CS480/CS680 Problem Set 1

Due in class Tuesday, 4 October at the beginning of lecture.

Please prepare the answers to these questions, neatly written or typed, on separate paper.

1. (a) (4 points) Write a $4 \times 4$ homogeneous transform matrix $M$ that when applied to a point $(x, y, z, 1)$ yields $(x', y', z', w')$ where

   \[
   x' = \frac{2}{\sqrt{2}} x + \frac{2}{\sqrt{2}} z \\
   y' = -y + a \\
   z' = \frac{2}{\sqrt{2}} x - \frac{2}{\sqrt{2}} z \\
   w' = 1
   \]

   (b) (16 points) In words, what four basic computer graphics transforms occur when we apply $M$ to a 3D point? Give a homogeneous transform matrix for each, and show the order in which they are multiplied.

2. (30 points) Problem 5-13 in the Hearn and Baker text.

3. (30 points) You must use quaternions (see Eq. 5-106 in the text) in this problem. We first rotate a 3D point $p$ around line $l_1$ to obtain $p'$. We then rotate $p'$ around a different line $l_2$ to obtain $p''$. Assume that both lines pass through the origin. Prove that these rotations do not commute in general.

4. (20 points) We are given a seesaw shown below.

   The seesaw rocks (rotates) on its fulcrum at point $C$. The seesaw’s width is denoted $w$. If we apply a rotation of $\theta$ degrees at the seesaw’s fulcrum, the point $P$ moves to a new position, denoted $P'$. Derive the 2D homogeneous transform $M$, such that $P' = MP$.

5. (CS680 required 20 points, CS480 extra credit 10 points) We can specify a 2D rotation by considering the location of a small number of points before and after the points have undergone a pivot rotation. How many points are needed to specify the transform uniquely? Are there any special conditions that these points must satisfy? Give the mathematical expression for computing the 2D homogeneous pivot rotation matrix given such a set of points.