Cache-and-Relay Streaming Media Delivery for Asynchronous Clients*

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ABSTRACT

We consider the problem of delivering popular streaming media to a large number of asynchronous clients. We propose and evaluate a cache-and-relay end system multicast approach, whereby a client joining a multicast session caches the stream, and if needed, relays that stream to neighboring clients which may join the multicast session at some later time. This cache-and-relay approach is fully distributed, scalable, and efficient in terms of network link cost. In this paper we analytically derive bounds on the network link cost of our cache-and-relay approach, and we evaluate its performance under assumptions of limited client bandwidth and limited client cache capacity. When client bandwidth is limited, we show that although finding an optimal solution is NP-hard, a simple greedy algorithm performs surprisingly well in that it incurs network link cost that is very close to a theoretical lower bound. When client cache capacity is limited, we show that our cache-and-relay approach can still significantly reduce network link cost. We have evaluated our cache-and-relay approach using simulations over large, synthetic random networks, power-law degree networks, and small-world networks, as well as over large real router-level Internet maps.

1. INTRODUCTION

The delivery of streaming media objects presents a formidable strain on server and network capacity due to the long duration and high bandwidth requirement which are characteristic of streaming media workloads. For highly popular streaming media objects, it is especially desirable to utilize truly scalable delivery techniques. As such, multicast solutions are attractive. Multicast reduces both network link cost and server bandwidth requirement for serving a large number of clients. Previous studies along these lines have assumed that client requests are synchronous [9, 8], or that only server bandwidth [30, 18, 17, 23] is to be minimized. In this paper, we depart from both of these assumptions. Specifically, we consider the delivery of popular streaming media objects to a large number of *asynchronous* clients with the ultimate goal of minimizing the *network link cost* subject to various constraints on client bandwidth and storage capacity.

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1.1 Motivation

Client requests for streaming media objects are likely to be asynchronous. This is true for requests to stored streaming media objects (e.g., on-demand delivery of popular movie clips or news briefs to clients), as well as for requests to buffered live streams (e.g., playout of a webcast to a large number of clients requesting that webcast asynchronously but within a short interval).

To enable asynchronous access to streaming media objects, various periodic broadcasting and stream merging techniques [30, 18, 17, 23] have been proposed. Using these techniques, scalability in terms of network link cost is assured by virtue of multicast messaging, whereas scalability in terms of server bandwidth requirement is achieved by ensuring that a relatively small number of multicast sessions (possibly coupled with short unicast sessions) are enough to cater to a large number of asynchronous client requests.

Periodic broadcasting and stream merging techniques assume the availability of a network infrastructure that is supportive of multicast delivery—IP multicast, for example. While such an assumption may be practical within the boundary of a multicast-enabled intranet, it is not a viable alternative in today's Internet. This realization has led to a large body of work on application layer (or end system) approaches. However, existing end system multicast solutions [9, 8] have focused on synchronous real-time delivery—i.e., all clients receive the same content at the same time. As such, these techniques can not be used to service asynchronous clients.

There are several requirements for asynchronous streaming delivery.

- *Scalability:* Consider a large number of clients requesting a streaming media object—for example a popular video or news clip—at different times. Also assume that clients require immediate on-demand service of the objects. For such applications, one requirement of any acceptable delivery solution is scalability: even when the number of concurrent clients is high, there should not be a single service bottleneck in the system, and the overall overhead should be kept as low as possible. Clearly, unicast service is not scalable since it consumes server bandwidth and network resources too fast.
- *Deployability:* As we discussed earlier, existing multicast-based broadcasting and stream merging techniques minimize server bandwidth requirement and assume that low-level multicast support is available to mitigate network link cost. This assumption restricts the usefulness of these techniques to IP-multicast-enabled networks, and thus makes them of limited value for Internet deployment. Therefore, the second desirable property of any acceptable delivery solution is deployability.
- *Client heterogeneity:* Clients (or client-side proxies) can be

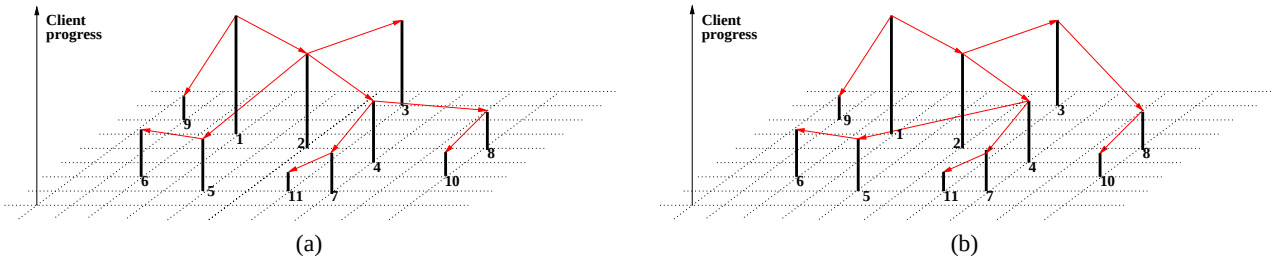


Figure 1: Illustrations of the cache-and-relay approach. Earlier clients temporarily keep the objects and relay them to later clients. (a) Scenario with unconstrained clients. (b) Scenario with bandwidth-constrained clients, whereby client 2 is limited to receive and send at most three streams.

