MA/CS 109 Lecture 4
So far....

We have answered the original question... There is no Euler circuit for the network given by the bridges of Konigsberg. (The degrees of the nodes for this network are all odd—never had a chance!).
Template for Doing Mathematics

Problem

Model---------Repeat-------------------|

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Examples/Conjectures

Proof---Did we answer the question?---No

Yes—Fame $$ $$
Next steps...

Change the problem and start on something new.

Since we are now SURE about the truth of our theorem, we can use it to prove more theorems...

An Euler path is a path the uses every edge of a network exactly once.
Note: Every Euler circuit is an Euler path, but an Euler path doesn’t have to be an Euler circuit...

New Question: Which networks have Euler paths?

Not so clear...We will use the same model (same terminology about networks), so let’s look at some examples and see if we can make a conjecture.
For example: The network below does Not have an Euler circuit (what are the degrees of the nodes?).

But it does have an Euler path...
Another example:

Conjecture: If a network does not have an Euler circuit, but does have an Euler path then it must have exactly 2 nodes of odd degree.
Proof?

We could try to do a proof with the same ideas as for Euler circuits...but maybe we can make our lives easier. We already know

Theorem: If a network has an Euler circuit then every node must have even degree.

Can we use this??
Suppose we have a network that has an Euler path that is NOT an Euler circuit—so the Euler path starts and ends at different nodes.

What can we do to make this into a network with an Euler circuit? (Remember, we are in our model world—we can do what we want!!).
In particular, we can add an edge! Why not, this is our world. If we add an edge from the start node to the end node of the Euler path, we get...
Try that again...

Add an edge to obtain a network with an Euler circuit....
Questions:

1. Can we always do this? Given a network that has an Euler path, can we always add an edge to obtain a network that has an Euler circuit? How do we do this?

2. Once we have added this edge, what do we know about the new network? (Remember the Theorem).

3. Now if we take the extra edge out, what do we know about the original network?
Problem: Prove our conjecture for Euler paths. 
(Remember—we are trying to prove a general statement, so do NOT refer to examples. Say why the conjecture is true in every case.)
What next??

We have been through the process of solving a problem in Mathematics...We know we were doing Mathematics because we proved our conjectures. That is, we gave arguments showing that our conjectures must be true in every case.

The Konigsberg bridge problem used to be “pure mathematics”—but networks appear in many applied fields, particularly Computer Science, so it is now an “applied mathematics” area.
But we do lots of other things with mathematics...

Next goal is to use the same ideas and “template” to consider a problem in population biology.

To consider this problem we need to think about “functions”.