Variation of frequency with blowing pressure for an air-driven free reed

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Summary: In free reed instruments an approximately linear decrease of playing frequency with increasing blowing pressure is normally observed. Laboratory measurements on harmonium-type reeds from an American reed organ have shown additionally that at very low pressure there is a small region of increasing frequency with increasing blowing pressure, and at extremely high blowing pressures, the frequency of reed vibration increases rapidly with increasing pressure. Measurements of growth and damping rates confirm the previously reported result (4) that at low air flow rates aerodynamic forces add to the mechanical damping of the vibrating reed, but at higher flow rates, the aerodynamic forces contribute negative damping resulting in self-sustained oscillations. Incorporation of these values along with other appropriate parameters in Fletcher's model of reed vibration (1) permit theoretical calculation of the variation in frequency with pressure for the air-driven free reed. The results of these calculations agree well with the experimental data.

THE MODEL: EQUATIONS AND REED PARAMETERS

Simple linear models are often effective in describing to a good first approximation systems that are inherently nonlinear. In this paper an attempt is made to see if some of the essential characteristics of the behavior of the air-driven free reed can be derived as a consequence of the linear model for reed instruments formulated by Fletcher.(1) Particular attention will be given to harmonium-type reeds from American reed organs, since a substantial number of laboratory measurements have been made on individual reeds of this type, facilitating comparison with theory. An example of the type of reed studied is shown in Figure 1. It is typical for the American organ reed to have the curvature near the reed tip as shown. This curvature is purported to improve response as well as modify tone quality.

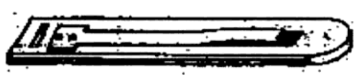

FIGURE 1. Example of a reed used in this study. Length of tongue is about 4 cm

Fletcher has given a general model for reed behavior in which the reed is modeled linearly as a harmonic oscillator and has obtained for this model an expression for the complex reed admittance Y_r. The equations for the amplitude and phase of Y_r in Fletcher's model are given below. The notation used here for the reed parameters is that used by Johnston in a paper on the harmonica.(2) A necessary condition for self-sustained oscillations is that Re[Y_r]<0. In the absence of a pipe resonator we expect the operating point to be at the frequency corresponding to the (negative) minimum of Re[Y_r]. Figure 2 shows a plot of Re[Y_r] vs. frequency from a calculation using Fletcher's equations and appropriate parameters for a C_3 reed from a reed organ. It can be noted that this curve resembles that for a "closing" (inward striking) reed, with the minimum of Re[Y_r] below the reed resonance frequency.

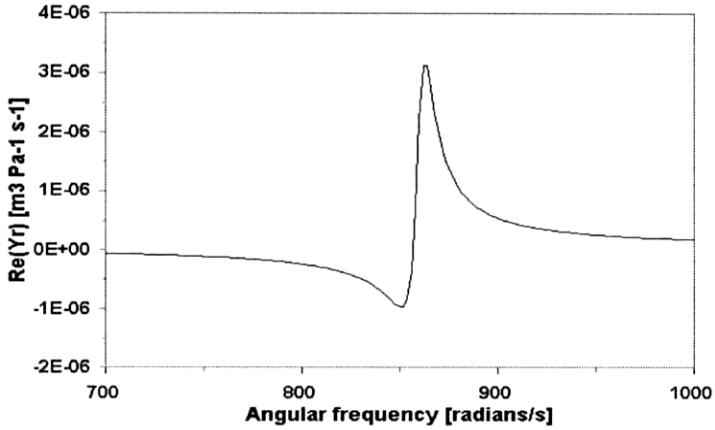

FIGURE 2. Re[Y_r] calculated as a function of frequency for the C_3 reed at P_0 = 0.6 kPa.

The equations are as follows:

$$Y_r = \frac{1 - B}{(P_0/zU_0)\cos\phi - \omega(R_0 a/bX_0)\sin\phi}$$

$$A = \frac{S_r(P_0 y/zX_0)\omega\omega_r D}{M_r[(\omega_r^2 - \omega^2) + (D\omega\omega_r)^2]}$$

$$B = \frac{S_r(P_0 y/zX_0)(\omega_r^2 - \omega^2)}{M_r[(\omega_r^2 - \omega^2) + (D\omega\omega_r)^2]}$$

$$\tan\phi = \frac{(P_0/zU_0)A + \omega[R_0(a/b)X_0(B-1)]}{(P_0/zU_0)(B-1) + \omega[R_0(a/b)X_0 A]}$$

The parameters used in the calculations are listed below, along with a brief description of the method of measurement or calculation and an indication in brackets of a typical value used:

P_0 = blowing pressure	varied as part of the calculation [0-3000 Pa]
U_0 = constant part of volume flow	varies: $U_0 = P_0^z$, with $z = 0.67$ (empirical)
ω_r = angular frequency of vibration	varied as part of the calculation [600-1000 s^{-1}]
ω_r = resonance frequency of reed	measured [800-900 s^{-1}]
a = effective length of reed	determined from $S_r = ab$
b = width of reed	measured width of reed tongue [4 mm]
X_u = unblown displacement of reed	estimated from measured values
X_o = equilibrium displacement of reed	$X_o = X_u - (S_r/M_r\omega_r^2)P_0$
M_r = effective mass of the reed	from ω_r and measured spring constant
$S_r = ab$ (effective area of the reed)	fitting empirical data with $X_o = X_u - (S_r/M_r\omega_r^2)P_0$
y, z = reed parameters	from measured pressure-volume relation
D = reed damping coefficient	varies: See Figure 4.
R_0 = density of air	

