

MA/CS 109 Lecture 1

What makes “Math” math?

What does it mean to say you are
“doing mathematics”

Template for Doing Mathematics

Problem

Template for Doing Mathematics

Problem: Always start by asking a question

The more interesting the question,
the more potential for an interesting
answer!...so we will start with
questions that are interesting

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Template for Doing Mathematics

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to us.

You should apply these ideas to what is
interesting to you.

Template for Doing Mathematics

Problem

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Model

Template for Doing Mathematics

Problem

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Model: Idealization of the world in which the problem lives. Model must be precise (so we know what we are talking about!). It should be as simple as possible, (life is hard enough!) and it is probably “abstract”, so there is hope of solving many problems at once.

Template for Doing Mathematics

Problem

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Model

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Examples/Conjectures

Template for Doing Mathematics

Problem

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Model

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Examples/Conjectures: This is what we do when study our problem (in the framework of the model). Scientists call examples “experiments” or “observation” and conjectures they call “theories”.

Template for Doing Mathematics

Problem

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Model

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Examples/Conjectures

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Proof

Template for Doing Mathematics

Problem

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Model

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Examples/Conjectures

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Proof: This step is unique to Mathematics—it is why Mathematics is so successful and useful and permanent.

Template for Doing Mathematics

Problem

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Model

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Examples/Conjectures

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Proof---Did we answer the question?

Template for Doing Mathematics

Problem

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Model-----Repeat-----|

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Modify

Examples/Conjectures

Model

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Proof---Did we answer the question?---No

Yes—Fame + \$\$\$

Let's see this process in action:

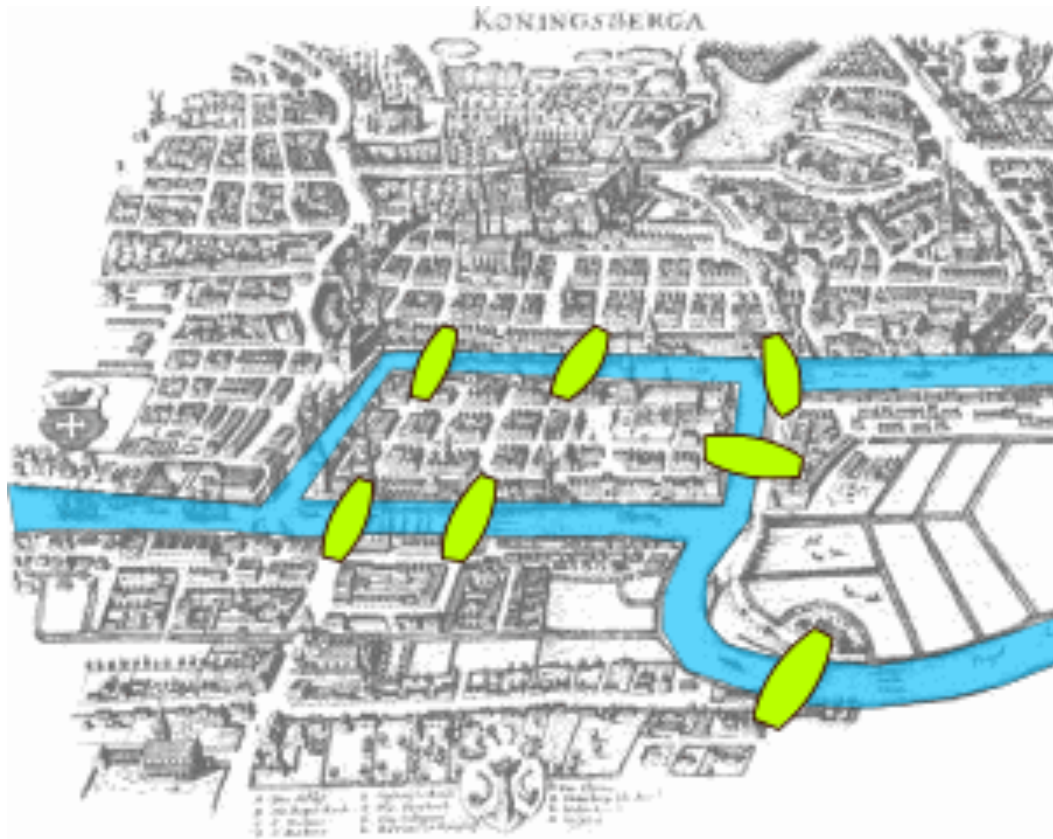
Problem: The 7 bridges of Königsberg.

