Vanishing Points,
Computer Vision to Analyze Paintings
Thin Lens Model

Lecture by Margrit Betke, CS 585, February 2024

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Vanishing Point

Definition:

Point at which receding parallel lines viewed in perspective appear to converge

Photo by Wikipedia user: MikKBDFJKGeMalak

Vanishing Point

Definition:

Point at which receding parallel lines viewed in perspective appear to converge

train tracks

Photo by Wikipedia user: MikKBDFJKGeMalak

Vanishing Point

Definition:

Point at which receding parallel lines viewed in perspective appear to converge

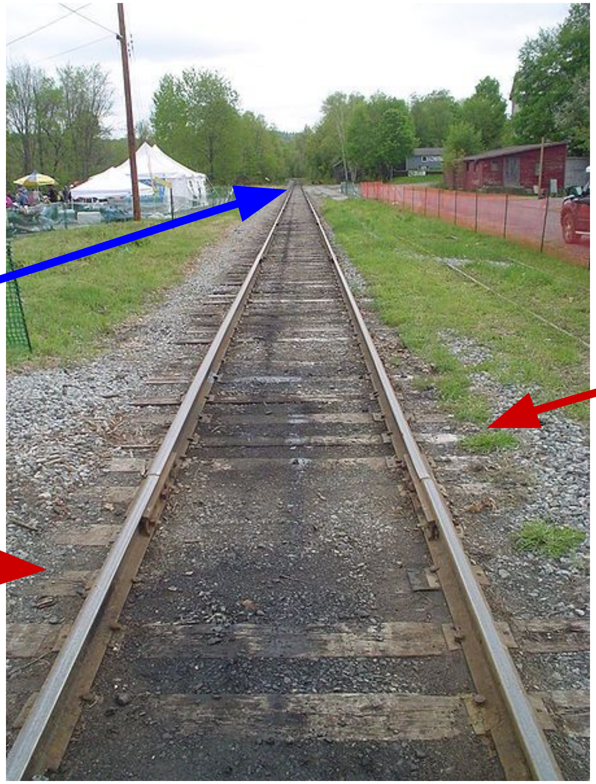

Photo by Wikipedia user: [MikKBDFJKGeMalak]

Why do parallel lines intersect when projected?

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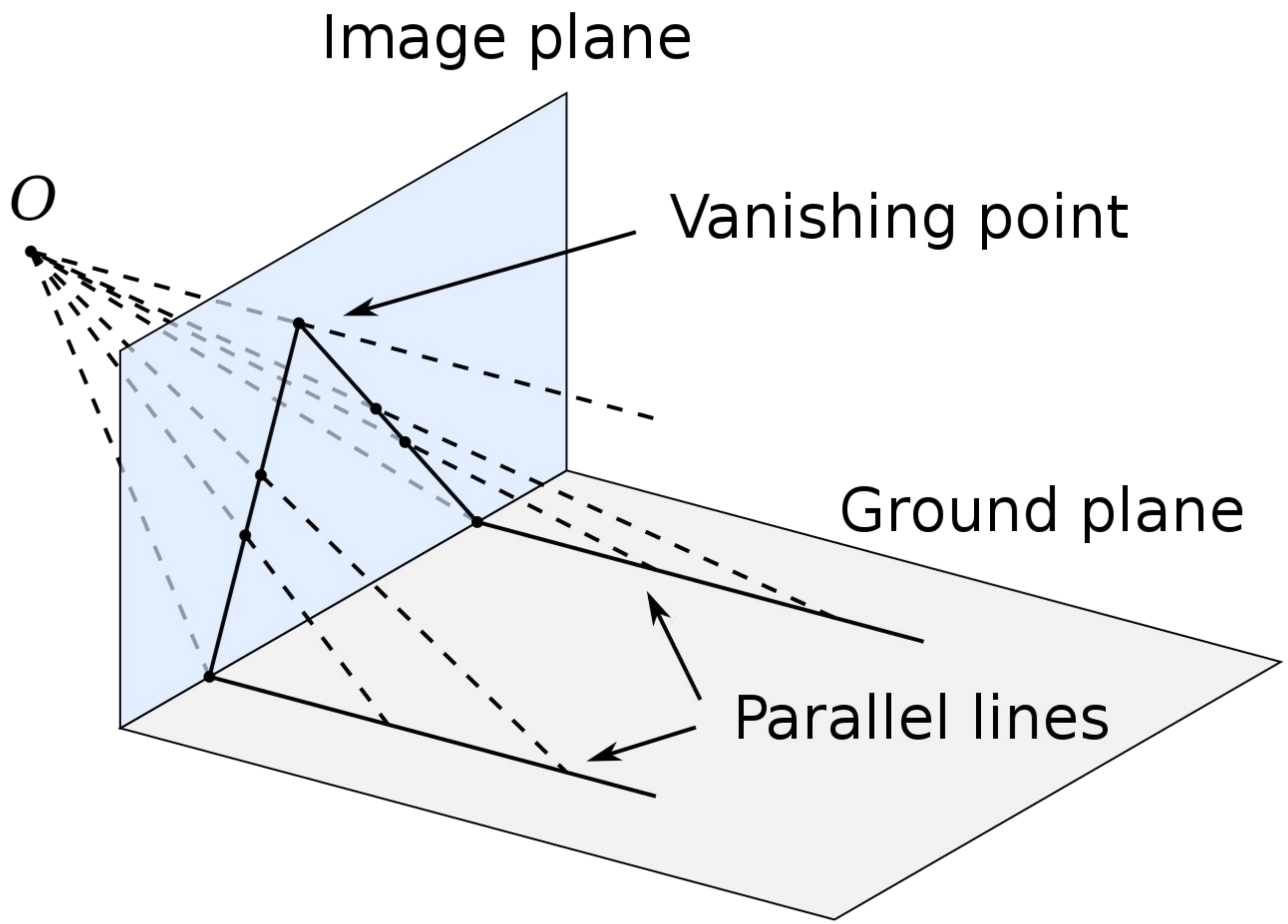


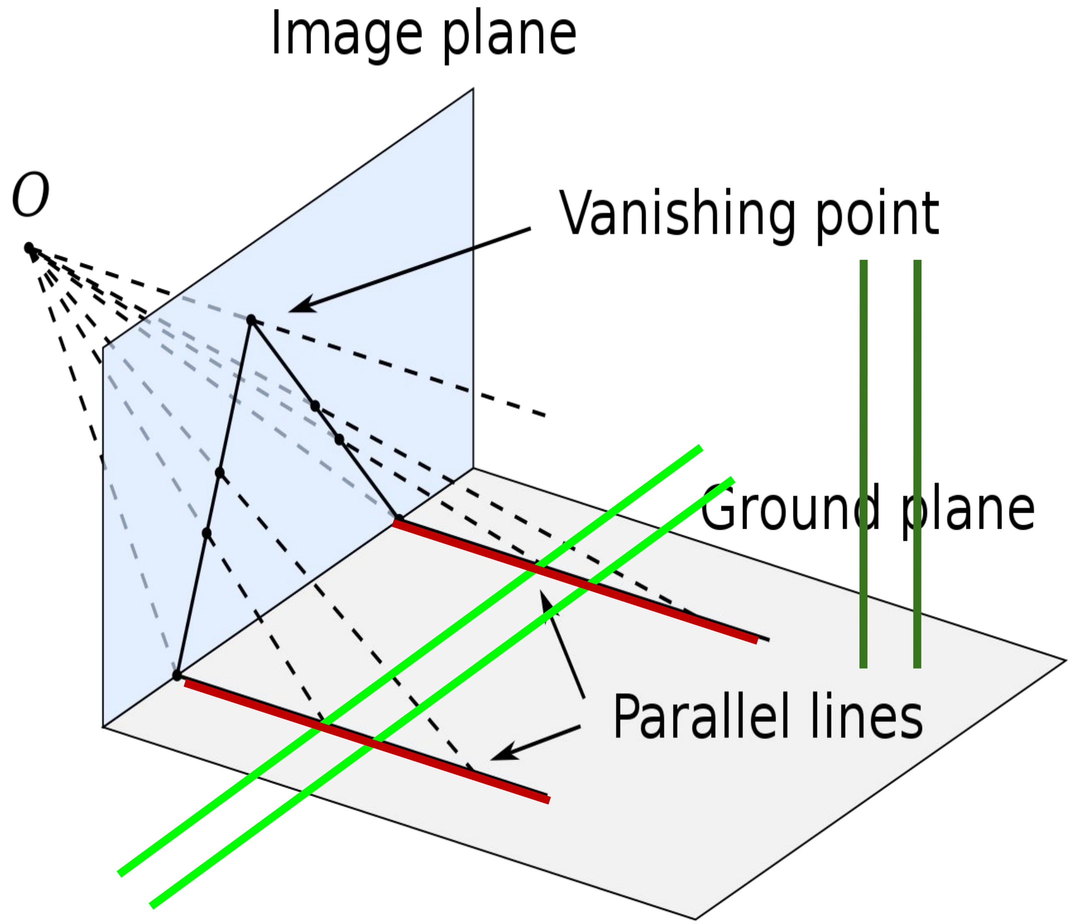
Illustration by Wikipedia user: Mgunyho

One Vanishing Point

Image Source: Betke

One-point Perspective: Only 1 Vanishing Point

One-point Perspective: When the image plane is parallel to two world-coordinate axes, lines parallel to the axis that is cut by this image plane will have images that meet at a single vanishing point. Lines parallel to the other two axes will not form vanishing points as they a parallel to the image plane.

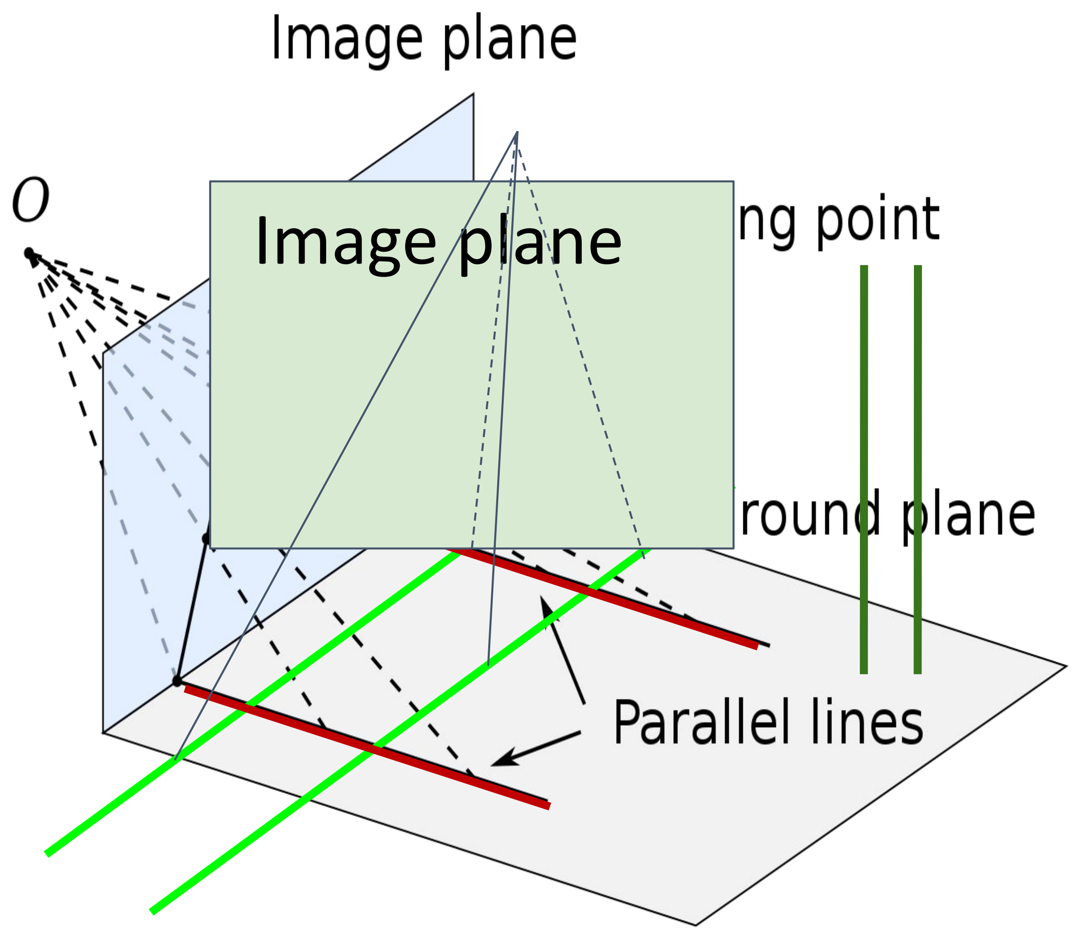


Source: Wikipedia

Two-point Perspective: 2 Vanishing Points

Two-point Perspective:

The image plane intersects two world-coordinate axes. Lines parallel to those planes will form two vanishing points in the image plane.