heterogeneous. For example, they may reside at different parts of the Internet and may have very different connection speeds and may command different cache capacities. It is desirable that different clients be able to join the system. Therefore, the third property of any acceptable delivery solution is that it would tolerate client heterogeneity.

1.2 Paper Contributions and Overview

In this paper, we propose and evaluate a scalable "cache-and-relay" end system multicast protocol for the asynchronous delivery of streaming media objects. Unlike existing periodic broadcasting and stream merging techniques, our protocol relies only on unicast messaging, and unlike existing end system multicast techniques, our protocol supports asynchronous delivery. Using our approach, upon joining an ongoing end system multicast session, a client caches the stream either partially or entirely, and if needed, relays that stream to neighboring clients which join the multicast session at some later time. The paper mainly analyzes the network link cost of this approach, studies the effect of limited client-side bandwidth and limited client cache capacity, and uses simulations to validate our findings.

The remainder of this paper is organized as follows. In Section 2, we describe the cache-and-relay approach. In Section 3, we derive the network link cost of the cache-and-relay approach and present its specific instances under assumptions of limited client bandwidth and limited client cache capacity. In Section 4, we evaluate our approach using simulations over large synthetic random networks, synthetic power-law degree networks, and synthetic small-world networks, as well as over large real router-level Internet maps. In Section 5 we present an overview of related works. In Section 6, we conclude the paper with a summary of findings.

2. THE CACHE-AND-RELAY APPROACH

Under the cache-and-relay approach, the network (including routers) does not support multicast functionalities. Instead, end-hosts are responsible for the caching and distribution of streaming media. Here, end-hosts can be client machines or proxies thereof. End-hosts keep retrieved media objects in their local caches temporarily, as the results of client requests. If another client requests the media objects later, the original server can redirect the request to those end-hosts who are geographically closer to the client.

We illustrate our cache-and-relay approach using the example in Figure 1.1(a). In that example, there are 11 clients requesting an object. These clients are placed on a two-dimensional grid to visualize network "distance" between these clients. Clients arrive at different times. In Figure 1.1(a), each client is marked with a number denoting the order of its arrival. Also, the value of the z-axis for a given client indicates the progress of the playout for that client. The figure shows how an object is forwarded from "earlier clients" to "later clients". Clearly, such an approach assumes that clients (or client-side caching proxies) have enough cache space to temporarily keep the received media objects either partially or entirely. Also, it assumes that a non-leaf client, while receiving and playing out an object, has additional bandwidth to forward that object to (one or more) neighboring clients who may arrive later.

Without loss of generality,[1] we assume that the objective of our cache-and-relay approach is to minimize the total network link cost, or hop-distance. It is not difficult to establish that the cache-and-relay solution shown in Figure 1.1(a) is optimal when client-side bandwidth and client cache capacity are unlimited. Each client receives the object from the nearest on-going peer. The total hop-distance is 28.

Despite the seeming simplicity of this approach, there are several issues that need to be addressed in order for an implementation of this approach to be practical. We discuss these below.

- One requirement of our cache-and-relay approach is the need for an effective discovery mechanism for close-by peers who can satisfy a request—that is, how to find the closest client with a cached copy of the requested stream and with sufficient bandwidth to serve that stream. One simple peer discovery mechanism works as follows. The server maintains a set of IP address of on-going clients. When a new client requests the media object, the server provides a subset of candidates (from its list of on-going clients) based on efficient clustering techniques, for example, the one in [22]. The new client may then choose one of these candidates based on measurements of the characteristics of its paths to those candidates.
- Another concern regarding our cache-and-relay approach is reliability. For example, if one client is relaying some media object to a another client, then if the first client dies, the latter one needs to figure out how and from where to receive the remainder of the object. One solution for this vulnerability is for the latter client to contact the original server, and establish a connection with another client. In the worst case, the client may have to download the remainder of the object from the original server. To minimize the implications of this "switch-over" on real-time playout, clients may wish to actively maintain a list of alternate sources, and to factor in the delay of a switch-over into their buffering requirements.
- Finally, security is a common concern of application layer approaches, including our proposed cache-and-relay protocol. Since clients can access the caches at other end-hosts,

[1] Specifically, our discussion and results can be easily adapted to allow for the minimization of other metrics such as delay, packet loss rates, etc.

the system must prevent unauthorized accesses, for example access without digital rights. Security support can be implemented by either the original server (alone or assisted by trusted clients).