The town of Königsberg is today called Kaliningrad, is now a (disconnected) part of Russia.

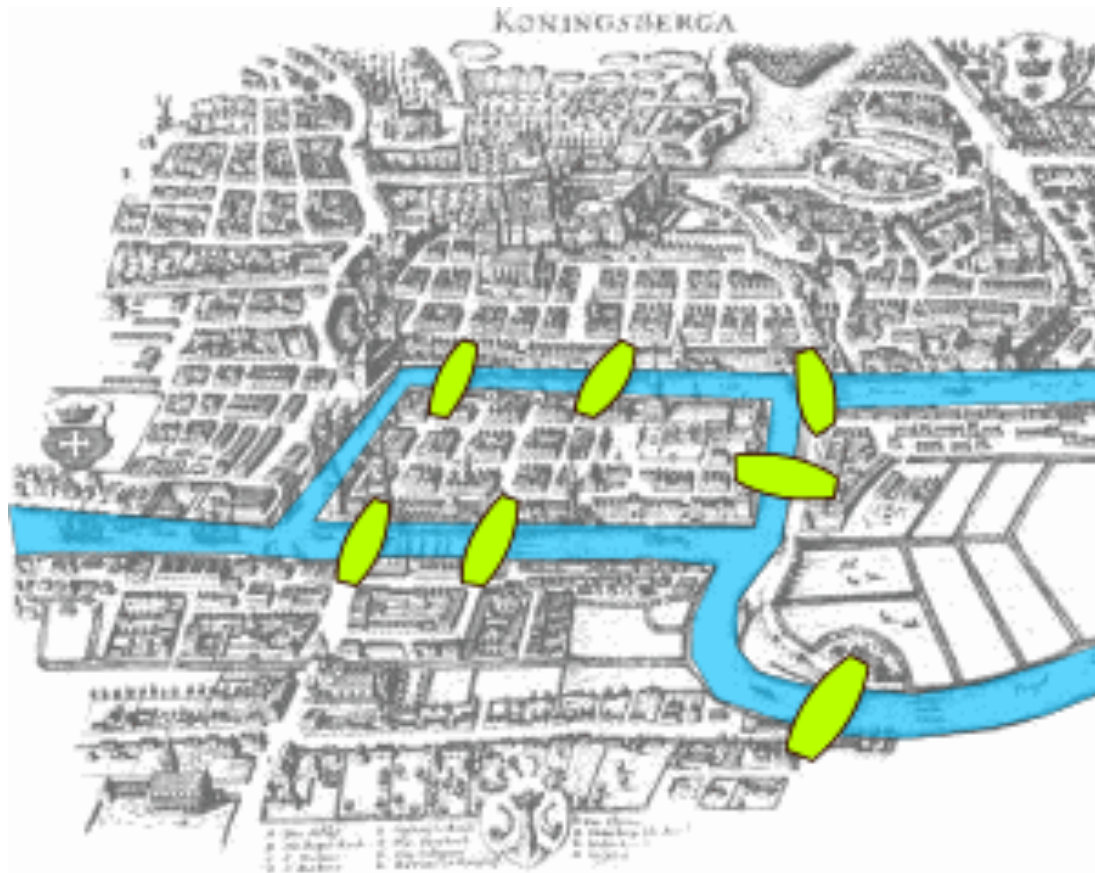
(map from BBC.com)



In the early 1700's, the story goes, the locals would amuse themselves on Sunday afternoons by trying to solve a puzzle. (Map from Wikipedia)



Is it possible to walk through the city, crossing each bridge exactly once and ending where you began?