Two-point Perspective: 2 Vanishing Points

The image plane intersects two world-coordinate axes. Lines parallel to those planes will form two vanishing points in the image plane.

Photo Source: Betke

Two-point Perspective: 1st Vanishing Point

Photo Source: Betke

Two-point Perspective: 2 Vanishing Points

Photo Source: Betke

Two-point Perspective: 2 Vanishing Points

These parallel lines do not intersect

Photo Source: Betke

Can a vanishing point be somewhere far in the image plane, without showing up in the actual image (i.e., the part of the image plane that is outside the field of view)?

Can a vanishing point be somewhere far in the image plane, without showing up in the actual image (i.e., the part of the image plane that is outside the field of view)?

Yes, see snowy house example

Two-point Perspective: 2 Vanishing Points

These parallel lines do not intersect.

The vanishing points are on the image plane but not in the image frame in this example.

How many vanishing points can an image have?

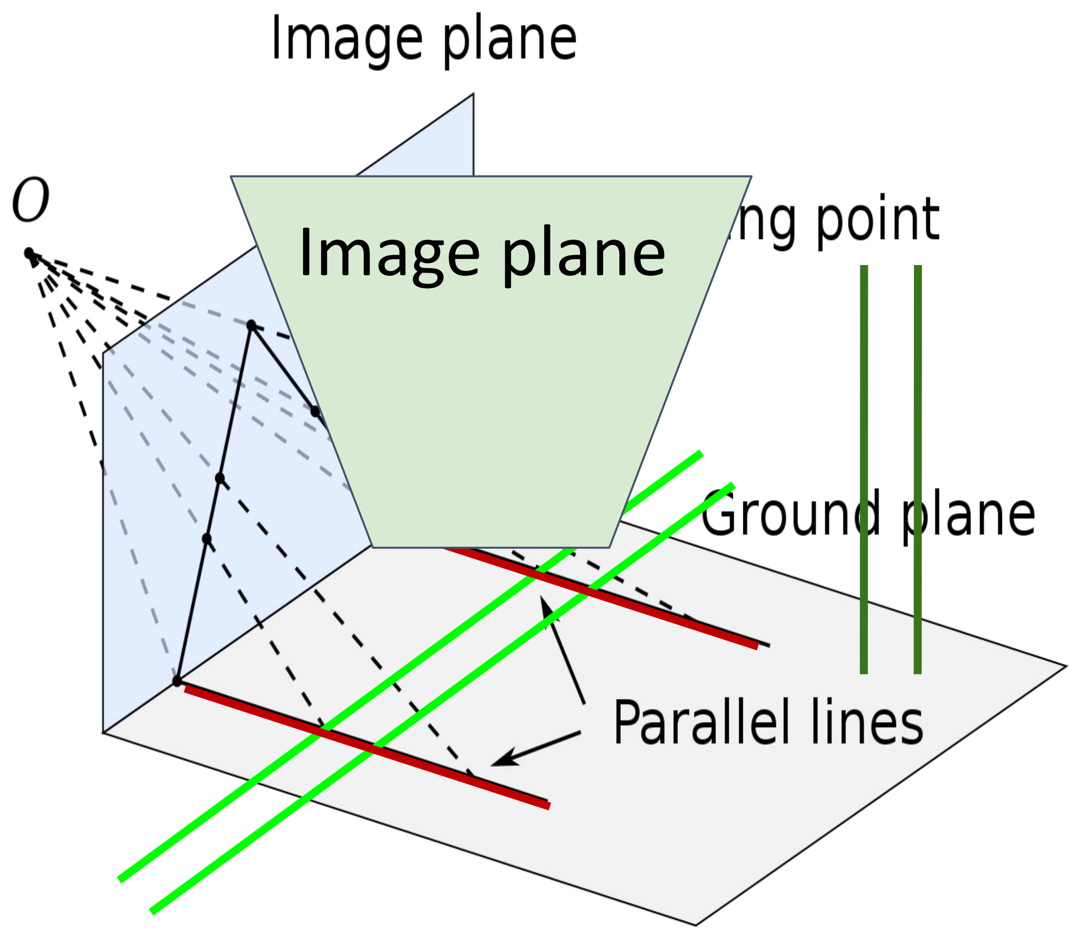
How many vanishing points can an image have?

Up to 3

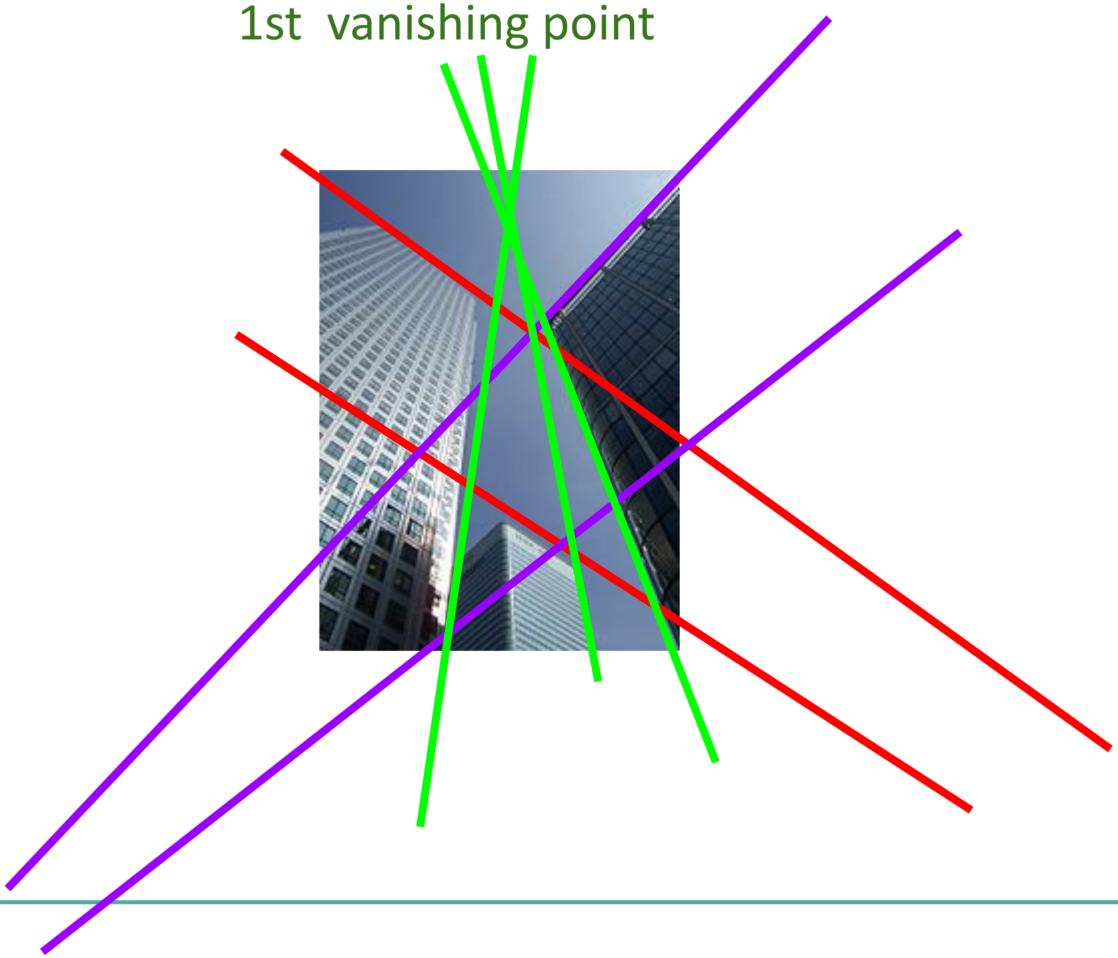
Three-point Perspective: 3 Vanishing Points

Three-point Perspective:

The image plane intersects the x, y, and z world-coordinate axes and therefore lines parallel to these axes intersect, resulting in three different vanishing points.



Three-point Perspective: 3 Vanishing Points



The image plane intersects the x, y, and z world-coordinate axes and therefore lines parallel to these axes intersect, resulting in three different vanishing points.

Photo by Wikipedia: Robin Heymans

How can a Computer Vision System use Vanishing Points to Make Sense of a Scene?

We can use them to derive real-world distances!!

Example: How wide is this sidestreet?

- Case 1:

  Assume you know the width of *The Jericho Cafe* and *Brancha,* 7 and 6 meters, respectively.

- Case 2:

  You only know the width of *The Jericho Cafe* (7 m)

Example: How wide is this sidestreet?

- Case 1:

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Photo by Wikipedia: Cmglee

Solution uses length ratios along a line that ends in a vanishing point

- Case 1:

  You know the width of *The Jericho Cafe* and *Brancha,* 7 and 6 meters, respectively.