A full consideration of each of the above dimensions of our cache-and-relay approach is interesting and potentially quite challenging. However a comprehensive study of these issues is beyond the scope of this paper, which mainly focuses on the promise of the cache-and-relay approach in reducing network link cost.

3. SCALABILITY AND INSTANTIATIONS

In this section, we first show how effectively the cache-and-relay approach can reduce network link cost, given unlimited client-side bandwidth and cache space. Then we formalize the problem when either client-side bandwidth or cache size is limited. In each case, a specific instantiation of the cache-and-relay algorithm is proposed.

3.1 Lower Bound on Network Link Cost

Assuming unlimited client-side bandwidth and cache capacity, a new client can always fetch the object from the nearest ongoing peer client—a peer that started receiving that object but have not finished. We define the cost of serving the new client to be the hop-distance between that client and the nearest ongoing peer. Let $L(n)$ denote the total network link cost for n consecutive client arrivals within a unit time, whereby each client fetches the object from the nearest ongoing peer. Here, a unit time is defined as the duration of the media object, and hence n is the average client concurrency level. $L(n)$ reflects how the cache-and-relay approach scales (in terms of network link cost) as the level of client concurrency n increases.

In random networks, which have exponential neighborhood expansion functions, we have computed the following asymptotic scaling behavior (see the Appendix for a detailed derivation):

$$L(n) \sim n\left(1 - \frac{\ln n}{\ln N}\right), \tag{1}$$

where N is the total number of nodes in the network. This result implies that the increase in network link cost is a sub-linear function of the client arrival rate n. This underscores a clear reduction in network link cost compared to unicast service whose cost is linear in n.

The key to the derivation of $L(n)$ is the neighborhood expansion function $E(d)$ of the network, which is defined as the average fraction of vertices reachable in d hops, starting from an arbitrary vertex. In random networks, this function is approximated by an exponential function. For example, Figure 2 shows the neighborhood expansion function of a random network with 119,259 vertices and with an average degree of 3.2. The network was generated using the ER model [14]. The function is well fitted to $E(d) = \frac{3.2^d}{119259}$ for a wide range of values, except when the actual function reaches a saturation point, i.e., the edge effect of the network is reached.

While the derivation of $L(n)$ for an arbitrary network is impossible, we have also considered networks whose neighborhood expansion functions follow a power-law. As detailed in the Appendix, we found that if the neighborhood expansion function is a power-law with exponent H, then $L(n)$ increases asymptotically as

$$L(n) \sim n^{1-\frac{1}{H}} \tag{2}$$

Example networks with power-law neighborhood expansion function include two-dimensional and three-dimensional grids.

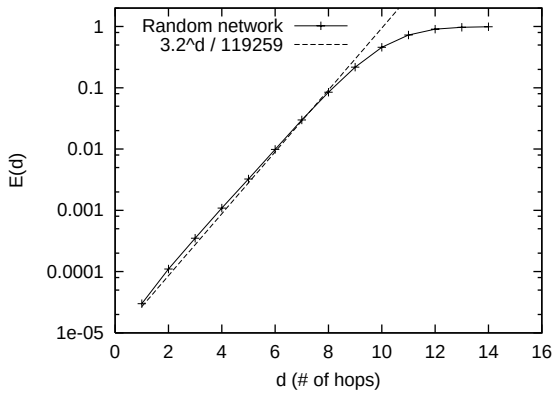


Figure 2: In random networks, neighborhood expansion functions are approximately exponential.

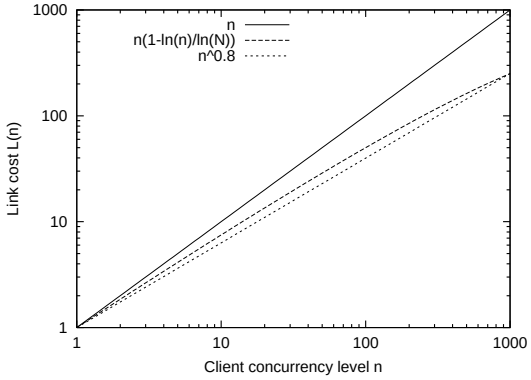


Figure 3: In limited scales, network link costs under different neighborhood expansion functions appear to be close.

These results show that the effectiveness of the cache-and-relay approach is largely determined by the topological properties of networks. With different neighborhood expansion functions, the network link cost reductions are much different.

