Second Quiz Solutions – CS 237 – Fall 2015

The quiz was graded out of 15 points total, i.e., 5 points per question.

Problem One. Suppose you have 3 red balls and 2 black balls in an urn. You select 2 balls with replacement and count the number of red balls. Consider this to be a random variable X and give (a) the precise distribution, (b) Range(X), (c) p_X, and (d) E(X).

Solution:

(a) This is a binomial distribution B(2,0.6)

(b) Range(X) = { 0, 1, 2 }

(c) \( p_X = \{ 1 \times (2/5)^2 \times (3/5)^0, 2 \times (2/5)^1 \times (3/5)^1, 1 \times (2/5)^0 \times (3/5)^2 \} \)

= \{ 4/25, 12/25, 9/25 \}

= \{ 0.16, 0.48, 0.36 \}

(d) \( E(X) = N \times p = 2 \times 0.6 = 1.2 \)

Problem Two. A player tosses two fair coins. The player wins $3 if 2 heads occur and $1 if 1 head and 1 tail occur. For the game to be fair (i.e., the expected payoff is $0), how much should the player lose if no heads occur?

Solution:

\[ E(X) = 3 \times \frac{1}{4} + 1 \times \frac{1}{2} = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}. \]

We are asking for x such that \( \frac{3}{4} + \frac{1}{2} + x/4 = 0 \) or \( x/4 = -\frac{5}{4}, \) so \( x = -5. \)

So, the player should lose $5, since then

\[ E(X) = 3 \times \frac{1}{4} + 1 \times \frac{1}{2} + -5 \times \frac{1}{4} = \frac{3}{4} + \frac{1}{2} - \frac{5}{4} = 0. \]
Problem Three. A family has 5 children. Find the probability that there are fewer boys than girls. (Assume the probability of any particular child being a boy is 0.5.) You MUST do this using an appropriate distribution and the appropriate formula.

Solution:

If we assume that “success” is a girl, then this is B(5,0.5) and we are looking for

\[ P(X \geq 3) = \binom{5}{3} \times 0.5^3 \times 0.5^2 + \binom{5}{4} \times 0.5^4 \times 0.5 + \binom{5}{5} \times 0.5^5 \]

\[ = 10 \times 1/32 + 5 \times 1/32 + 1 \times 1/32 \]

\[ = 16/32 = 1/2 \]

This is intuitively obvious, because if we exchange “boys” and “girls” we don’t change the problem. However, the point was to do the calculation!