- Case 2:

  You only know the width of *The Jericho Cafe* (7 m)

Photo by Wikipedia: Cmglee

Cross-ratio Rule

Given four points A, B, C, and D on a projective line, their cross ratio is defined as

$$\frac{AC \cdot BD}{BC \cdot AD}$$

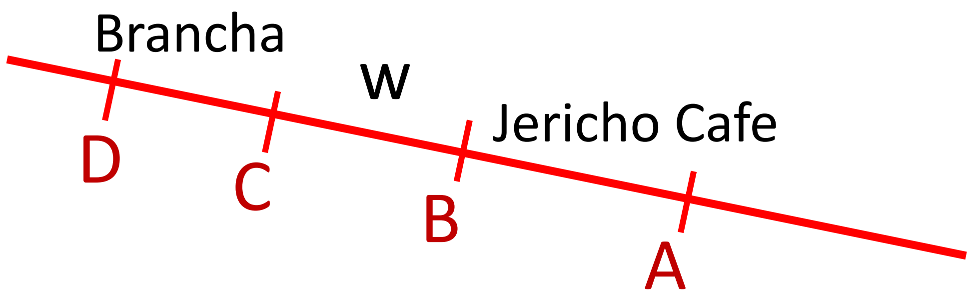
If one of the points is a vanishing point V, the cross ratio of (V, B, C, D) reduces to

$$\frac{D-B}{C-B}$$

Cross-ratio Rule: Case 1

Given four points D, C, B, and A on a projective line, their cross ratio is defined as

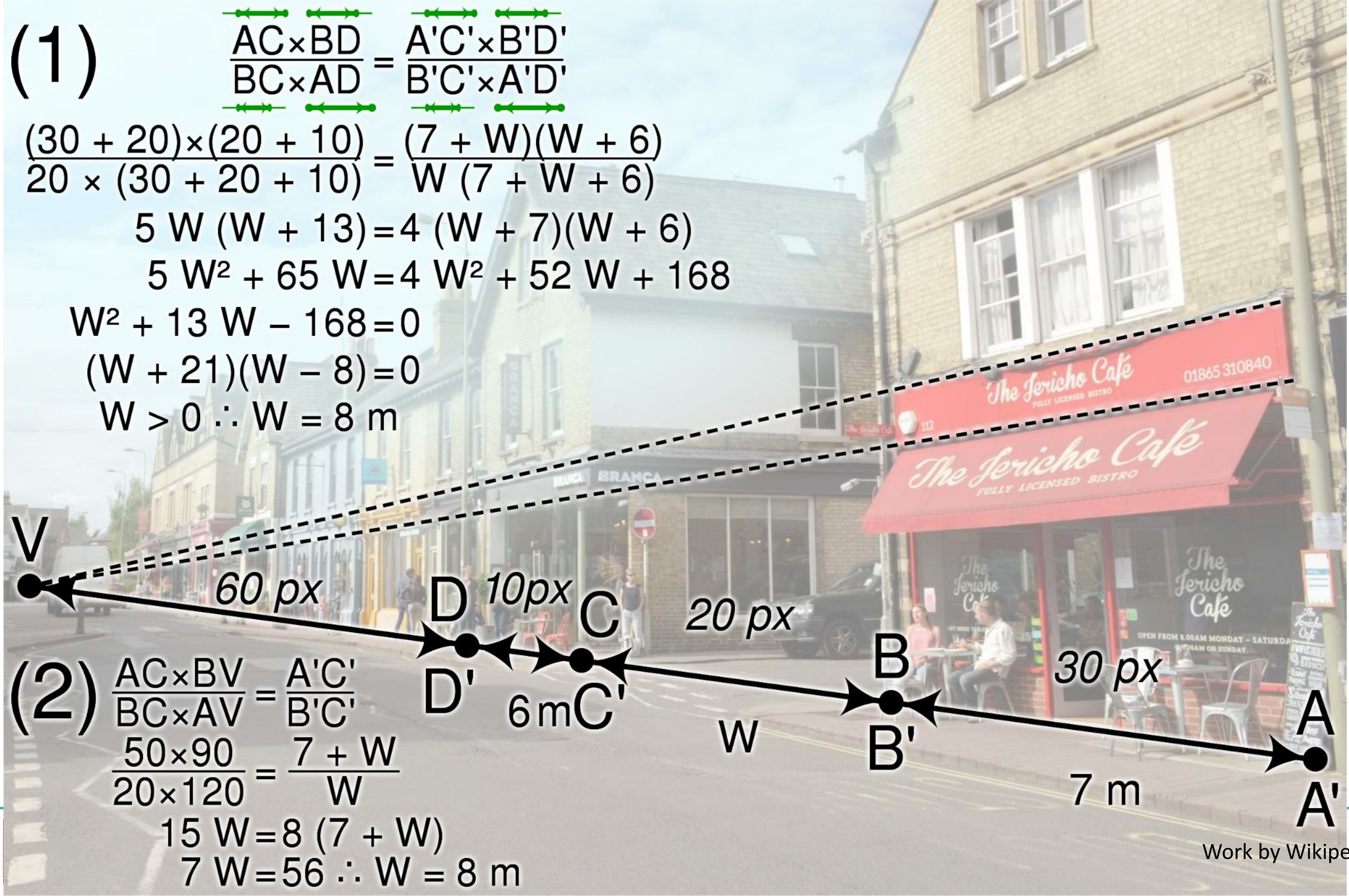
$$\frac{AC \cdot BD}{BC \cdot AD}$$



Here:

[(Jericho Cafe + w)·(w + Brancha)] / [w ·(Jericho Cafe + w + Brancha)]

Ratios are equal in both real world meters and image plane pixels



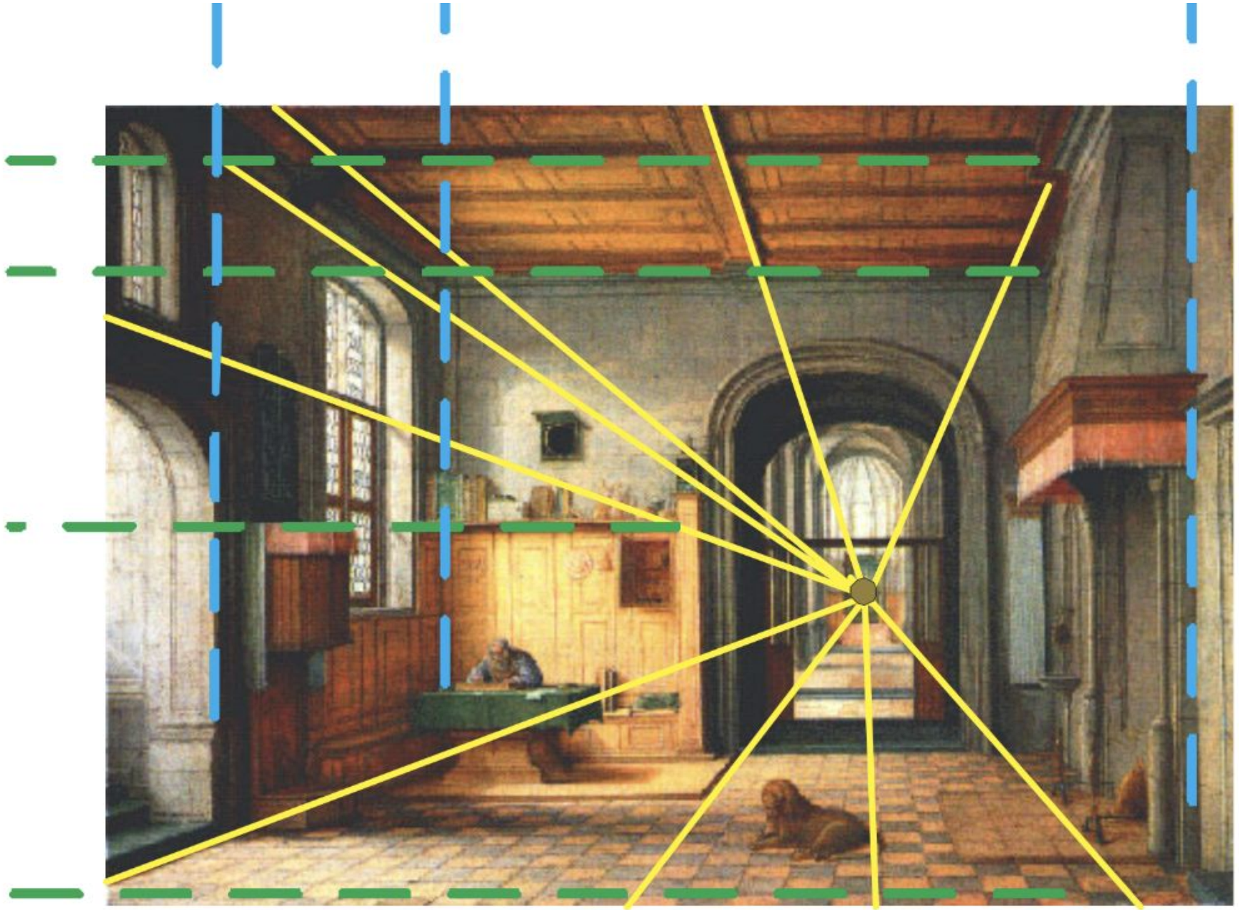
Work by Wikipedia's [Cmglee]

How can a Computer Vision System use Vanishing Points to Make Sense of a Scene?

1) We can use them to derive real-world distances

2) We can use them to analyze paintings

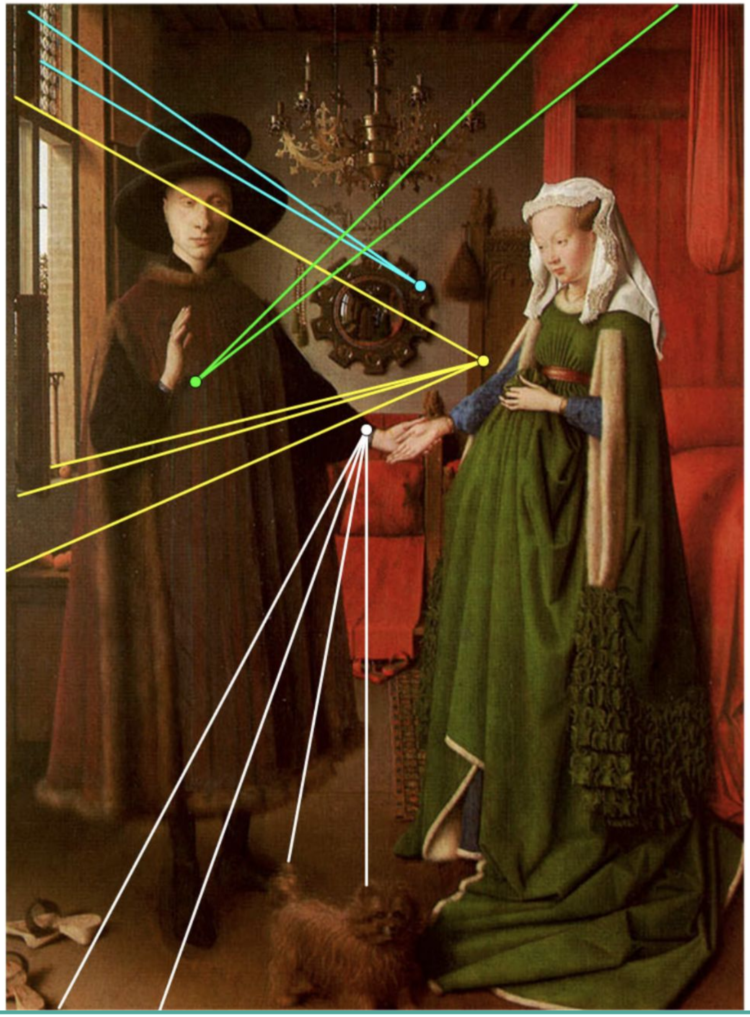
Computer Vision to Analyze Paintings



St. Jerome in his Study (1630) by Hendrick Steenwick

Paper by [Criminisi et al.]

