A flow of $w=2$, is treated like 2 flows of $w=1$
  - Network would choose one of these
    - $\max_{r \in R} \min x_r \quad \Rightarrow \quad$ Max-min Fairness
    - $\max_{r \in R} \sum \log x_r \quad \Rightarrow \quad$ Rates are proportionally fair
    - $\max_{r \in R} \sum w_r \log x_r \quad \Rightarrow \quad$ Rates are weighted proportionally fair

Fairness criterion (3/3)

- **In our previous example**
  
  **Maximizing total throughput**
  - $0 \quad 6 \quad 6$
  - Proportional allocation ($w=1$)
    - $2 \quad 4 \quad 4$
  - Max-min allocation
    - $3 \quad 3 \quad 3$
  
  **General Parameterized Utility [MW00]**
  $$U(x) = \frac{x^{1-\alpha}}{1-\alpha}$$
  - $\alpha \rightarrow 0$ (linear utility)
  - $\alpha \rightarrow 1$ (log utility)
  - $\alpha \rightarrow \infty$ (min utility)
Kelly [K97,K99,KMT98]

\[
\text{NETWORK}(d, C; x) : \\
\max \sum_{r \in d} w_r \log x_r \\
\text{subject to} \\
\sum_{r \in d} x_r = \mu' (C - d) \\
\text{over} \\
x_r \geq 0.
\]

- Proof outline: Theory of constrained convex optimization and using Lagrange multipliers
  - \( \mu' \) = cost incurred or shadow price of additional capacity
  - \( \lambda \)'s in earlier slides
- A solution exists
  - \( x \) = weighted proportionally fair
  - Solves Network, User and System for log utility functions

Discussion

- Just to recap
  - Interested in maximizing the aggregate utilities
  - Network wouldn’t know the utilities
  - Broke the problem into users and one network problem
  - So, we introduced the vector \( W \) as a mediator
  - Shown that a solution exists
  - Fairness criterion depends on the network maximization function

Discussion

- But, we need to address few issues:
  - Network does not know \( W \)
    - Network implicitly determines \( W \) from the user’s behavior along its path, which is chosen by the network on behalf of the user
    - Or, Network puts an implicit weighting for relative utilities of different users
  - No central controller to know \( W \) and allocate rates

Look into individual controllers for the users and for the resources
Network Dynamics & Control Theory Preliminaries

System Modeling and Feedback Control
- TCP
- AQM
- TCP + RED

Control Problem
- The basic control problem: Control the output (results) for a given input

  ![Control System Diagram](image)

- Examples:
  - Price → User → Rate
  - Rates (Demand) → Resource → Prices

Questions to ask
- Steady state
  - What is the long range value of the output?
  - How far is it from the reference value?
- Transient Response
  - How does the system react to perturbations?
- Stability
  - Is this system stable?
- Stability Margins
  - How far is the system from being unstable?
Open-loop Control

- There is no feedback
  - Controlled directly by an input signal
- Simple
- Example: Microwave
  - Food will be heated for the duration specified
- Not as common as closed-loop control

Feedback (Closed-loop) Control

- Feedback control is more interesting ...
- Multiple controllers may be present in the same control loop

Feedback control makes it possible to control well even if
- We don’t know everything
- We make errors in estimation/modeling
- Things change
- Flow/congestion control example:
  - No need to EXACTLY know
    - Number of users
    - Connections’ arrival rate
    - Resource’s service rate
  - Continually measure & correct
Feedback (Closed-loop) Control

- Feedback delay is usually associated with feedback control.

- Feedback delay: Time taken from the generation of a control signal until the process reacts to it and this reaction takes effect at the resource and effect is observed by the user/controller.

- Feedback delay can compromise stability!!
  - The process may be reacting to some past condition that is no longer true.

System Models

- Deterministic vs. Stochastic
  - Are stochastic effects (noise, uncertainties) taken into account?

- Time-invariant vs. Time-varying
  - Do system parameters change over time?

