Homework Due IN CLASS Tuesday Oct 17.

USE THE EQUALLY LIKELY OUTCOMES MODEL FOR ALL PROBLEMS BELOW.

1. Suppose you flip a coin 3 times. The coin can not land on its edge, so can only be heads(H) or tails(T).

   (a) How many outcomes are in the sample space (list them and then think about more efficient ways to count them without listing!).

   Well, HHH, THH, HTH, TTH, HHT, THT, HTT, TTT is the list. Note that there are 2 ways for the first coin to land. Then 3 ways for the second coint to last (so 4 ways for the first two to land) then 2 ways for the third coin to land. So there are $2^3 = 8$ total outcomes in the sample space.

   (b) What is the probability of getting heads first then tails (so HT*)?

   Well, there are 2 outcomes that start HT so the probability is 2/8=1/4.

   (c) What is the probability of getting tails for the last two flipps (so *TT)?

   This is the same as the previous problem. There are 2 outcomes so 2/8=1/4.

   (d) Are these two events independent? (Note: Answering this question requires computation!)

   Well, the outcomes in both these events is HTT. Since there is just one, the probability is 1/8. However, the product of the probabilities is $(1/4)\cdot(1/4) = 1/16$, so these are NOT independent events.

2. Suppose you flip a coin 5 times. The coin can not land on its edge, so can only be heads(H) or tails(T).

   (a) How many outcomes are in the sample space (last time you did this question it was about fruit salad...)

   As above, each additional flip has 2 possible results no matter what the preceeding flips, so each additional flip multiplies the number of possible outcomes by 2. The total for five slips is $2^5 = 32$.

   (b) What is the probability of getting one H and one T in the first two flips (i.e., start with HT*** or TH***)?

   Well, HT*** has 8 outcomes and TH*** has 8 outcomes and these events are disjoint, so there are 16 outcomes including both cases. So the probability of this event is 16/32=1/2.

(c) What is the probability of having the last flip be H (i.e., ****H)??

Since the first 4 can be anything, there are $2^4 = 16$ possible outcomes and the probability is 16/32=1/2.

(d) Are the two events of above independent? (Note: Answering this question requires computation!)

Well, the outcomes in both are HT**H and TH**H. There are 4 of the first type and 4 of the second type (and they are all distinct) so there are 8 outcomes in total in both events. Hence the probability of the b and the c event is 8/32=1/4.

The probability of the b event is 1/2 while the probability of the c event is 1/2 and $(1/2) \cdot (1/2) = 1//4$. So the events are independent.

3. Suppose you flip a coin 10 times. The coin can not land on its edge, so can only be heads(H) or tails(T).

(a) How many outcomes are in the sample space?

As above, $2^{10} = 1024$ outcomes.

(b) What is the probability of the sequence HTHHTTHHHT?

This is just one outcome so the probability is 1/1024.

(c) If you repeat the experiment of flipping a coin ten times 10,000 times, (so 100,000 flips in all), about how many times do you expect to get the sequence HTHHTTHHHT?

You would expect to get this sequence about 10 times because probability gives long-term frequency and 10,000/1024 is about 10.

(d) Suppose you do just one experiment flipping a coin ten times and you get the sequence HHHHHTTTTT. What is the probability that you got the sequence HHHHHTTTTT? (Read this question carefully!)

The probability is one. You know you got the sequence, so there is no uncertainty!.

(e) Suppose a friend flips a coin ten times and gets HTHTHHTTHH. If your friend says to you "Wow, that was an incredible coincidence. Of all the sequences, I got HTHTHHTTHH. The probability of that sequence is tiny, there must have been something wrong with the coin because I got this incredibly unlikely sequence."

What is the problem with your friend's statement–in a couple of sentences, explain the error in their thinking.

You know something is going to happen...once you know what it is, the probability that it did happen is one (no uncertainty). It is only surprising when a low probability event is predicted and the event happens after the prediction in a single trial.

(f) People make the error about probability in the problem just above all the time...give an example that you have heard or experienced (or even made yourself!).

Happens all the time–almost whenever you hear the sentence "What a coincidence that..."