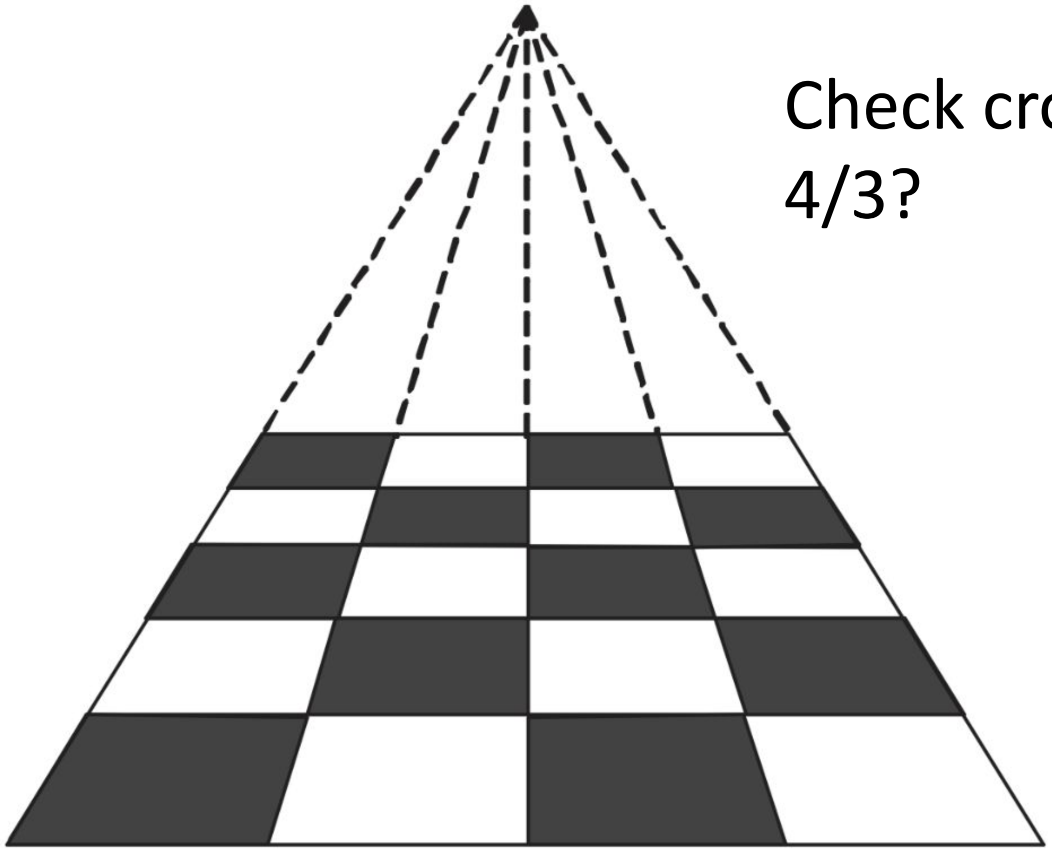
Computer Vision to Analyze Paintings



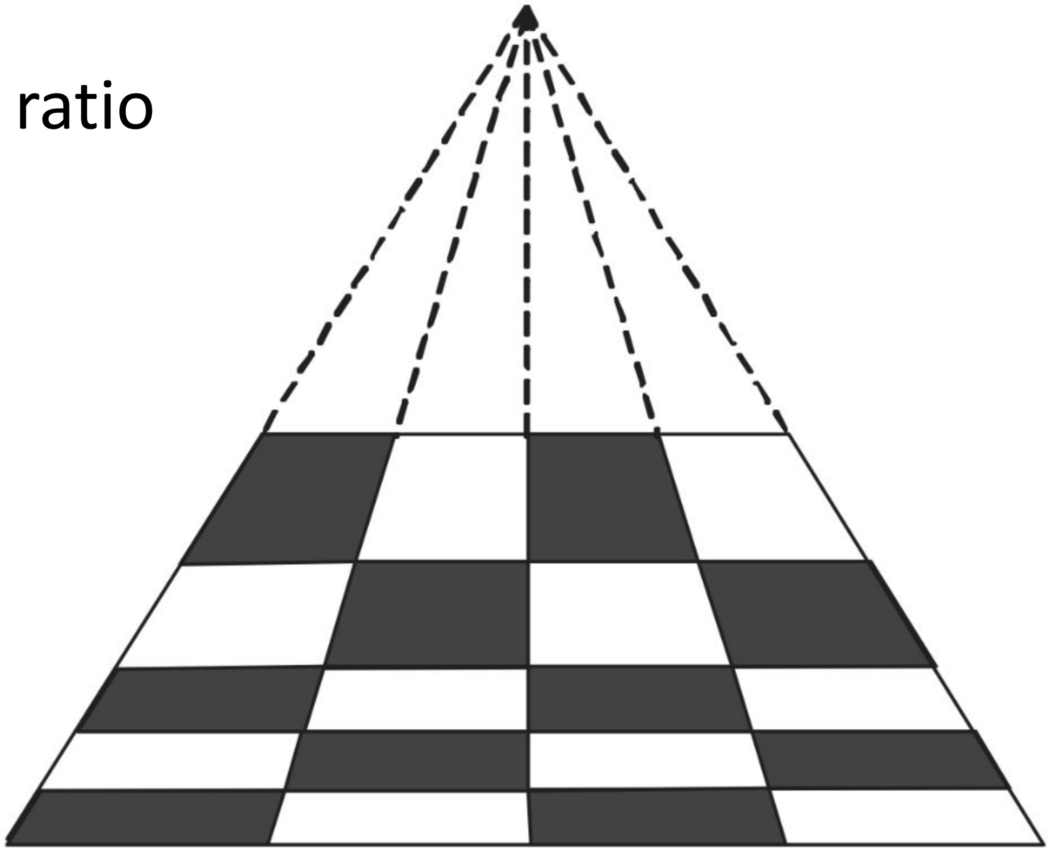
The Arnolfini Portrait by Van Eyck (1434)

Too many intersecting lines…

Computer Vision to Analyze Floor Tiles in Paintings

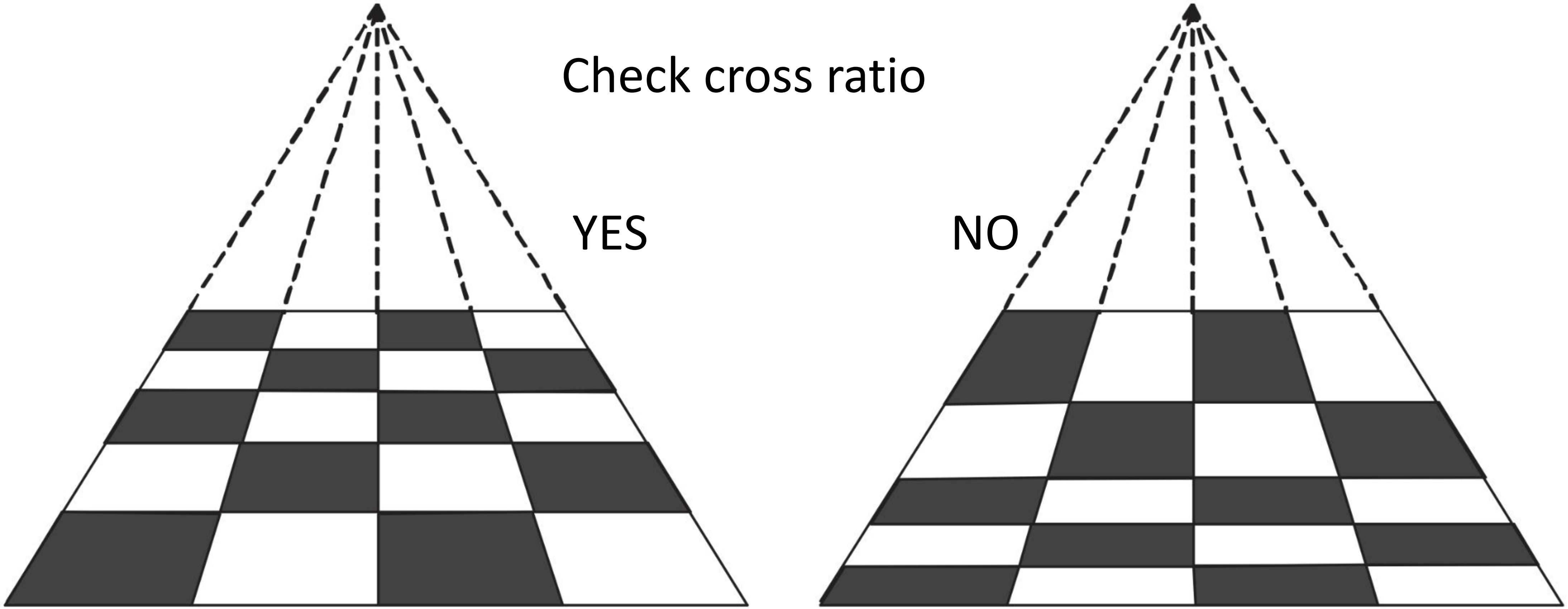


Check cross ratio
4/3?



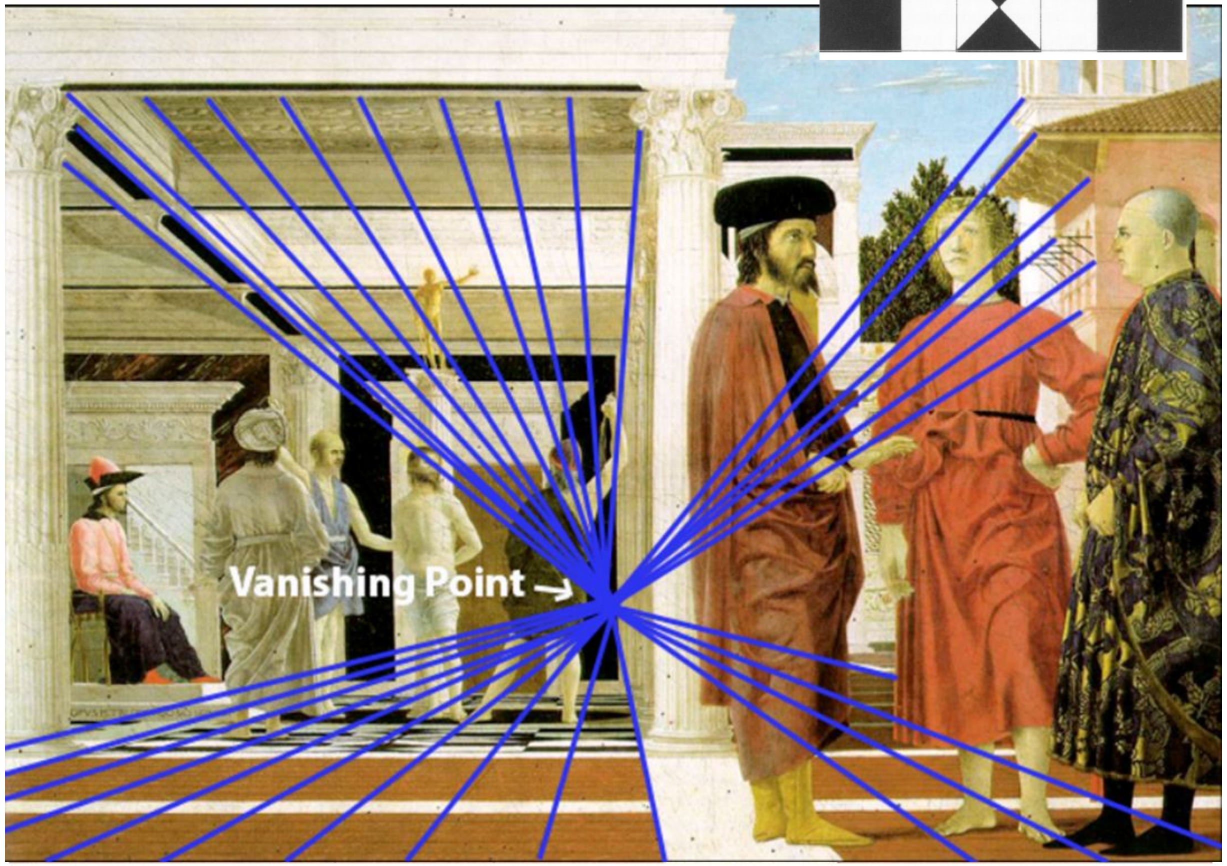
Paper by Criminisi et al.