- Continuous-time vs. Discrete-time
  - Is time divided into discrete-time steps?

- Linear vs. Non-linear
  - Do dynamic equations contain non-linear terms?

System Modeling

- Characterize the relationships among system variables as a function of time.

\[
\begin{align*}
  u(t) & \rightarrow \text{System} (x) \rightarrow y(t) \\
  \dot{x} & = f(x, u) \\
  y & = h(x, u)
\end{align*}
\]

- In general, f and h are nonlinear functions.
Instatations

TCP & RED
- One of the instantiations that received a lot of attention
  - Neither TCP nor RED [FJ93] was introduced from a control theoretic framework

TCP Modeling [K99]
- Think about an aggregate of m TCP flows, MuTCP [CO98]
- Congestion window changes:
  \[
  \frac{m \frac{(1-p)}{p} - \frac{cwnd}{2m} \frac{p}{p}}{T/cwnd} \times = \frac{cwnd}{T} \\
  \frac{d}{dt} x_r(t) = \frac{m_r}{T_r} - \left( \frac{m_r}{T_r^2} + \frac{x_r(t)^2}{2m_r} \right) p_r(t)
  \]
TCP Modeling [K99]

- Depending on the total traffic passing through a resource, a congestion signal is generated with probability:
  \[ p_r(t) = p_j \left( \sum_{i \in R} x_i(t) \right) \]

\[ \frac{d}{dt} x_r(t) = \frac{m_r}{T_r^2} - \left( \frac{m_r}{T_r^2} + \frac{x_r(t)^2}{2m_r} \right) p_r(t) \]

TCP-Reno Utility Function

\[ \frac{d}{dt} x_r(t) = \frac{m_r}{T_r^2} - \left( \frac{m_r}{T_r^2} + \frac{x_r(t)^2}{2m_r} \right) p_r(t) \]

- For \( m=1 \) and small \( p \), we have:
  \[ \frac{d}{dt} x(t) = \frac{1}{T} - \frac{x(t)^2}{2} - p \]
  \[ \frac{d}{dt} x(t) = x(t)^2 \left( \frac{2}{T^2 x(t)} - p \right) \]
  \[ U(x) = \frac{2}{T^2 x(t)} \quad U(x) = \frac{-2}{T^2} \]

E2E Congestion Avoidance

TCP Vegas

- End-to-end, dynamic window, implicit
- Expected throughput = \( \text{transmission_window_size/propagation_delay} \)
- Numerator: known
- Denominator: measure smallest RTT
- Also know actual throughput, measure it every RTT
- Difference = how much to reduce/increase rate
- New Congestion Avoidance Algorithm
  - \((\text{expected} - \text{actual}) \times \text{RTT packets in bottleneck buffer}\)
  - adjust sending rate linearly if this is too large or too small

- Generally loses to TCP Reno!
**TCP-Vegas Utility Function**

- At steady state:
  \[ x = \frac{\alpha D}{T_q} \]
  \[ T_q = \frac{b}{C}, \quad \hat{T}_q(t) = \frac{\dot{b}(t)}{C} = \frac{1}{C} (y(t) - C), T_q = \text{price} \]
  \[ \dot{x} = \frac{\alpha D}{x}, \quad U = \alpha D \log(x) \quad \text{WPF allocation} \]

**RED Modeling**

- Buffer evolution
  \[ \frac{d}{dt} b(t) = \sum x(t) - C \]
  - RED averaging
    \[ \dot{v}(t) = -3C(v(t) - b(t)) \]
  - RED marking
    \[ p_v(t) = \begin{cases} 
    6 & \text{if } v(t) - C \leq B_{\text{min}} \\
    0 & \text{if } B_{\text{min}} < v(t) - C < B_{\text{max}} \\
    1 & \text{if } v(t) - C \geq B_{\text{max}} 
    \end{cases} \]

