Bernoulli Distribution: Bern(p)

**Motivation:** The Bernoulli distribution describes the outcomes of a single trial with two outcomes, success or failure, where $p$ = probability of success.

**Definition:** If $1 = \text{success}$ and $0 = \text{failure}$, then $X \sim \text{Bern}(p)$ when

$\text{Rng}(X) = \{0, 1\}$

$f(k) = (1 - p)p$

$E(X) = p$

$\text{Var}(X) = (1 - p)p$

Binomial Distribution: B(N,p)

**Motivation:** The binomial distribution describes the number of successes which occur among $N$ independent Bernoulli trials.

**Definition:** $X \sim B(N, p)$ when

$\text{Rng}(X) = \{0, \ldots, N\}$

$f(k) = \binom{N}{k}p^k(1-p)^{N-k}$

$E(X) = Np$

$\text{Var}(X) = N(1-p)p$

Geometrical Distribution: G(p)

**Motivation:** This counts the number of Bernoulli trials until the first success occurs.

**Definition and Example:** $X \sim G(p)$ if

$\text{Rng}(X) = \{1, 2, \ldots\}$

$f(k) = (1-p)^{k-1}p$

$E(X) = \frac{1}{p}$

$\text{Var}(X) = \frac{1-p}{p^2}$

$P(X > k) = (1-p)^k$

$P(X \leq k) = 1 - (1-p)^k$

Negative Binomial Distribution: NB( r, p )

**Motivation:** This is a generalization of the Geometric: it counts the number of Bernoulli trials until the $r^{th}$ success occurs.

**Definition and Example:** $X \sim NB(r, p)$ if

$\text{Rng}(X) = \{r, r+1, \ldots\}$

$f(k) = \binom{k-1}{r-1}(1-p)^{r-p}p^r$

$E(X) = \frac{r}{p}$

$\text{Var}(X) = \frac{r(1-p)}{p^2}$

Poisson Distribution: Poi(λ)

**Motivation:** If we have a process in which events arrive (hence, called arrivals) independently through time, with some mean rate $λ = \#$ arrivals/unit time, then the Poisson characterizes how many arrivals will occur in a randomly chosen time unit.

**Definition:** $X \sim \text{Poi}(\lambda)$ if

$\text{Rng}(X) = \{0, 1, 2, \ldots\}$

$f(k) = \frac{e^{-\lambda} \lambda^k}{k!}$

$E(X) = \lambda$

$\text{Var}(X) = \lambda$

where $e = 2.71828183 \ldots$ (Euler's constant).

Continuous Uniform Distribution: U(a,b)

**Motivation:** The continuous uniform distribution equiprobably selects a real number in the range $[a, b]$.

**Definition:** $X \sim U(a, b)$ when

$\text{Rng}(X) = \{a, \ldots, b\}$

$f(k) = \frac{1}{b-a}$

$E(X) = \frac{a+b}{2}$

$\text{Var}(X) = \frac{(b-a)^2}{12}$

$P(X < x) = x-a$

Exponential Distribution: Exp(λ)

**Motivation:** If we have a process in which events arrive (hence, called arrivals) independently through time, with some mean rate $λ = \#$ arrivals/unit time, then the exponential characterizes the inter-arrival time, e.g., "how long until the next arrival?"

**Definition:** $X \sim \text{Exp}(\lambda)$ if

$\text{Rng}(X) = [0, \infty)$

$f(k) = \lambda e^{-\lambda t}$

$F(t) = 1 - e^{-\lambda t}$

$E(X) = \frac{1}{\lambda}$

$\text{Var}(X) = \frac{1}{\lambda^2}$

$P(X > t) = e^{-\lambda t}$

$P(X \leq t) = 1 - e^{-\lambda t}$

where $e = 2.71828183 \ldots$ (Euler's constant).