Nobody succeeded...but that doesn't mean it couldn't be done. There are many possible paths(!) There are 38 choices for just the first 2 bridges crossed...

While it would be possible to list all the possible ways to attempt to find a path, this would take a long time and you would have to start over if a bridge was closed or if a new bridge was built.

Can we find a better way to think about the problem?

Template for Doing Mathematics

Problem: Interesting??

Well, maybe to the Konigsbergians...

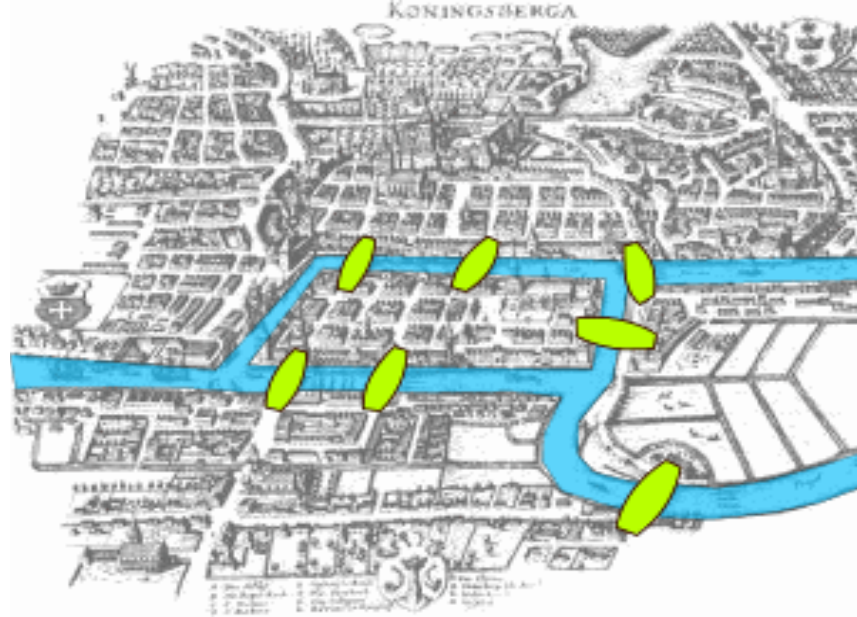
We'll see the ideas that started with this problem have many applications from Google searches to anti-terrorism.