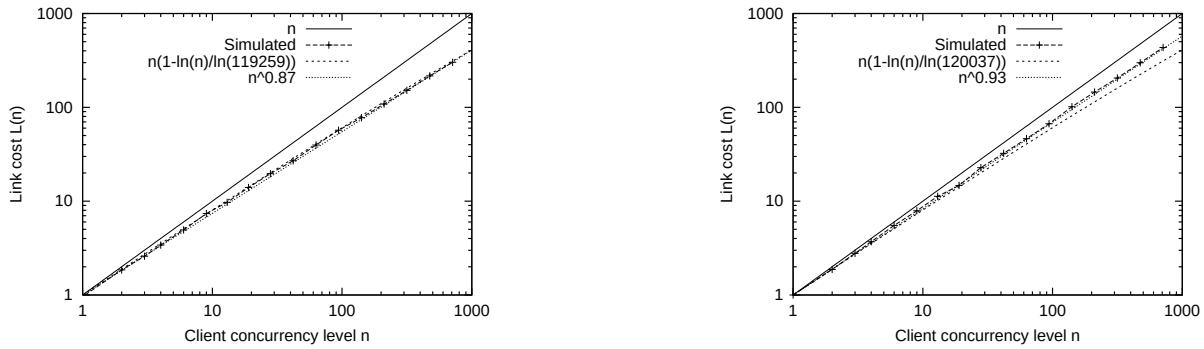
We should also point out that the above theoretical results are valid only in asymptotic cases. In limited scales (small network size N and low client concurrency level n), the difference between these equations is less pronounced. Figure 3 plots an example to show that equation (1) and (2) are close when $N = 10,000$.

3.2 Handling Limited Client Bandwidth

In practice, clients may have limited bandwidth to receive and send streams. Therefore, it may be infeasible for a client to receive a stream from the nearest on-going peer. For example, in Figure 1.1(a), if we assume that each client can receive and send at most three streams, then the solution shown in the figure becomes infeasible since client 2 cannot receive and send four streams in total. In Figure 1.1(b), a feasible solution is shown.

It is difficult to find the optimal solution when bandwidth is limited. Indeed, even the off-line algorithm (with prior knowledge of client arrivals) is NP-hard. To explain why this is the case, it suffices to note that the construction of a degree-constrained spanning tree[2] is an NP-complete problem [16]. By restricting our problem

[2]Given a graph $G = (V, E)$ and a positive integer $K \leq |V|$, find a spanning tree for G in which no vertex has degree larger than K.



(a) Random network (b) Power-law network

Figure 4: Comparisons of theoretic network link cost and simulation results using synthetic networks.

to synchronous clients and integer bandwidth values, finding an optimal cache-and-relay solution is equivalent to finding a solution to the degree-constrained spanning tree problem.

A simple greedy solution for the bandwidth-constrained cache-and-relay problem works as follows. Each new client receives the object from the nearest ongoing peer client who still has abundant bandwidth. The solution in Figure 1.1(b) is obtained using this greedy algorithm. Client 5 receives a stream from client 4 and client 8 receives data from client 3. The total hop-distance is 32. Our simulation results suggest that this greedy algorithm usually finds good solutions.

3.3 Handling Limited Cache Capacity

In practice, clients may have limited cache capacity. For example, in a caching proxy, it might be unrealistic to cache a whole video whose size is up to Giga bytes, especially when many such media objects may be competing for cache space.

When cache space is limited, the solutions in Figures 1.1(a) and 1.1(b) may become infeasible. For example, if client 1 has a cache capacity that enables it to keep only 50% of the object, then when client 9 arrives, it is already too late for that client to fetch the object from client 1. Instead, a feasible solution is for client 6 to relay the object to client 9.

To handle cache capacity constraints, it is necessary to determine a *cache replacement* policy. It is straightforward to use a FIFO policy: a sliding window indicates the current segment in the cache. The client can relay the object to other clients who start slightly later (i.e., within that window). In the next section, we show how constraints on cache capacity impact the reduction of network link cost.

4. PERFORMANCE EVALUATION

In this section, we validate through simulation the network link cost of our proposed cache-and-relay approach, and study the effect of limited client-side bandwidth and cache capacity. Large, synthetic and real networks are used in simulation. Experimental results are compared to theoretical results.

4.1 Networks Used in Our Simulations

In our simulations, four synthetic and real networks were used. All four networks have approximately 110000-120000 nodes and have an average degree of 3.2. The topologies we consider are:

- A random network generated using the ER model [14]. In this model, there is a uniform probability of having an edge between any pair of vertices in the graph. The model does not guarantee that the network is connected. So, we use the largest connected component with 119,259 vertices.
- A random power-law degree network with 120,037 vertices generated using the model in [2]—namely, the probability of having node degree larger than d is proportional to $d^{-\alpha}$ (we set $\alpha = 2.5$).
- A small-world network with power-law degree distribution, generated using the model in [21]. The network has 120,000 vertices. The resulting topology is different from random power-law degree networks as it features a large clustering coefficient. In generating this network, we not only used power-law vertex degree with $\alpha = 2.5$, but also considered the physical distance of the vertices in creating edges.
- A router-level Internet map (Lucent) available from [29]. This map has 112,269 vertices and it less strictly follows a power-law degree distribution. In addition, it has a high clustering coefficient [21]. We have found that our small-world network is the closest to this real Internet in terms of average path length and clustering coefficient.