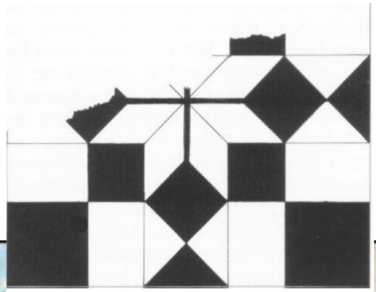
Computer Vision to Analyze Floor Tiles in Paintings



Paper by Criminisi et al.

Computer Vision to Analyze Paintings





Flagellation of Christ (1453) by Piero della Francesca

Paper by Criminisi et al., Brueckner: Image source

Poll: Are these real or fake photos?

A: Pumpkin picture is fake
B: Tower of Pisa picture is fake
C: Both are fake
D: Both are real
E: We don’t know

Image credit:
S. Lazebnik

Image credit:
Seniorforums.com

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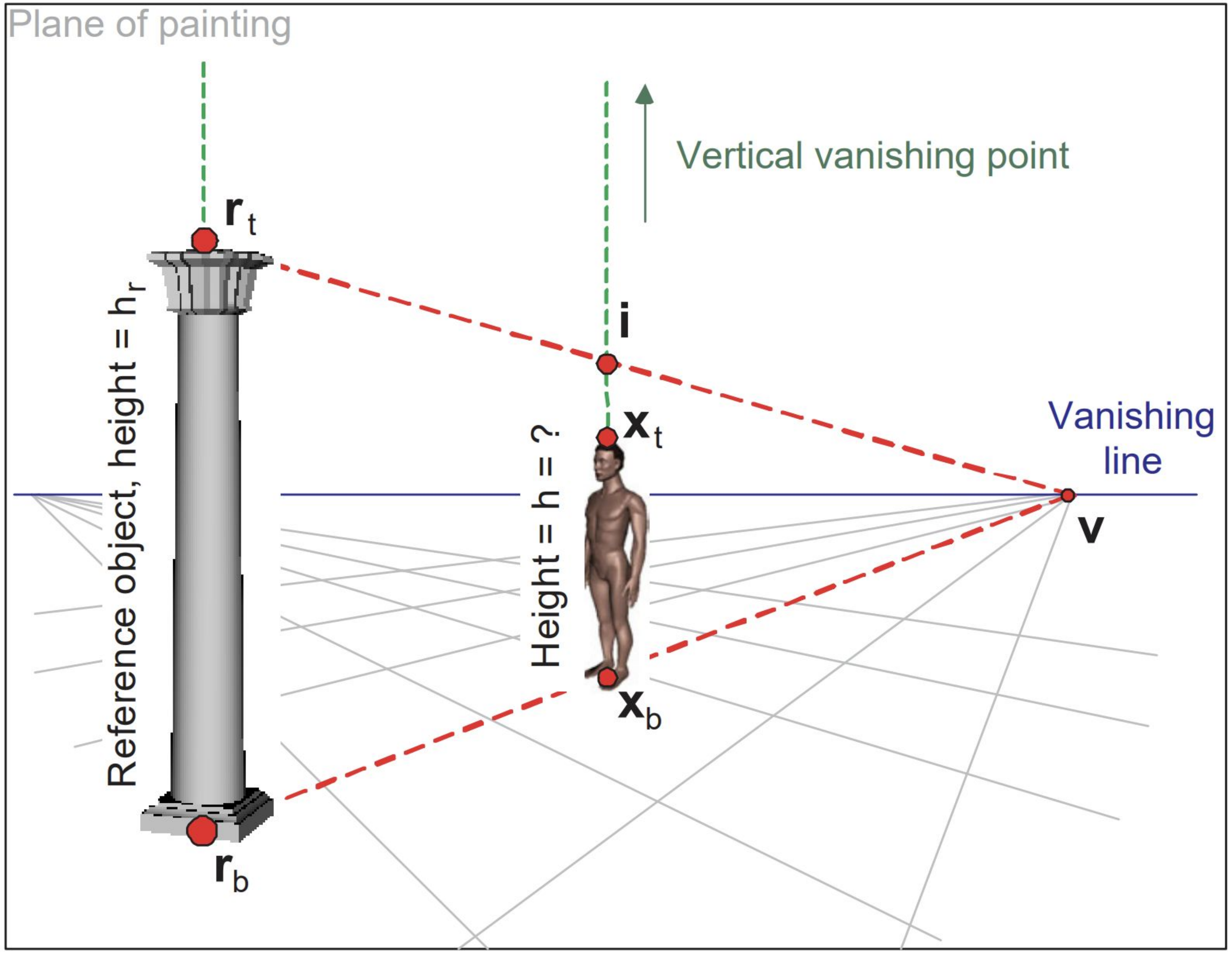
How to Check for Realism with a Vanishing Point



A person's height *h* can be checked by comparing it to reference height h_r :

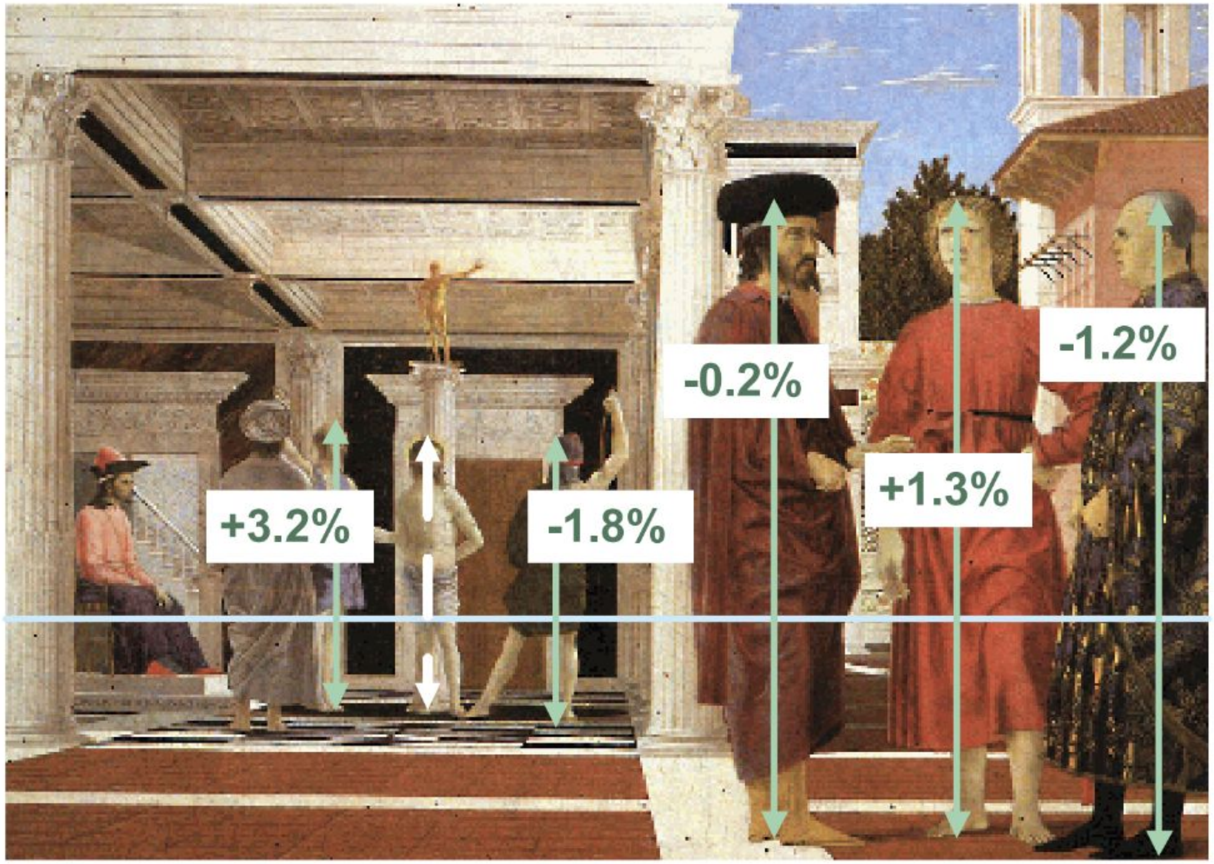
$$\frac{h}{h_r} = \frac{d(\mathbf{x}_t, \mathbf{x}_b)}{d(\mathbf{i}, \mathbf{x}_b)}$$

Paper by Criminisi et al.

Unrealistic Placement of People or Genius Painting?

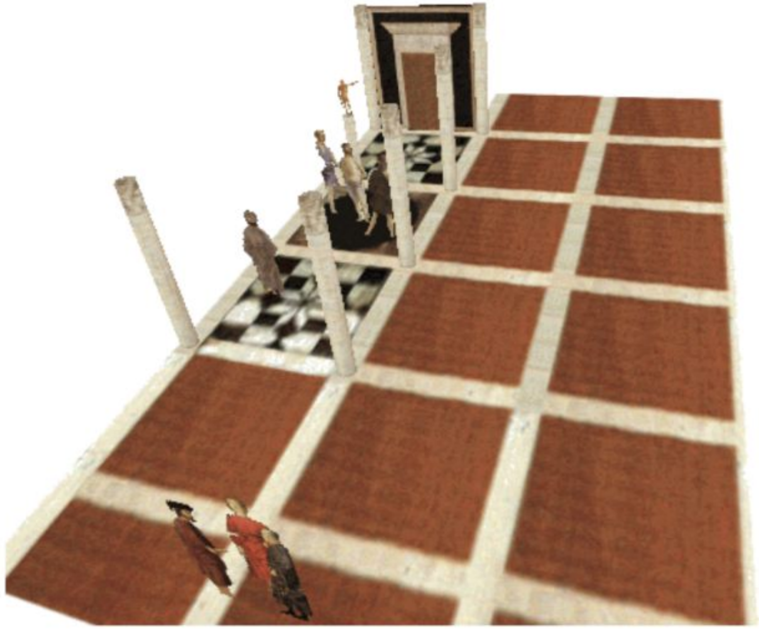
Flagellation of Christ (1453) by Piero della Francesca