Template for Doing Mathematics

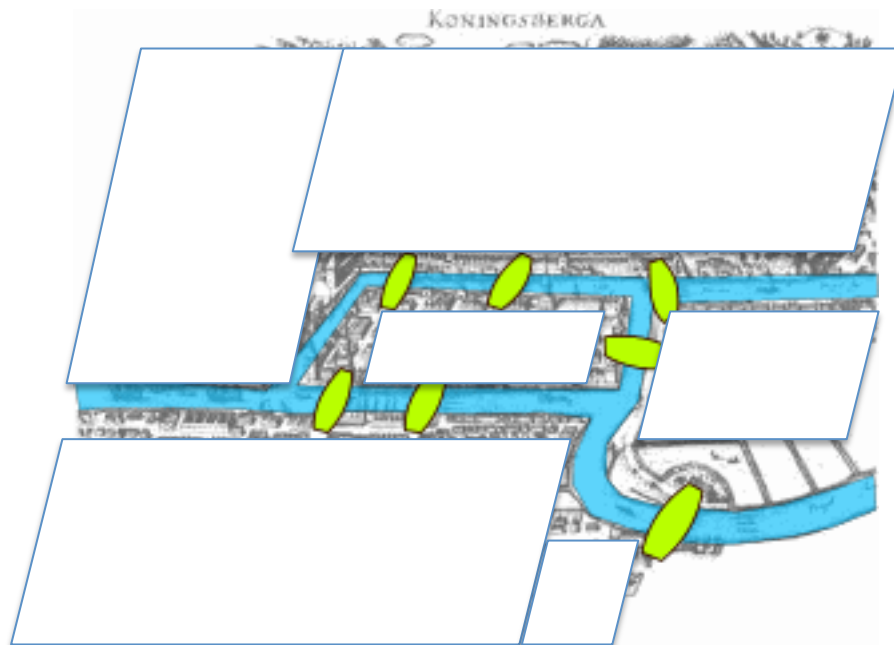
Problem

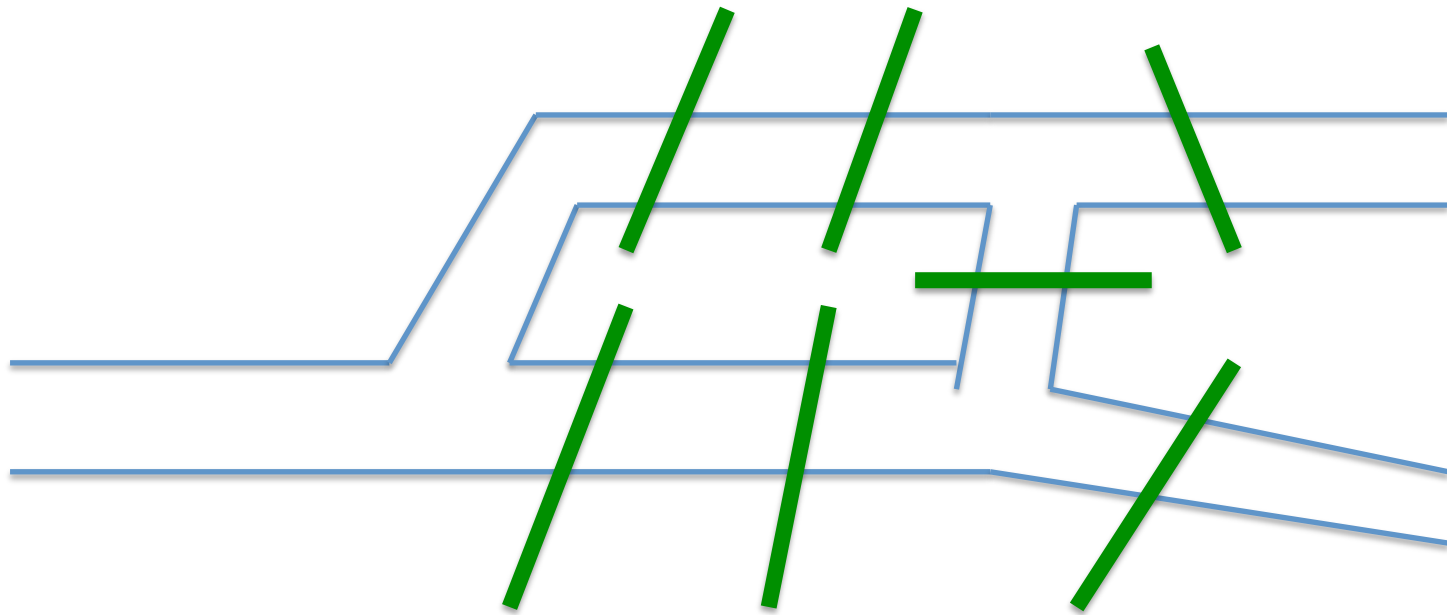
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Model: What are the essential features of this problem? What do we really need to talk about in order to state the problem?



Don't care about the building, only the brigdes.