4.2 Network Link Cost Validation

Our simulation proceeds as follows. Client arrivals are Poisson, with each client residing at a random node of the simulated network. We vary the client arrival rate (or concurrency level n) and obtain the corresponding network link cost $L(n)$ scaling as a function of n.

We first assume unlimited client bandwidth and cache capacity in validating the network link cost presented in the last section. Here only the results using the random network and using the power-law random network are presented.

Figure 4(a) shows that when the random network is used, the network link cost is well-predicted by Equation (1). In addition, it appears that Equation (2) also provides a good fit. This is explained by our discussion in the last section, i.e., in limited scales, this two equations are close.

Figure 4(b) shows that the network link cost is clearly higher than that predicted by Equation (1). Notice the log-scale of $L(n)$ in the figure. This is because power-law random networks do not have an exponential neighborhood expansion function.

4.3 Effect of Limited Client Bandwidth

Figure 5 shows the resulting scaling behavior when the client-side bandwidth is chosen in different ways. Also, for comparison purposes, it shows the cost of unicast delivery.

When client-side bandwidth is chosen uniformly between object

playback rate and four times that rate, the network link cost is not significantly higher than what is achievable with infinite bandwidth. This result suggests that our approach is effective even when client-side bandwidth is low.

Again, the simulation results using power-law networks appear to be rather different from those obtained using router-level Internet maps. In the power-law network, the network link cost reduction is less than that in router-level maps. However, we found that the simulation results using a small-world, power-law network is close to that obtained using router-level Internet maps. This underscores the importance of capturing small-world behaviors in Internet topologies–namely *clustering in networks is important to the scaling behavior of multicast delivery* [21].

4.4 Effect of Limited Client Cache Capacity

Figure 6 shows the resulting scaling behavior when the client cache capacity is constrained. In our simulations, we choose different cache capacities, corresponding to 10%, 30%, and 100% of the object size. Buffer management uses a FIFO replacement policy.

The results in Figure 6 indicate the follows. First, even when cache space is limited, the reduction in network link cost is still significant compared to that of unicast delivery. Second, there is still a room for improvement when cache capacity is small. Notice that we use a simple FIFO policy which can be less efficient than others. In addition, it is also possible to combine prefetching techniques to better utilize limited cache space. For example, assume the client cache can only store a S-minute segment of the object. When a later client starts, it may prefetch the object from any client who started less than $2S$ minutes earlier. Therefore, it works as if the cache size is doubled.

5. RELATED WORK

End system multicast was advanced by the authors of [9] as a deployable alternative to IP multicast. In their Narada protocol, end systems self-organize into an overlay network using a fully distributed protocol, with fairly low delay and bandwidth overheads. Recently, the same research group conducted an extensive evaluation of schemes for constructing overlay networks on a wide-area testbed [8]. This study demonstrated that end system multicast is promising for conferencing applications in a dynamic and heterogeneous Internet environment, and highlighted the importance of adapting to latency and bandwidth while constructing overlays optimized for the real-time delivery of content to synchronous clients.

Delivery of content to asynchronous clients is the focus of many recent studies, including periodic broadcasting [30, 18, 17, 23] and stream patching/merging techniques [6, 15, 11, 12]. These approaches are targeted mainly at video-on-demand applications. In periodic broadcasting techniques, segments of an object (with increasing sizes) are repeatedly transmitted on dedicated channels, and asynchronous clients simply join one or more broadcasting channels to receive this data. Using stream patching/merging techniques, asynchronous clients are merged into larger and larger groups that share a single multicast channel. Both techniques assume the availability of a lower-level multicast delivery infrastructure. In that respect, they are scalable in minimizing server bandwidth requirement [13, 20], but do not specifically attempt to optimize for network link cost.

The idea of utilizing client-side cache space was also developed in several previous work [28, 27]. Their main objective was to reduce server load, but did not evaluate their network link cost reduction. A network level scheme was presented in [19], which caches data at routers in the network to service subsequent requests. It is therefore different from our application layer approach. In addition, it also aimed at lightening the demand on the server bandwidth.

Another class of content delivery techniques originated with the use of periodic broadcasting of encoded content as was done over broadcast disks [3] using IDA [26], and later using the Digital Fountain approach which relied on more efficient Tornado encoding [5, 4]. These approaches enable end-hosts to efficiently reconstruct the original content of size n from a subset of any n symbols from a large universe of encoded symbols. Such approaches enable reliability and a substantial degree of application layer flexibility. The primary weakness of these techniques is their inability to efficiently deal with real-time (live or near-live) streaming media objects due to the necessity of encoding/decoding rather large stored data segments.

