Problem One. How many committees of 3 men and 3 women can be formed from 7 men and 6 women if a particular man (e.g., Wayne) and a particular woman (e.g., Elizabeth) can NOT be on the committee together (i.e., if one of them is on the committee, that is fine, they just can’t be on it together).

Solution: Find the total number of committees, C(7,3) * C(6,3), and the subtract those where Wayne and Elizabeth are two of the members (effectively forming a committee of 2 men and 2 women from 6 men and 5 women), C(6,2)*C(5,2). Thus:

\[ C(7,3) * C(6,3) - C(6,2)*C(5,2) = 550. \]

Problem Two. A player tosses two fair coins. The player wins $3 if 2 heads occur and $1 if 1 head and 1 tail occur. For the game to be fair (i.e., the expected payoff is $0), how much should the player lose if no heads occur?

Solution: \( E(X) = 3 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} = \frac{3}{4} + \frac{1}{2} = \frac{5}{4} \). The player should lose $5, since then \( E(X) = 3 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + -5 \cdot \frac{1}{4} = \frac{3}{4} + \frac{1}{2} - \frac{5}{4} = 0 \).

Problem Three. Simultaneously and independently, each of \( N \) people make a single toss of a (biased) coin with probability \( p \) of a head. What is the probability of an "odd man"--that is, one person gets a different result from all the other \( N-1 \) people?

Solution: This is \( B(N, p) \); you an have either \( N-1 \) tails and 1 head, or 1 tail and \( N-1 \) heads, so the formula is

\[ C(N, 1)(1-p)(N-1)p + C(N, N-1)p(N-1)(1-p). \]

But \( C(N, 1) = C(N, N-1) = N \), so we have

\[ N \cdot [ (1-p)(N-1)p + p(N-1)(1-p) ] \]

Problem Four. Consider the following events for a family with children: \( A = \"the family has children of both sexes\" \) and \( B = \"the family has at most one boy.\" \) Show that \( A \) and \( B \) are independent events if the family has 3 children.

Solution: We must show that \( P(AB) = P(A)*P(B) \). The sample space is \{BBB, BBG, BGB, BGG, GBB, GGB, GBG, GGG\}, so \( A = \{ BBG, BGB, BGG, GBB, GGB, GBG, GGG\} \), \( B = \{ BGG, GBG, GGB, GGG\} \), and \( AB = \{ BGG, GBG, GGB\} \); thus \( P(A)*P(B) = \frac{6}{8} \cdot \frac{4}{8} = \frac{24}{64} = \frac{3}{8} = P(AB) \). Alternately, you could show that \( A|B = \{ BGG, GBG, GGB\} \), so \( P(A|B) = \frac{3}{4} = P(A) \), or that \( B|A = \{ BGG, GBG, GGB\} \), so \( P(B|A) = \frac{3}{6} = \frac{4}{8} = P(B) \).