~~Unrealistic Placement of People~~ or Genius Painting

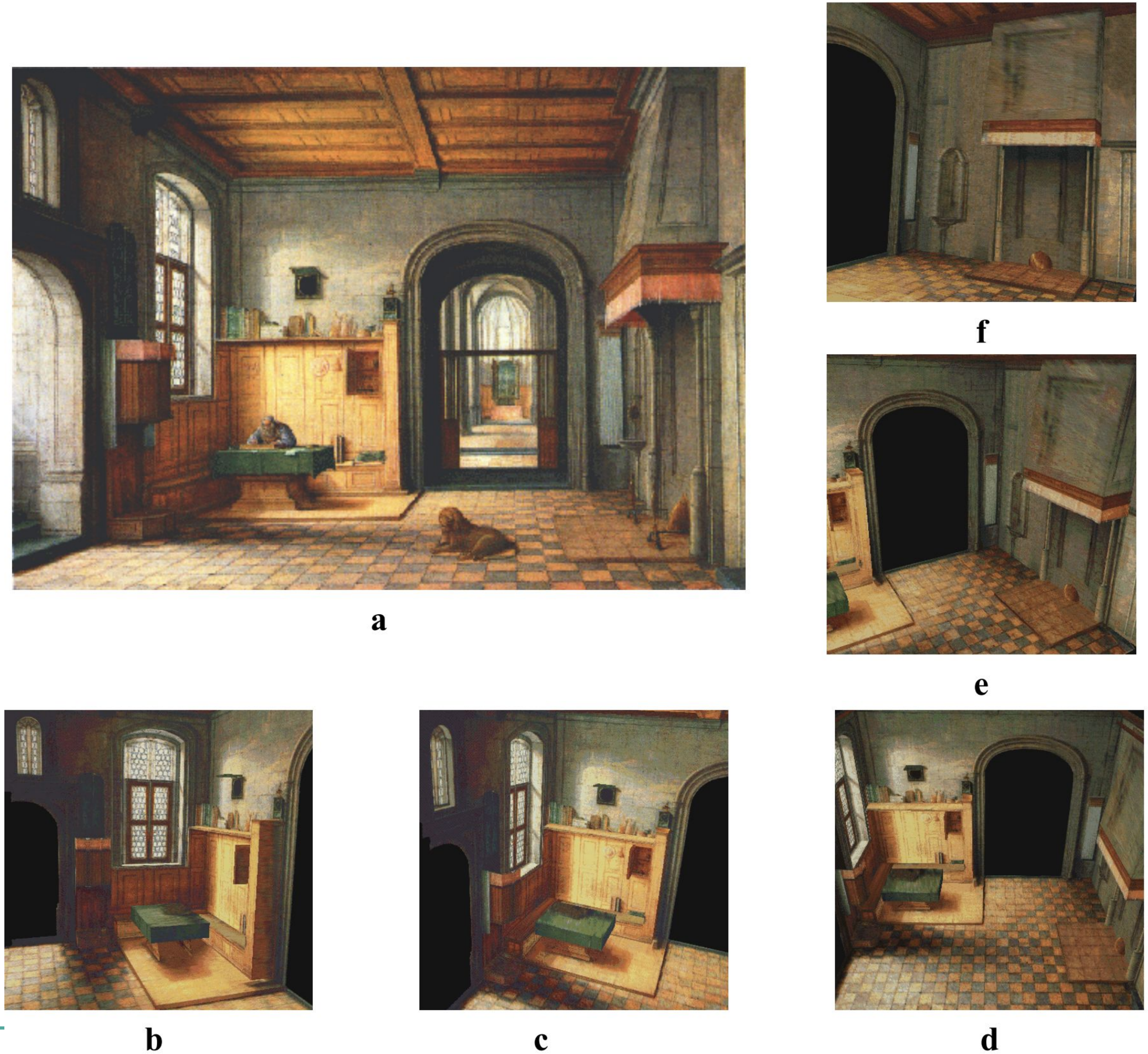


Flagellation of Christ (1453) by Piero della Francesca

From 2D Painting to 3D Scene



From 2D Painting to 3D Scene



Paper by [Criminisi et al.]

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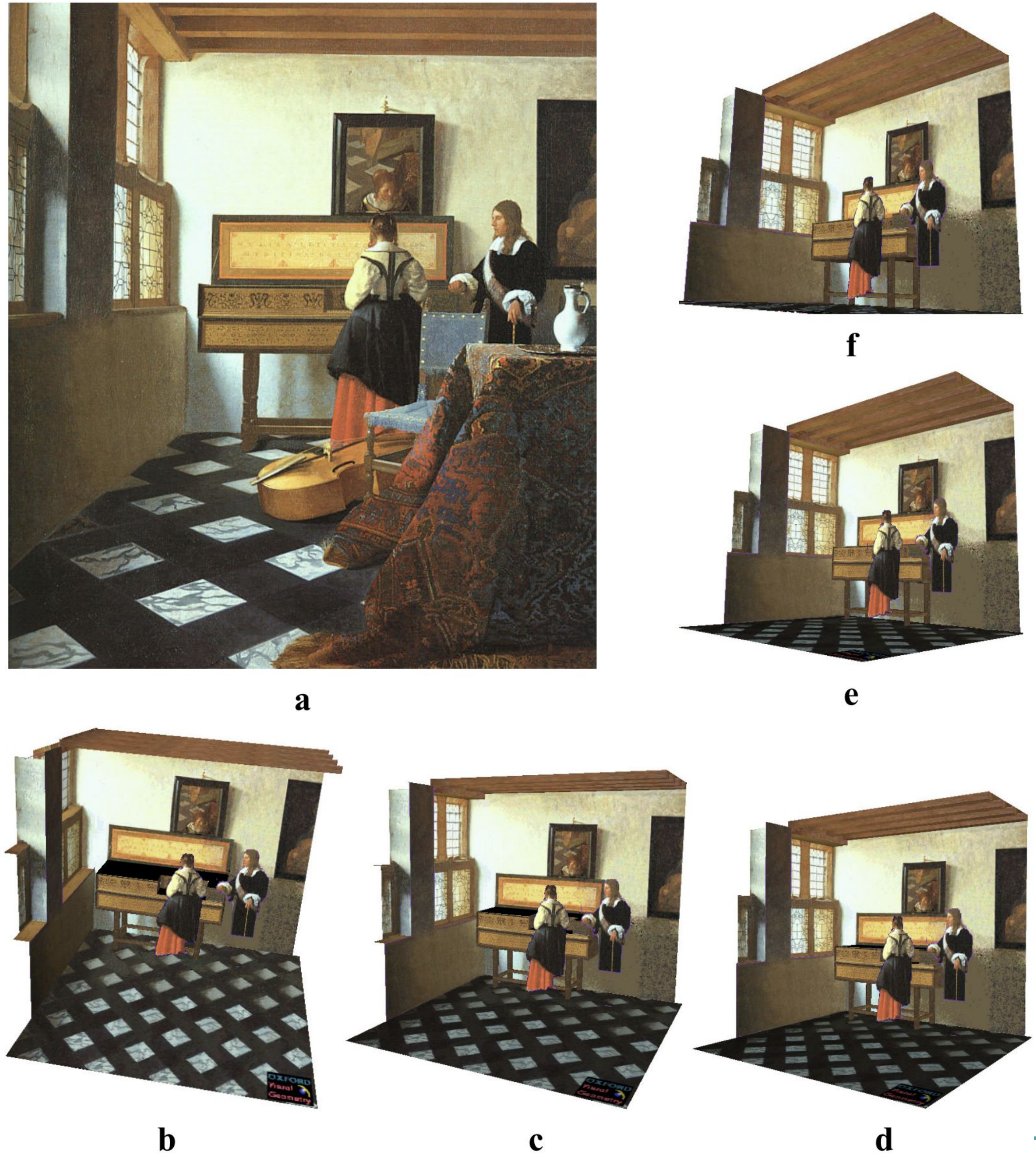
From
2D Painting
to 3D Scene



Paper by [Criminisi et al.]

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Pinhole camera

=

Box with a hole

Image is projected upside down on side opposite to hole.

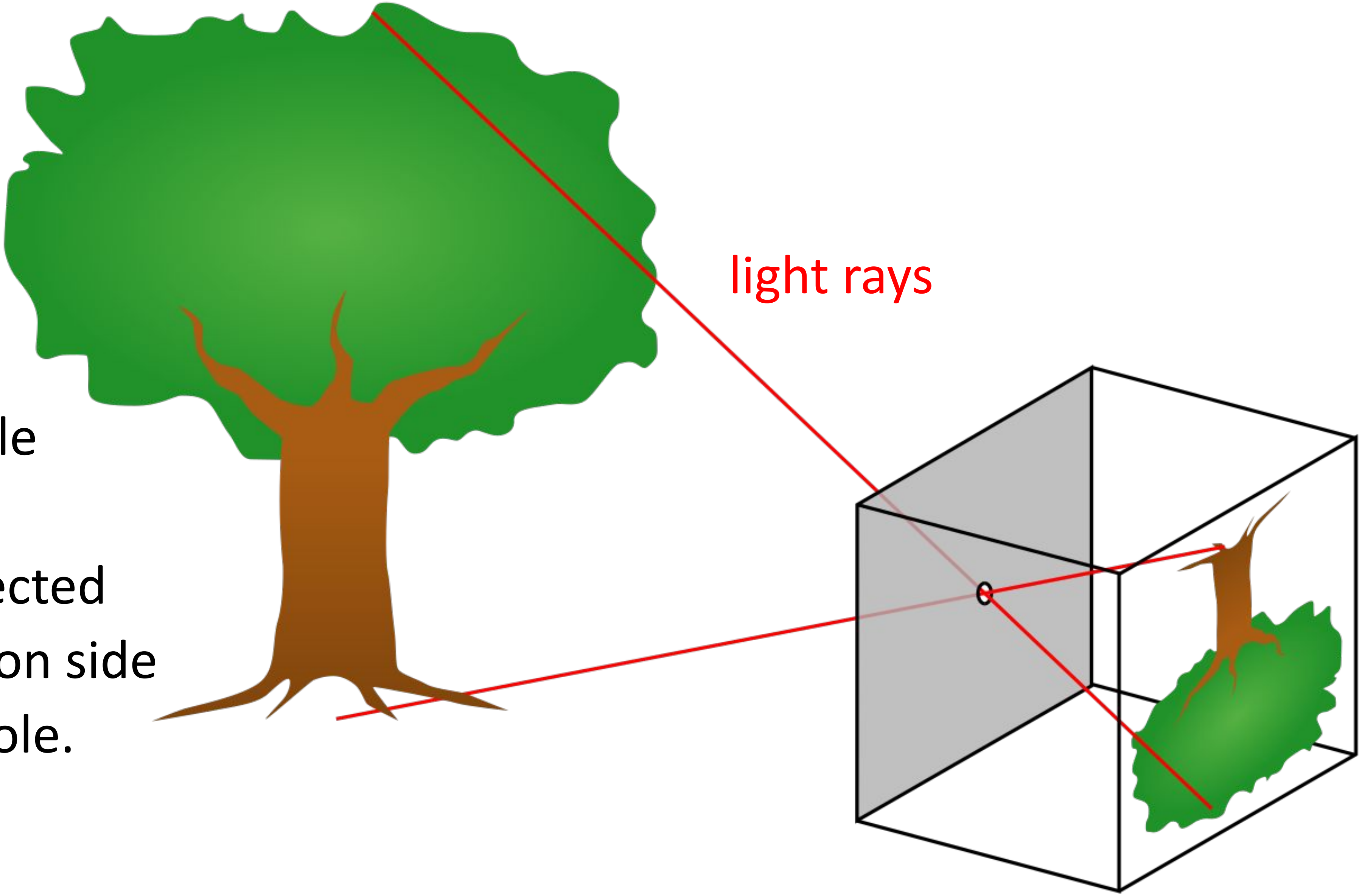
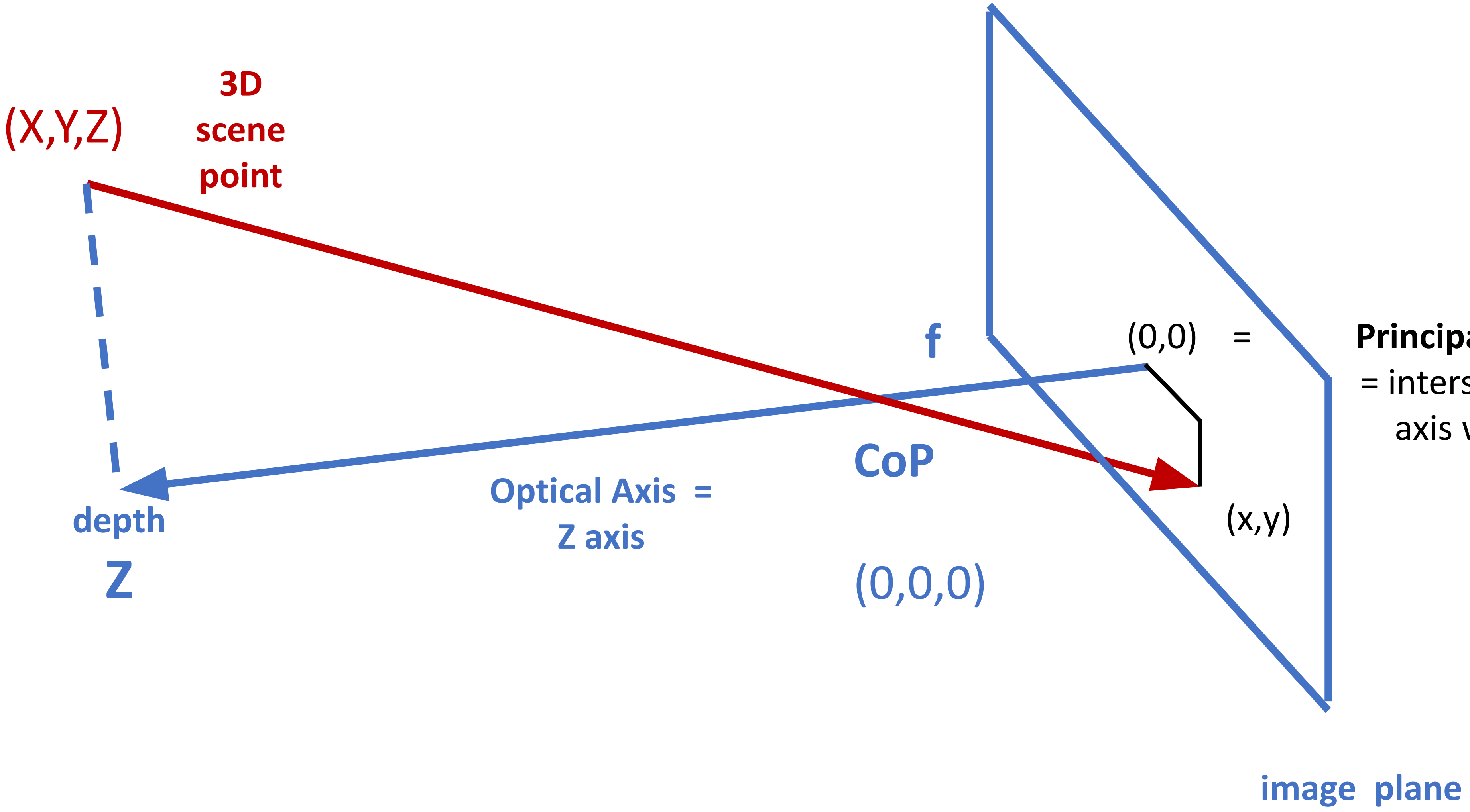