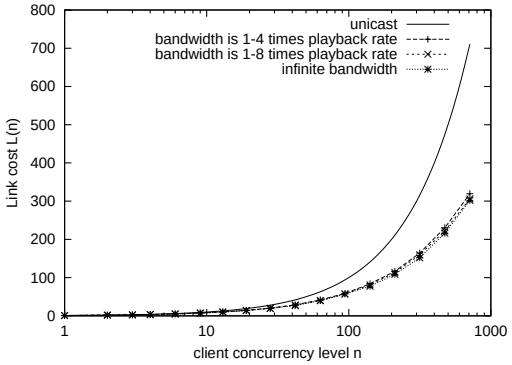
Several recent studies have addressed the impact of topology on IP multicast using shortest path trees. The authors of [25] showed how the size of a multicast tree increases with the size of the multicast group, primarily under an assumption of exponentially increasing network neighborhood (the number of vertices within a certain distance). They provided a theoretical result which *roughly* obeys the Chuang-Sirbu law for IP multicast scaling [10]. This law asserts that multicast tree size increases as $n^{0.8}$, where n is the group size. This was generalized in [7] for more realistic multicast tree shapes. The authors of [24] considered a stricter approximation of the link cost reduction. Their results show that the Chuang-Sirbu law is not asymptotic for random graphs and k-ary trees. Recently, [1] also re-examined the analysis in [25] and provided precise asymptotic scaling behavior of tree size. This asymptotic term scales as $n(1 - \frac{\ln n}{\ln N})$ where n is the group size and N is the network size. More importantly, they showed that by replacing the k-ary complete tree topology by a self-similar tree, multicast tree size satisfies a power law. This finding strongly supports the claim that the essence of the problem lies in the modeling assumptions on the topology. Our results have shown that the network link cost of cache-and-relay approach also largely depends on the network topology.

6. SUMMARY

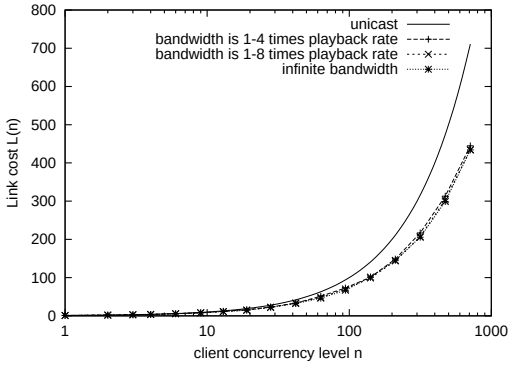
We proposed and evaluated a cache-and-relay application layer multicast delivery mechanism for streaming media objects. Our cache-and-relay approach minimizes the total network link cost and is especially tailored for applications featuring asynchronous client requests to popular streaming media objects. This approach has several salient features:

- It is fully distributed as it enables the replication of media objects in a demand-driven fashion, keeping the server and the end systems lightly-loaded. It thus provides a highly scalable solution.
- It is flexible to implement and easier to deploy, as it relies on an application layer multicast delivery technique, by utilizing client-side cache space and abundant bandwidth.
- It allows heterogeneous clients to join in the multicast delivery asynchronously. The clients may have different available bandwidths and different cache spaces devoted to peer cooperation.

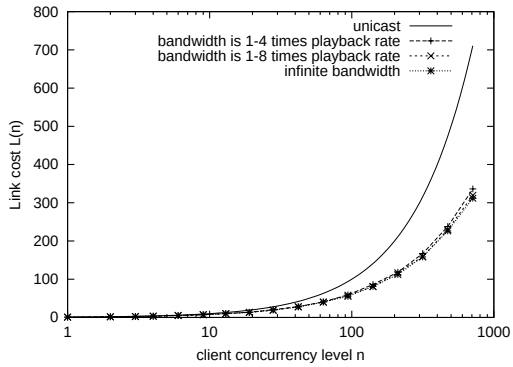
We derived the theoretical network link cost. In a random network, the link cost asymptotically scales as $n(1 - \frac{\ln n}{\ln N})$ where n is the client concurrency level and N is the network size. The link cost is different under different network topology assumptions. In addition, we found that the network cost is close to its theoretical lower bound when client-side bandwidth is limited; compared to unicast service, the link cost is still significantly reduced when client-side cache size is limited.



