Problem 1

Give the decimal for the following binary numbers:

a) \(11011\) = \(16 + 8 + 2 + 1\) = 27
b) \(1111\) = \(8 + 4 + 2 + 1\) = 15
c) \(1001001\) = \(64 + 8 + 1\) = 73

Problem 2

Give the binary numbers for the following decimal numbers:

a) \(23\) = \(16 + 4 + 2 + 1\) = 10111
b) \(128\) = 10000000

c) \(72\) = \(64 + 8\) = 1001000

Problem 3

Add the following two binary numbers:

a) \(101101 + 010111\) = 1000100 (check: 45 + 23 = 68)
b) \(101010 + 111\) = 110001 (check: 42 + 7 = 49)
c) \(111111111 + 1\) = 1000000000 (check: 511 + 1 = 512)

Problem 4

What is the largest number that can be represented in 4 bits? How about in \(N\) bits, where \(N\) is an arbitrary number (come up with a general mathematical formula)?

4 bits can represent \(2^4 = 16\) possibilities, 0 – 15, so the largest number is \(1111 = 15\).

In general the largest number representable in \(N\) bits is \(2^N – 1\). This is just a sequence of \(N\) 1s.

We have studied in class the various units for computer storage. Answer the following questions about the correspondence between typical amounts of information produced by humans and computer memories.

You should use www.wolframalpha.com to do the calculations: it can evaluate very large numbers. For exponents use the notation “\(10^4\)” for “\(10^4\)” and use asterisk “\(*\)” for multiplication. Round to the nearest integer for smallish numbers and use scientific notation for large numbers.
For example, here is how you would find how many copies of the U.S. Constitution (with Amendments) you can store on a CD (it has 50,534 characters total):

So your answer would be 15,293.

**Problem 5**

There are about 3,566,480 characters in the Bible and 320,015 in the Koran.

A standard CD can hold about 737 MB (MB = $2^{20}$ bytes) of data. In comparison, a single-layer DVD-ROM can hold 4.7 GB (GB = $2^{30}$ bytes) of data.

a) How many copies of the Bible could you store on a typical CD, assuming one byte for each character?

$$\frac{737 \times 2^{20}}{3566480} = 216.$$  

b) How many copies of the Koran could you store on a typical CD, assuming two bytes for each character (since storing Arabic letters must be done in Unicode)?

$$\frac{737 \times 2^{20}}{(2 \times 320015)} = 1207.44.$$  

**Problem 6**

Currently you can buy on Amazon a Seagate 8 TB (TB = $2^{40}$) external disk drive for your Mac on for $199.99.
(a) The Library of Congress stores about $3 \times 10^{15}$ bytes of data. How much would it cost to store the whole of the Library of Congress on these disks? (Round to the nearest penny!)

$199.99 \times \frac{(3 \times 10^{15})}{(8 \times 2^{40})} = \$68,208.69.$

(b) Human DNA holds about 725 MB of data, and there are 7.6 billion people on Earth. How much would it cost to store all the data in human DNA on the planet on Seagate external disks?

That would be $725 \times 2^{20} \times 7.6 \times 10^9$ bytes to store. So it would cost

$199.99 \times \frac{(725 \times 2^{20} \times 7.6 \times 10^9)}{(8 \times 2^{40})} = \$131,362,068.65$ (ok to approximate)

Problem 7

If you are a sophomore, you will graduate BU (hopefully) in 2020, at which point it is predicted that there will be $44 \times 10^{21}$ bytes of data stored on this planet.

(a) A sheet of book paper is about 0.1 millimeters ($10^{-4}$ meters) thick. If all the data in the world in 2020 were printed on paper and stacked up, and assuming about 3000 characters each side (two sided) of a page, and two bytes per word (in Unicode, and forget about images and sounds, just pretend it is all text), and assuming the solar system (ending at Neptune) has a diameter of 9.09 billion kilometers, how many times larger would this stack of paper be than the diameter of the solar system?

Each two sided page can store $3000 \times 2 \times 2 = 12,000$ bytes, so each meter in the stack can store $12000 \times 10,000 = 120,000,000$ bytes.

Thus, the stack would be $(44 \times 10^{21} / 120000000)$ meters high, so the answer is:

$\frac{(44 \times 10^{21})}{(120000000)} / (9.09 \times 10^{12}) = 40.33.$

b) The Seagate disk drive mentioned above has dimensions $4.65'' \times 1.61'' \times 7.8''$, so it is $1.61''$ wide. If you stored all the data in 2020 in such drives, and lined them up side to side (each taking up 1.61 inches), how many times around the circumference of the earth ($24,901$ miles, with $5280$ feet per mile) would they go?

$\frac{(1.61 \times 44 \times 10^{21} / (8 \times 2^{40}))}{(24901 \times 5280 \times 12)} = 5.1$ times
c) A gram of DNA can hold \(215 \times 10^{15}\) bytes of data. How many kilograms of DNA would it take to store all the information on the planet in 2020? Would you be able to lift it?

DNA can hold \(215 \times 10^{15} \times 10^3 = 2.15 \times 10^{20}\) bytes of data per kilogram. So it would take \(44 \times 10^{21} / (2.15 \times 10^{20}) = 204\) KG of DNA. This is about 450 pounds, so no, you couldn’t lift it unless you are an Olympic weight lifter!

**Problem 8**

What would be the minimum bit depth (total number of bits) necessary to represent 512 colors? How about to represent 179 colors?

Since \(2^9 = 512\), then 9 bits would be necessary. We have \(2^7 = 128\) and \(2^8 = 256\), so you would need 8 bits for 179 colors (with some possibilities unused).

**Problem 9**

If you use 4 bits per color component and three components (red/green/blue), how many different colors can you represent?

There would be \(3 \times 4 = 12\) total bit depth with three colors, so \(2^{12} = 4096\) colors could be represented.

**Problem 10**

If a typical audio CD uses a sample rate of 44.1 KHz (KHz = 1000 / second) and stores each sample in 16 bits, how many hours and minutes of audio can be stored on a typical audio CD? Give your answer in whole number of hours and minutes (e.g. “1 hour and 34 minutes”).

16 bits is 2 bytes, so each second takes \(44100 \times 2 = 88200\) bytes, so a CD could hold \(737 \times 2^{20} / 88200 = 8,762\) seconds of music, which is 2 hours and 26 minutes (and 2 seconds, which we can ignore).