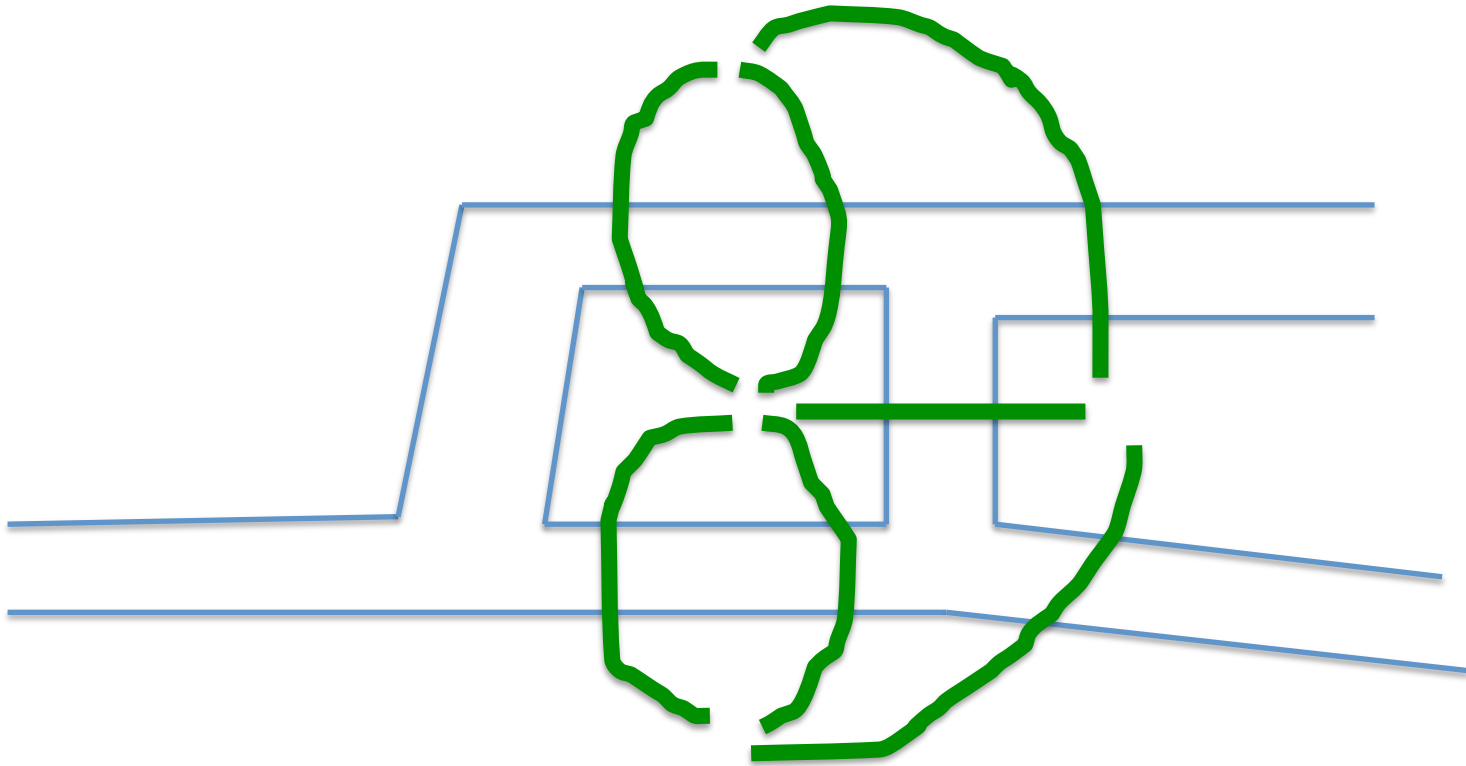


Doesn't matter how wide the bridges are or how curvy or how wide the river is.