**RED Pricing Function**

- Assume linear function of instantaneous queue length: \( p(t) = K q(t) \)
  \[ \dot{p}(t) = K \dot{q}(t) \]
  \[ \dot{q}(t) = y(t) - C \]
  \[ \dot{p}(t) = K (y(t) - C) \]
  - \( p = \text{Lagrangian multiplier (price)} \)
Nonlinear Models

- **Sources of nonlinearity**
  - Nonlinear components
    - Example: Rate Controlled MulTCP
      \[ \frac{d}{dt} p_c(t) = \frac{m_r}{2} - \left( \frac{m_l}{2} + \frac{s_c(t)}{2m_c} \right) p_c(t) \]
  - Different operating regions
    - Example: RED
      \[ p_c(t) = \begin{cases} \frac{a}{\rho(0) - c} \quad & v(t) \leq B_{max} \\ \frac{a}{B_{max} - v(t)} \quad & v(t) > B_{max} \end{cases} \]

- Hard Nonlinearities
- Soft Nonlinearities

Nonlinear Models

- Nonlinear control theory deals directly with nonlinear differential equations
  - Stability: Lyapunov functions
  - Transient Response: Numerical solutions

- Sometimes it gets very complicated

- Linearization: Process of transforming a nonlinear set of equations into a linear set of equations around a single point of operation

Linearization

- Concerned with local stability
- Assumes a single operating point
- Studies perturbations around this point
- Expands the nonlinear DE into Taylor series, then ignores high-order terms

\[ x = f(x, u) \quad \text{Linearization around } x_0 \]
\[ y = h(x, u) \]
\[ \dot{x} = Ax + Bu \quad y = Cx + Du \]
### Linear Models

- Once we have a Linear Model
  - Apply classical (first-course) control theory
- See Control Theory Primer slides & notes

### Linear vs. Nonlinear

- **Linear Control**
  - Rely on “small range of operation” assumption
  - Simple to use
  - Has a unique equilibrium point (if stable)
  - Satisfies the superposition property
- **Nonlinear Control**
  - Wide range of operation
  - Could be more complex to use
  - Multiple equilibrium points may exist
  - Most control systems are nonlinear

### Nonlinear Model of Sources’ and Network’s Adaptations

- **Kelly’s optimization framework**
  - Maximize users’ utilities subject to the network’s capacity constraints [K99]

\[
\frac{d}{dt} x_r(t) = \kappa \left( w_r - x_r(t) \sum_{I \in \mathcal{P}} p_I \left( \sum_{I \in \mathcal{P}} x_s(t) \right) \right)
\]

Additive Increase
Multiplicative Decrease
Steady-state and stability

- **Steady state**
  - Set the derivatives to 0, we get the steady-state point(s)
  - We have a single equilibrium point here

\[
\frac{dx(t)}{dt} = x(t) - \sum_{j \in C} \sum_{i \in S} p_{ji} x_i(t) \\
x_f = \frac{w_x}{\sum_{j \in C} p_{ji}}
\]

- **Stability**
  - Provided through a Lyapunov function

\[
\dot{u}(x) = \sum_{i \in S} w_{xi} \log x_i - \sum_{j \in C} C_j \left( \sum_{x_i \in S} x_i \right)
\]

Lyapunov

- Scalar function, strictly convergent
- Finding a function guarantees stability
- Not finding a function, doesn’t say anything
- Art to find one

\[
\frac{d}{dt} \dot{u}(x(t)) = \sum_{i \in S} \frac{d}{dt} \left( w_{xi} \log x_i - \sum_{j \in C} C_j \left( \sum_{x_i \in S} x_i \right) \right) \\
= \sum_{i \in S} \left( w_{xi} x_i(t) \right) - \sum_{j \in C} \sum_{x_i \in S} C_j \left( \sum_{x_i \in S} x_i \right) ^2 > 0
\]

(difficult road ahead…)

Difficult road ahead...