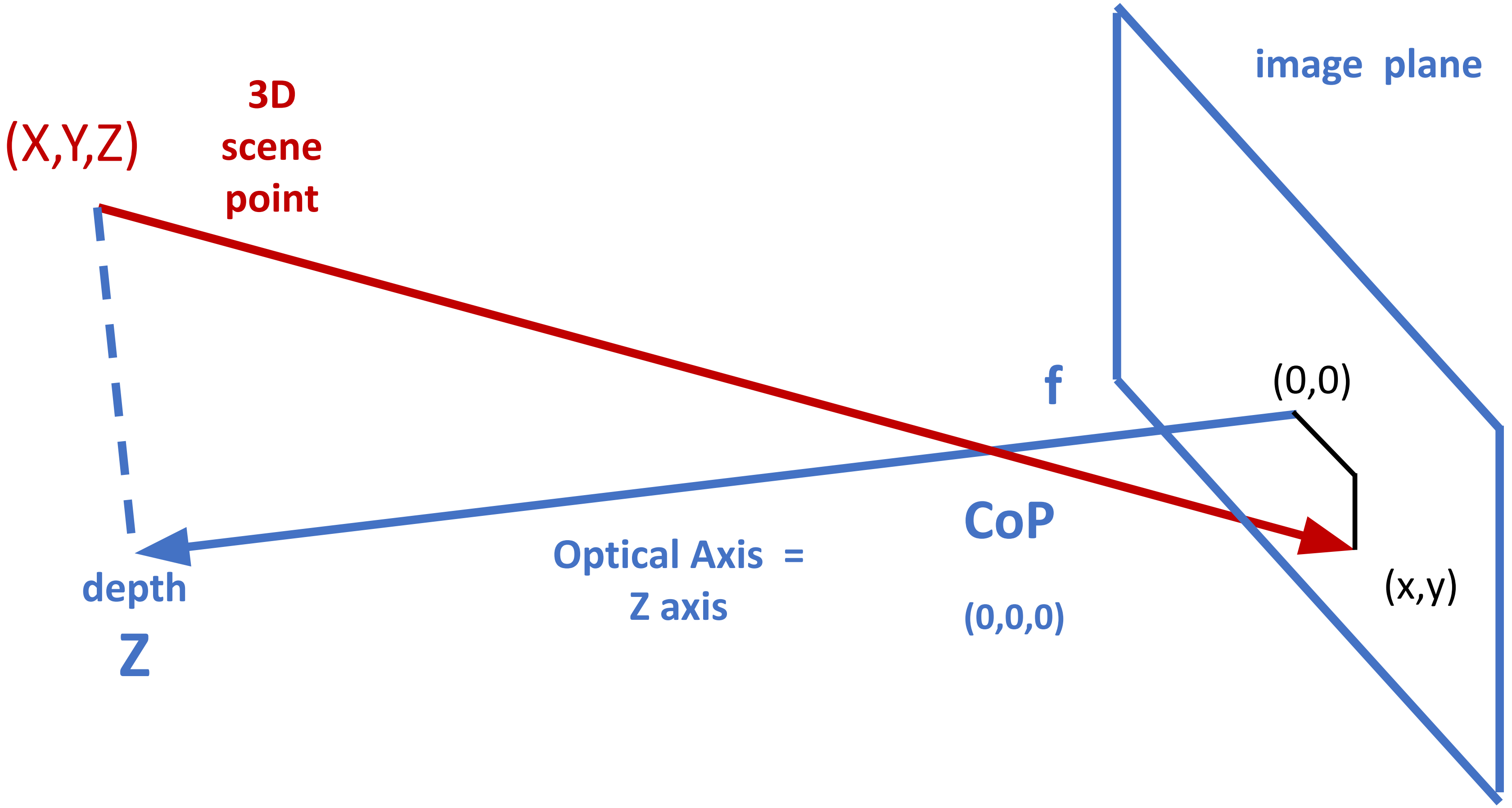
Image credit: public domain, Wikipedia

Ideal Pinhole Model



Principal Point
= intersection of optical axis with image plane

Perspective Projection Equations:



$X/Z = x/f$

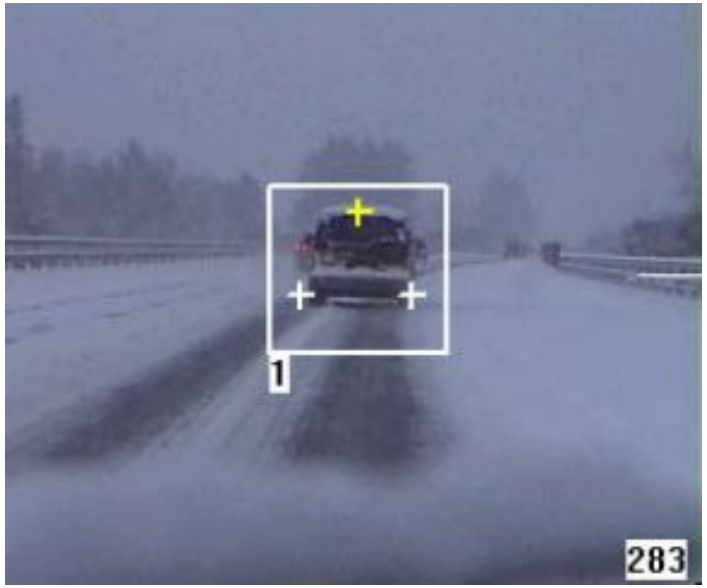
and

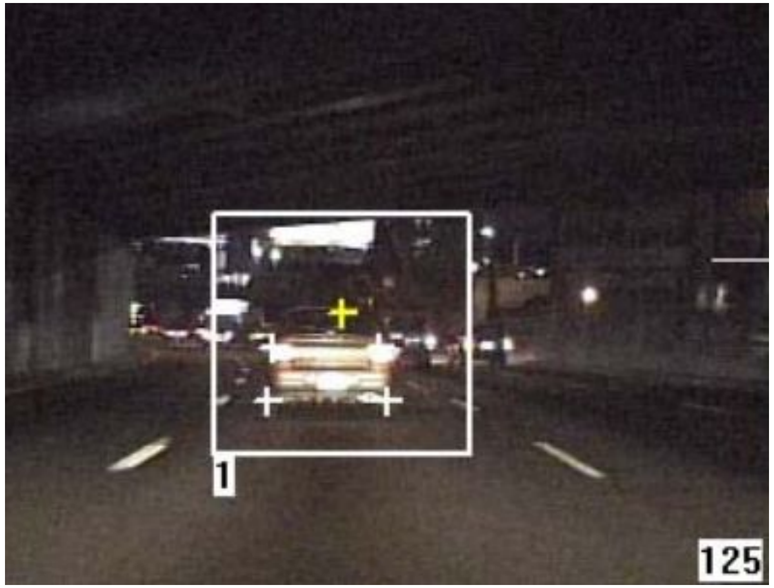
$Y/Z = y/f$

Relate scene (X,Y,Z) and image (x,y) coordinates

Example:
Self-driving Cars:

Estimate distance to car in front





Distance in meters: $Z = c_{horiz} f\, W / w = (22 \text{ pix/mm})(50 \text{ mm})(1.77 \text{ m})/(100 \text{ pix})$

$= 19.47 \text{ m}$

- Typical width of a car: $W = 1.77$ m
- Car width measured in image: $w = 100$ pixel
- focal length $f = 50$ mm
- 35-mm camera: pixel-to-mm conversion $c_{horiz} = 22$ pixel/mm

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Images from Betke & Nguyen, IEEE International Conference on Intelligent Vehicles, **1998**

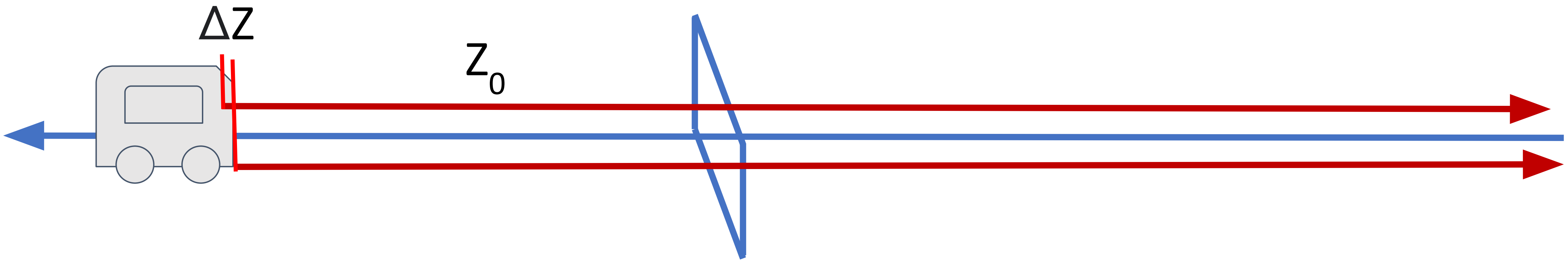
Orthographic Projection

Assume all Z's are approximately at fixed distance Z_0 and $\frac{|\Delta Z|}{Z_0} \ll 1$

i.e., distances of points in the scene do not differ much

Then $x = (f/Z_0)X$ and $y = (f/Z_0)Y$.

