Homework 1 Solution:

1. (a) Counting the regions, we get 8 regions. (Note that $8 = 2 \cdot 2 \cdot 2 = 2^3$.)

   (b) Counting the regions on a carefully drawn picture, we get 16 regions. (Note that $16 = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$.)

   (c) Since one point gives 1 region, two points gives $2 = 2 \cdot 1$ regions, three points gives $4 = 2 \cdot 2 = 2^2$ regions, four points gives $8 = 2 \cdot 4 = 2^3$ regions, the patern is that each additional point ends up giving twice the number of regions. As an experiment, we see that five points gives $16 = 2 \cdot 8 = 2^4$ regions. The pattern is if we have N dots then we will have 2 times itself $N - 1$ times or 2^{N-1} regions.

   (d) For six points, we get a surprise! If the points are arranged symmetrically then there are 30 regions and if the points are arranged asymmetircally (with different size arcs between them) we get 31 regions. In either case, we do not get $32 = 2 \cdot 16$ regions as we expected from the pattern.

   This problem is one reason the standard of acceptance for "facts" in mathematics is so high. A pattern can be dangerously deceiving, true for many cases, but not true in general. It is not until we have a proof, (that is, an explanation of why the statement must be true) that we can accept the statement as fact.

2. Person 1 says to show respect, person 2 must always be on time. Since there is one example of person 2 being late, person 2 is not always on time. One example is enough to show that a general rule is not ture.

   Person 2's response is just one example of being on time. One example never proves that a statement is always true (it just provides one example).

3. You can't really tell. If the teacher said the ONLY think students needed to do was all the homework to get an A, then perhaps, but that isn't stated by person 1.

   Person 2 doesn't say if the did all the homework, so again, insufficient information is available.

4. Person 2 is making a general statement based on examples–in this case only one example. While we need to learn from example (seeing one traffic accident on Commonwealth Avenue should be enough to make you extremely cautious), you should distinguish examples from proof! Proofs are arguments explaining why a statement must be true.