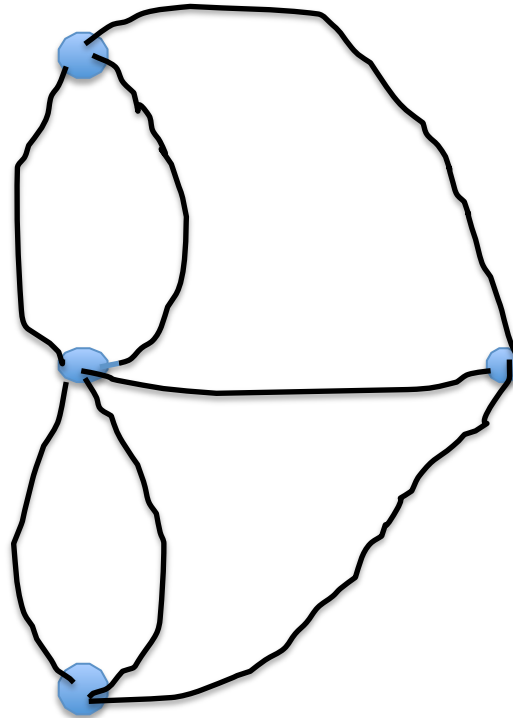




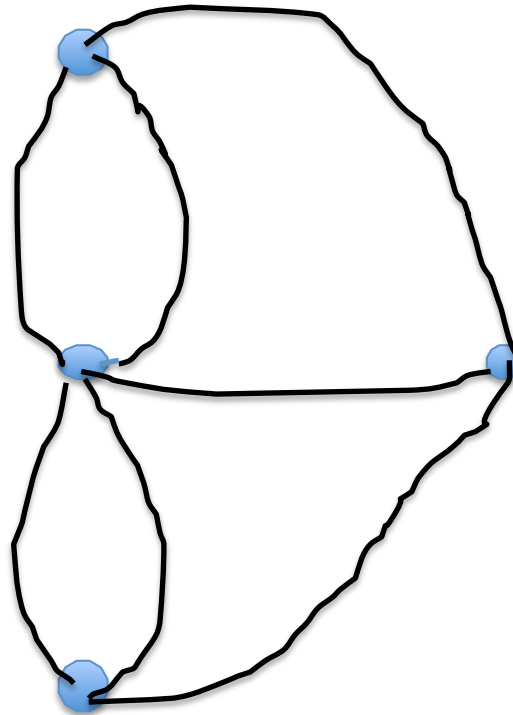
Where the bridges start and end doesn't matter (as long as it is on the same land mass).



Finally, the water doesn't matter...and how big the land masses are doesn't matter...and the color of the picture certainly doesn't matter...So all we really need to talk about the 7 bridges of Königsberg is the picture



That is, we need 4 dots (that correspond to the different pieces of land) and 7 curves connecting the dots (that correspond to the bridges).



Networks

A “network” is a collection of dots called “nodes”, and a collection of curves that start and end at nodes. The curves are called “edges”.

[NOTE: Nodes are sometimes called “vertices” and networks are sometimes called “graphs”—but we will use “Network” to avoid confusion with graphs with horizontal and vertical axes...]

