**Homework 2 Solution:**

Conjecture: A network that has an Euler path but does not have an Euler circuit must have exactly two nodes with odd degree.

Proof: Take a network that has an Euler path but no Euler circuit. Then the Euler path must start at some node and end at a different node.

Add an edge to the network with one end point at the ending node of the Euler path and the other at the starting node of the Euler path. The resulting network has an Euler circuit. We can see this by starting at the start node of the Euler path, following the Euler path which uses every edge of the original network exactly once. Once we reach the end node of the Euler path, we have used every edge of the network exactly once except the added edge. Now follow the added edge (using it exactly once) back to the start node. This is an Euler circuit.

Hence, our network with the added edge has an Euler circuit. By the theorem proven in class, this implies that every node must have even degree.

Finally, remove the added edge. We are back to the original network. We have reduced the degree of the start node and end node of the Euler path by one, so these two nodes (and only these two nodes) have odd degree in the original network and our proof is complete.

So our conjecture is now a Theorem.

Note: You could also build a proof by using an argument as in class for Euler circuits, but realizing that the start and end nodes of the Euler path have an extra leaving and arriving edges, respectively.