EXPERIMENTAL DATA AND CALCULATIONS FROM THE MODEL

It is well known to players and tuners that, under normal playing conditions, the frequency of oscillation of the reed decreases in an approximately linear fashion with increasing blowing pressure. Laboratory studies of individual reeds in recent years have shown that, although this is the pattern in the range of normal playing pressure, anomalous behavior occurs at extremely low and very high blowing pressure.(3) At the very lowest pressures at which the reed can be made to sound there is a small region of increasing frequency with increasing pressure. As the blowing pressure is increased well above normal playing levels the frequency levels off and eventually rises. A typical case is shown below in Figure 3. It will be shown that this characteristic pattern in frequency is predicted by the model, providing that aerodynamic forces are accounted for in the form of a damping/growth coefficient that varied with blowing pressure.

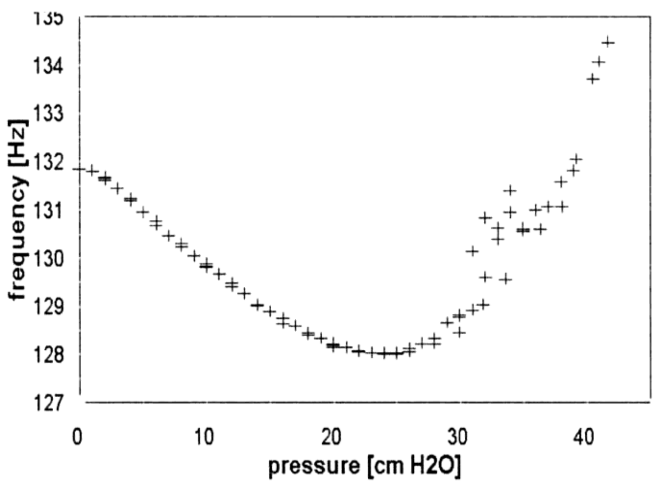

FIGURE 3. Frequency vs. blowing pressure for an organ C_3 reed (experimental data)

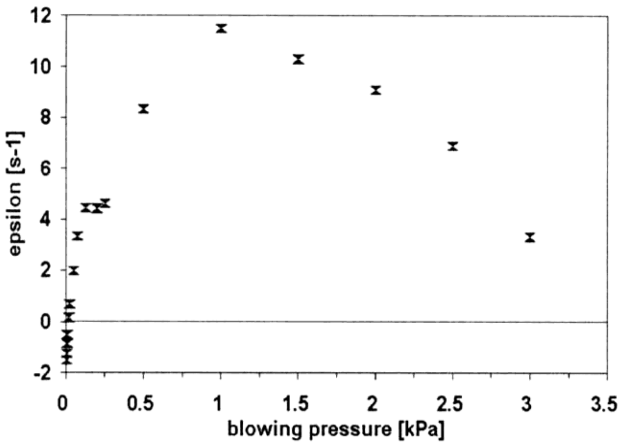

FIGURE 4. Experimental values of growth/damping coefficient as a function of blowing pressure (C_3 reed).

Realistic results can be obtained from the model if it is noted that, due to aerodynamic effects as discussed by St. Hilaire, *et al*, the reed damping/growth coefficient varies as a function of pressure.(4) This begins with negative values (damping) at low pressures, increases to maximum growth around 1.0-1.5 kPa, then gradually decreases at high pressures. Figure 4 shows experimental data from the C_3 reed. To simplify the computation, a quadratic approximation to this experimental pattern was used to obtain the calculated curve shown in Figure 5, which uses parameters close to those appropriate for the C_3 reed. The qualitative agreement with experiment is evident.

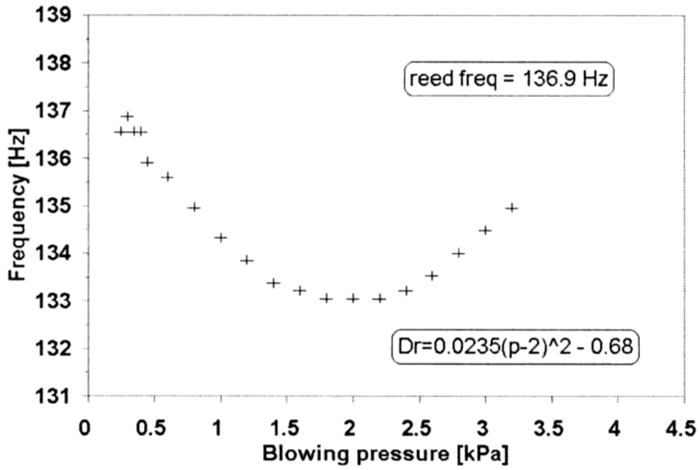

FIGURE 5. Frequency vs. pressure values calculated from the model for C_3 reed parameters.

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