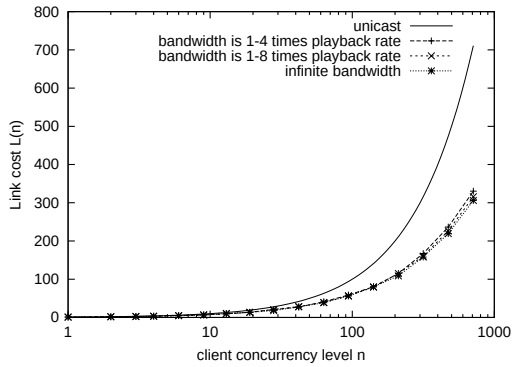
(a) Random network



(b) Power-law network

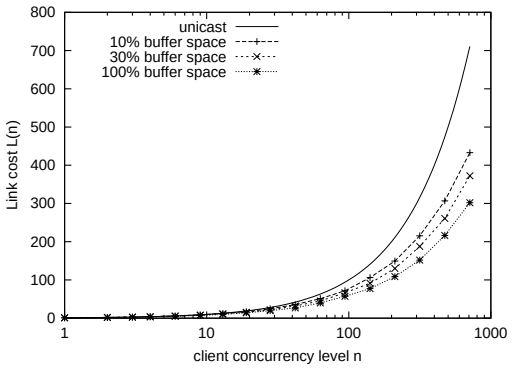


(c) Small-world power-law network

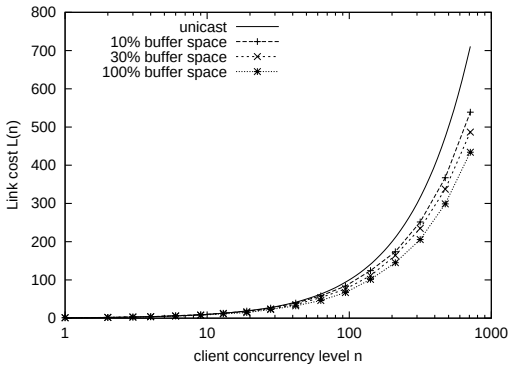


(d) Router-level Internet map

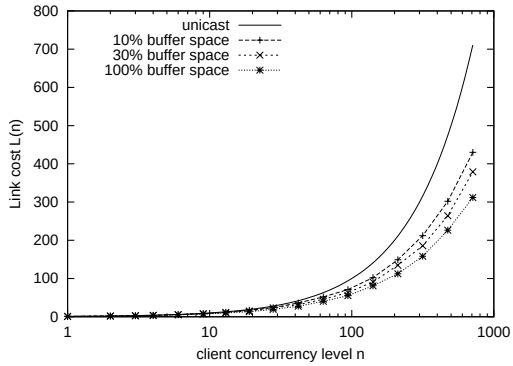
Figure 5: Simulation results when client-side bandwidth is limited.



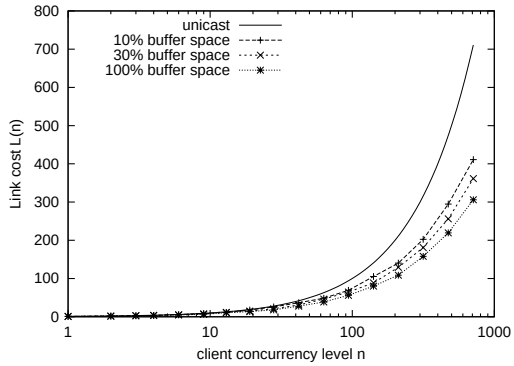
(a) Random network



(b) Power-law network



(c) Small-world power-law network



(d) Router-level Internet map

Figure 6: Simulation results when client-side cache space is limited.

APPENDIX

In this appendix, we derive the asymptotic scaling behavior of network link cost given in Section 3.

A.1 The General Case

The key to our derivation is the notion of network neighborhood size (or expansion) function $E(d)$, which is defined as the fraction of nodes in the network that are reachable in d hops.

Consider a general neighborhood expansion function $E(x)$, which is defined over $a < x < b$ in continuous form. Let us consider the probability that an arbitrary peer is not within distance x. By definition, this probability is $1 - E(x)$. Since there are n independent data sources in the network, the probability that none of them is within distance x is $(1 - E(x))^n$.

Let $F(x) = 1 - (1 - E(x))^n$. The probability density function $f(x)$ (i.e., the probability that the nearest peer is at distance x) is computed as $\frac{dF(x)}{dx}$. Let $g(n)$ denote the expected distance of the nearest peer.

$$\begin{aligned} g(n) &= \int_a^b x f(x) dx \\ &= \int_a^b x dF(x) \qquad \text{Substituting } y \text{ for } F(x) \\ &= \int_0^1 F^{-1}(y) dy. \end{aligned} \tag{3}$$