Other Examples of Networks

Other Networks: Facebook friends

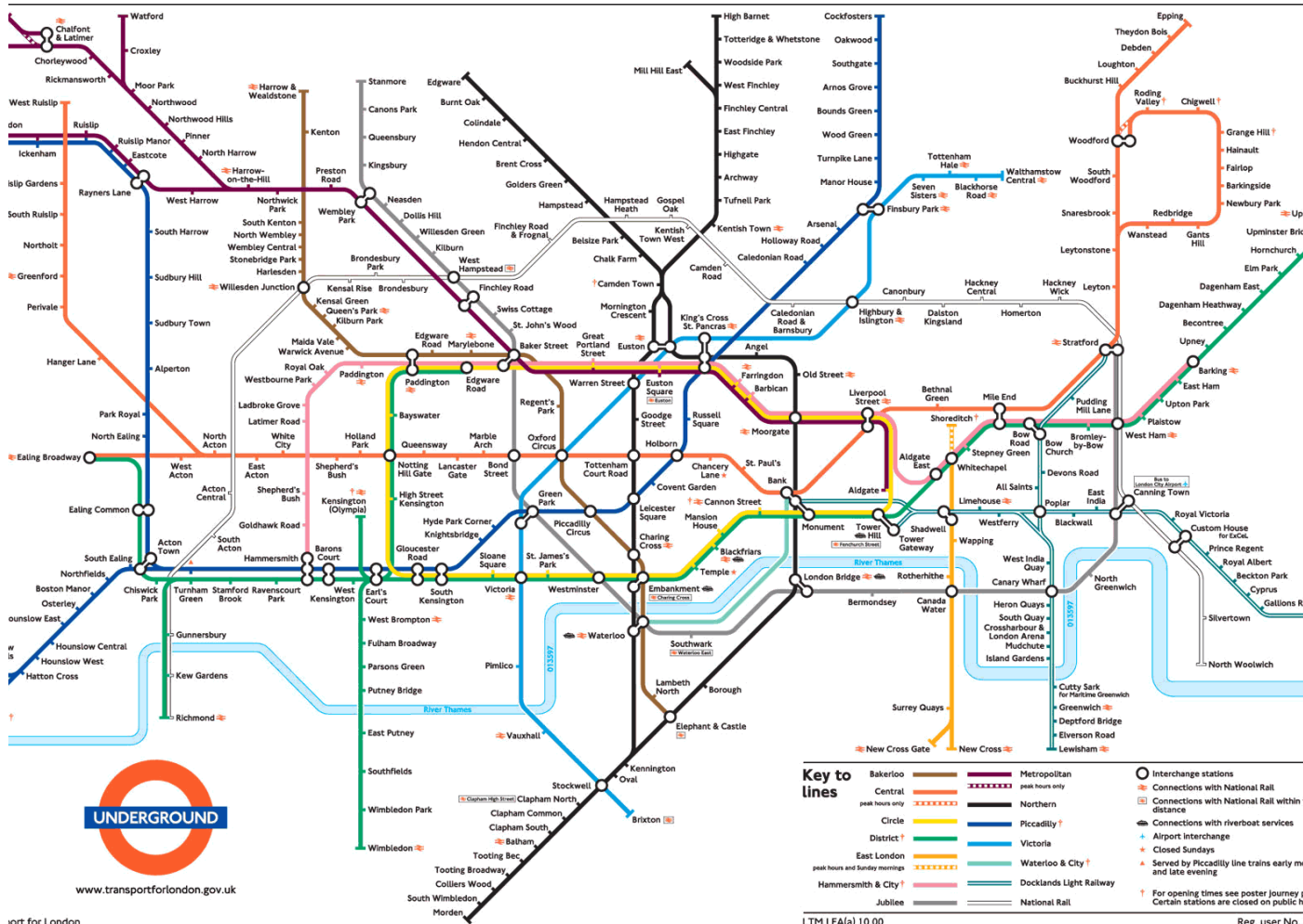
Image from <http://www.mcdougallinteractive.com/blog/social-media/facebook-graph-search-connections-matter/>

A marketing firm



London Subway Map

<http://worldofmap102.blogspot.com/2015/08/london-underground-map.html>



www.transportforlondon.gov.uk

ort for London

Key to lines

Bakerloo	Metropolitan	○ Interchange stations
Central	Northern	⊕ Connections with National Rail
Circle	Piccadilly †	⊞ Connections with National Rail within distance
District †	Victoria	⊞ Connections with riverboat services
East London	Waterloo & City †	✈ Airport interchange
Hammersmith & City †	Docklands Light Railway	⊞ Closed Sundays
Jubilee	National Rail	⊞ Served by Piccadilly line trains early mc and late evening

† For opening times see poster Journey 2
 ⊞ Certain stations are closed on public h

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Political relationships

Make a network representing “communication” in order to determine who are the “important nodes”...if you do this for the major players in the American Revolution, you see that Paul Revere was a key figure (not because of his ride, but because he wrote letters to members of otherwise disjoint groups in the rebellion)...