- Coming up with Lyapunov functions, even for simple models, is not easy
- As we move towards
  - More sophisticated models
  - Feedback delay
  - Different regions/aspects of TCP
  - Timeouts
  - Slow-start
  - Self-clocking
- Challenging environments
  - High bandwidth-delay product networks
  - Effect of exogenous losses (e.g., wireless)
- Accounting for different AQM at the resources
- Interference processes as in DoS attacks
- It gets harder very quickly
Linear Models

- Many sources of nonlinearity
  - Nonlinear components
    - Example:
      \[ \frac{d}{dt} x_i(t) = \frac{m_x}{2} \left( \frac{m_x}{2} + x_i(t) \right) \beta_i(t) \]
  - Different operating regions
    - Example: RED
      \[ p_i(t) = \begin{cases} 
        0 & s(t) < c \\
        \text{sgn}(s(t) - c) & B_{min} < s(t) < B_{max} \\
        1 & s(t) > B_{max} 
      \end{cases} \]
    - Need to study every point/region separately

Linearization

- Concerned with local stability
- Assumes a single operating point
  - Studies perturbations around this point
- Expands the nonlinear DE into Taylor series, then ignores high-order terms
  - Example: aggregating all sources and assuming one resource
    \[ \frac{d}{dt} x_i(t) = \left( \psi_i - x_i(t) \sum_{i=0}^{n} \sum_{j=0}^{m} x_j(t) \right) \]
    \[ \frac{d}{dt} s(t) = s(t) - x(t) \]
    \[ \frac{d}{dt} f(t) = -s(p + p') f(t), \quad f(t) = s(t) - x \]

Control Theoretic Analysis

- Linearized Model
  \[ \frac{d}{dt} f(t) = -s(p + p') f(t), \quad f(t) = s(t) - x \]
- Taking the Laplace Transform
  \[ sF(s) - f(0) = -s(p + p') F(s) \]
  \[ F(s) = \frac{f(0)}{s + s(p + p')} \]
- Stable if \( s = -s(p + p') < 0 \) (overdamped)
- For impulse perturbation, steady-state error = \( \lim_{s \to 0^+} sF(s) = 0 \)
How about feedback delay?

- What if the system has feedback delay \( T \) ?
- Use Nyquist stability criterion ...

Cauchy’s Principle

- \( Z \): number of zeros of \( F(s) \)
- \( P \): number of poles of \( F(s) \)
- \( N \): number of encirclements of origin
- For \( G(s)H(s) \), and contour around right-hand s-plane,
  - \( N \): encirclements around -1
  - \( P \): number of unstable poles of \( GH \)
  - \( Z \): number of unstable zeros of \( F \) = closed-loop poles
  - If \( P=0 \), and \( N=0 \), then \( Z=0 \) and system is stable

Nyquist Test

- What if the system has feedback delay \( T \) ?
- If the plot of the open-loop \( G(j\omega)H(j\omega) \) does not encircle the point -1 as \( \omega \) is varied from -inf to +inf, then the system is stable
- The number of unstable closed-loop poles \( (Z) \) is equal to the number of unstable open-loop poles \( (P) \) plus the number of encirclements \( (N) \) of the point \((-1, j0)\) of the Nyquist plot of \( GH \), that is: \( Z = P + N \)
Nyquist Test

- What if the system has feedback delay T?
- If the plot of the open-loop \( G(j\omega)H(j\omega) \) does not encircle the point -1 as \( \omega \) is varied from -\( \infty \) to +\( \infty \), then the system is stable
- Thus, we need to study the behavior of:
  \[ e^{-j\omega T} \]
  as \( \omega \) is varied
- Sufficient condition for stability:
  \[ \kappa T(p + p'x) < \pi / 2 \]

Routing is also a dynamical system!

- Link price functions reflect prices fed back to routing as the load on the links varies
- Convergence and stability can be proved using Lyapunov functions

Lyapunov for Routing

- Need to show that mapping function is contractive, i.e., range of function reduces
- Consider an adaptive routing system over two paths, with “N” total traffic, and fraction \( \alpha \) being re-routed based on path prices
- Find necessary condition for stability
- Show it is also sufficient
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