A.2 Exponential Neighborhood Expansion

Let $E(x) = k^{x-D}$, $0 \le x \le D$, where D is called the diameter of the network. Thus, $F(x) = 1-(1-E(x))^n = 1-(1-k^{x-D})^n$, and the inverse function $F^{-1}(y) = D + \frac{\ln(1-(1-y)^{\frac{1}{n}})}{\ln k}$. From equation (3), $g(n)$ is computed as

$$\begin{aligned} g(n) &= \int_0^1 F^{-1}(y) dy \\ &= D + \frac{1}{\ln k} \int_0^1 \ln(1-(1-y)^{\frac{1}{n}}) dy \\ &= D + \frac{1}{\ln k} \int_0^1 \ln(1-y^{\frac{1}{n}}) dy \end{aligned}$$

Consider the term $-\int_0^1 \ln(1-y^{\frac{1}{n}})dy$ in the above equation. Notice that $\ln(1+x)$ can be expanded to $x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$. Therefore,

$$\begin{aligned} -\int_0^1 \ln(1-y^{\frac{1}{n}}) dy &= \sum_{i=1}^{\infty} \int_0^1 \frac{y^{\frac{i}{n}}}{i} dy \\ &= \sum_{i=1}^{\infty} \frac{n}{(n+i)i} \\ &= \sum_{i=1}^{n} \frac{1}{i} \end{aligned}$$

Thus, $-\int_0^1 \ln(1-y^{\frac{1}{n}})dy$ is equal to the sum of a harmonic number series which is equal to $\ln n + 0.5772156 \cdots + \frac{1}{2n}$. This term is asymptotically close to $\ln n$. Therefore, $g(n) \approx D - \frac{\ln n}{\ln k}$. Notice that $\frac{\ln N}{\ln k} = D$. Thus, $g(n) \approx D(1 - \frac{\ln n}{\ln N})$, which grows asymptotically as $1 - \frac{\ln n}{\ln N}$.

A.3 Power-law Neighborhood Expansion

Let neighborhood expansion function $E(x) = (x/D)^H$, $1 \le x \le D$, where D is called the diameter of the network. $F(x) = 1 - (1 - E(x))^n = 1 - (1 - (x/D)^H)^n$. Inverse function $F^{-1}(y) = D(1 - (1 - y)^{\frac{1}{n}})^{\frac{1}{H}}$. From equation (3), $g(n)$ is computed as follows

$$\begin{aligned} g(n) &= \int_0^1 F^{-1}(y) dy \\ &= D \int_0^1 (1 - (1-y)^{\frac{1}{n}})^{\frac{1}{H}} dy \\ &= D \int_0^1 (1 - y^{\frac{1}{n}})^{\frac{1}{H}} dy. \end{aligned}$$

The computation of $g(n)$ is complicated (more details are in the next paragraph). It is easier to numerically solve $\frac{g(n)}{D}$ for different values of n. We have done so using different choices of H. The results are plotted in Figure 7. Interestingly, the results show that $\frac{g(n)}{D}$ match $n^{-\frac{1}{H}}$ very well for various choices of H when n is not too small. Notice that, the numerical lines are parallel to the corresponding $n^{-\frac{1}{H}}$ lines. It means their slight difference does not change the asymptotic scaling behavior.

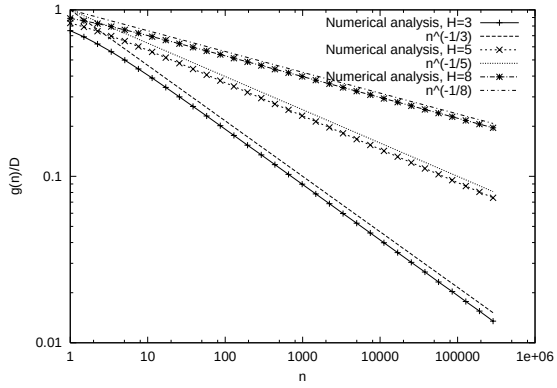


Figure 7: Numerical result of lower bound with power-law neighborhood expansion.

Now let us recall the computation of $\frac{g(n)}{D}$:

$$\begin{aligned} \frac{g(n)}{D} &= \int_0^1 (1 - y^{\frac{1}{n}})^{\frac{1}{H}} dy \qquad \text{Let } x = 1 - y^{\frac{1}{n}} \\ &= n \int_0^1 x^{\frac{1}{H}} (1-x)^{n-1} dx \\ &= n \frac{\Gamma(1+\frac{1}{H})\Gamma(n)}{\Gamma(n+1+\frac{1}{H})}, \end{aligned}$$

where $\Gamma(a)$ is the Gamma function defined as $\int_0^\infty x^{a-1}e^{-x}dx$. It appears, due to the factorial nature of Gamma function, $\frac{\Gamma(n)}{\Gamma(n+1+\frac{1}{H})}$ is roughly proportional to $n^{-1-\frac{1}{H}}$ when n is large. In addition, $\Gamma(1+\frac{1}{H})$ is a constant, corresponding to the displacement between the numerical lines $\frac{g(n)}{D}$ and $n^{-\frac{1}{H}}$ lines in Figure 7.

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