Or assume $x = X$ and $y = Y$. => Simplifies image analysis to 2D problem.



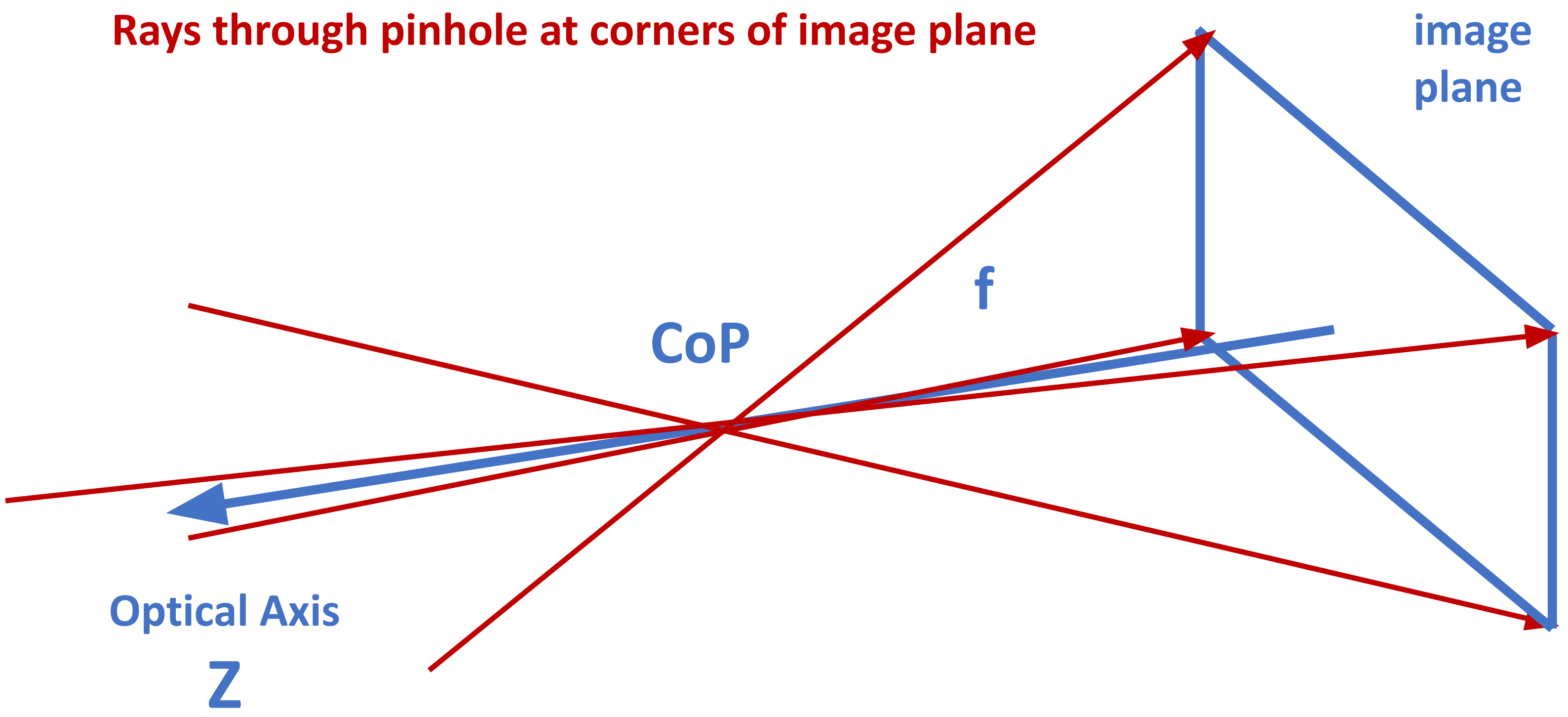
Orthographic vs. Perspective Projection

Orthographic projection is not an appropriate model if

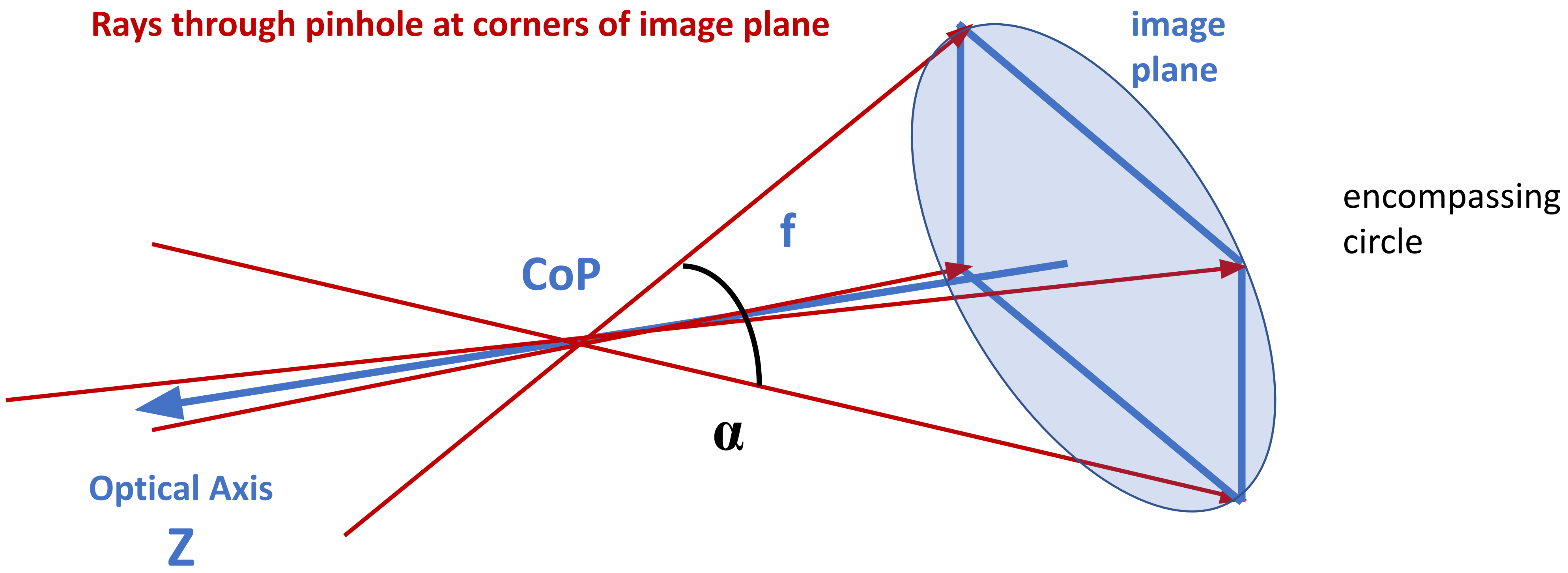
- objects are far from camera and
- objects are not at approximately the same distance

Use perspective projection instead!

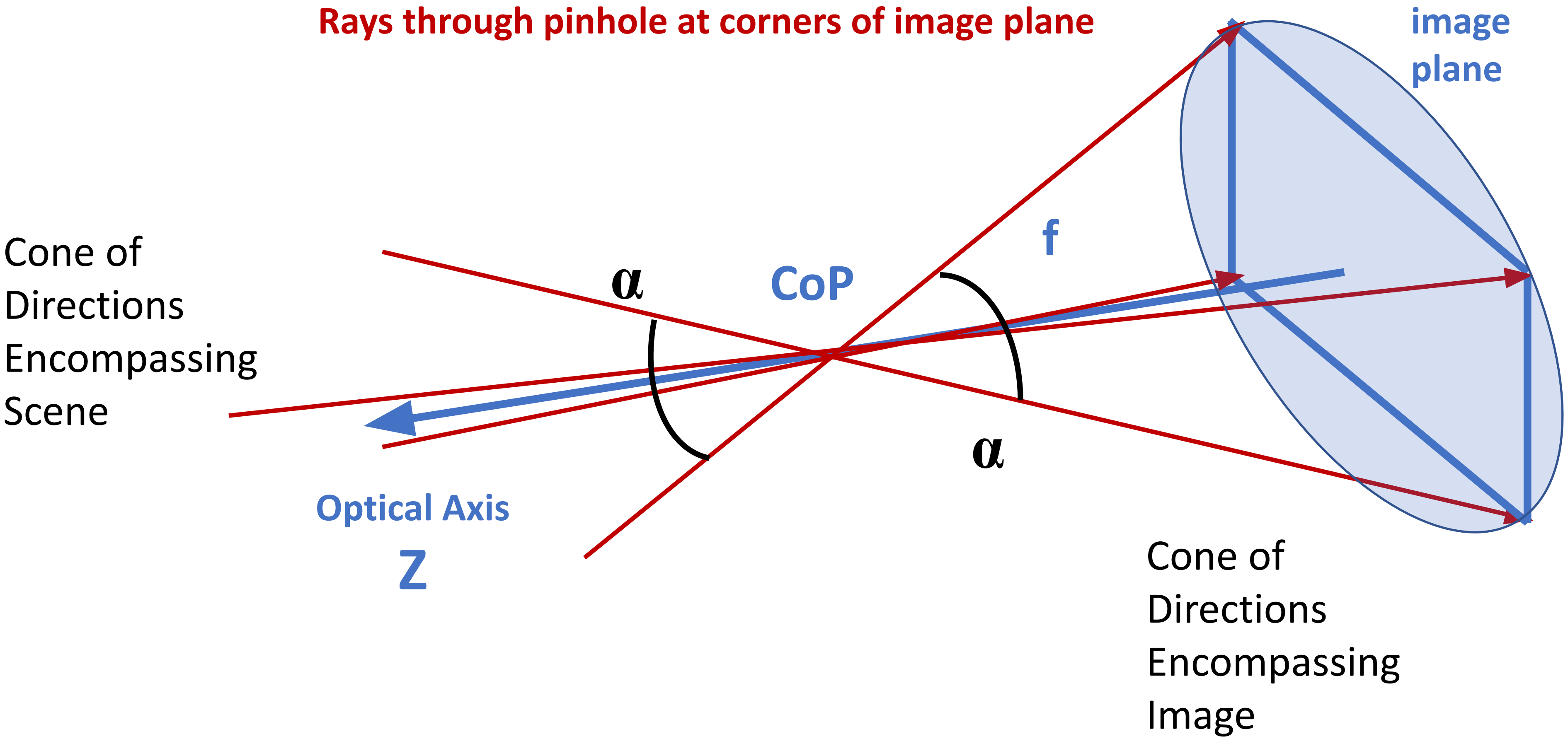
Field of View



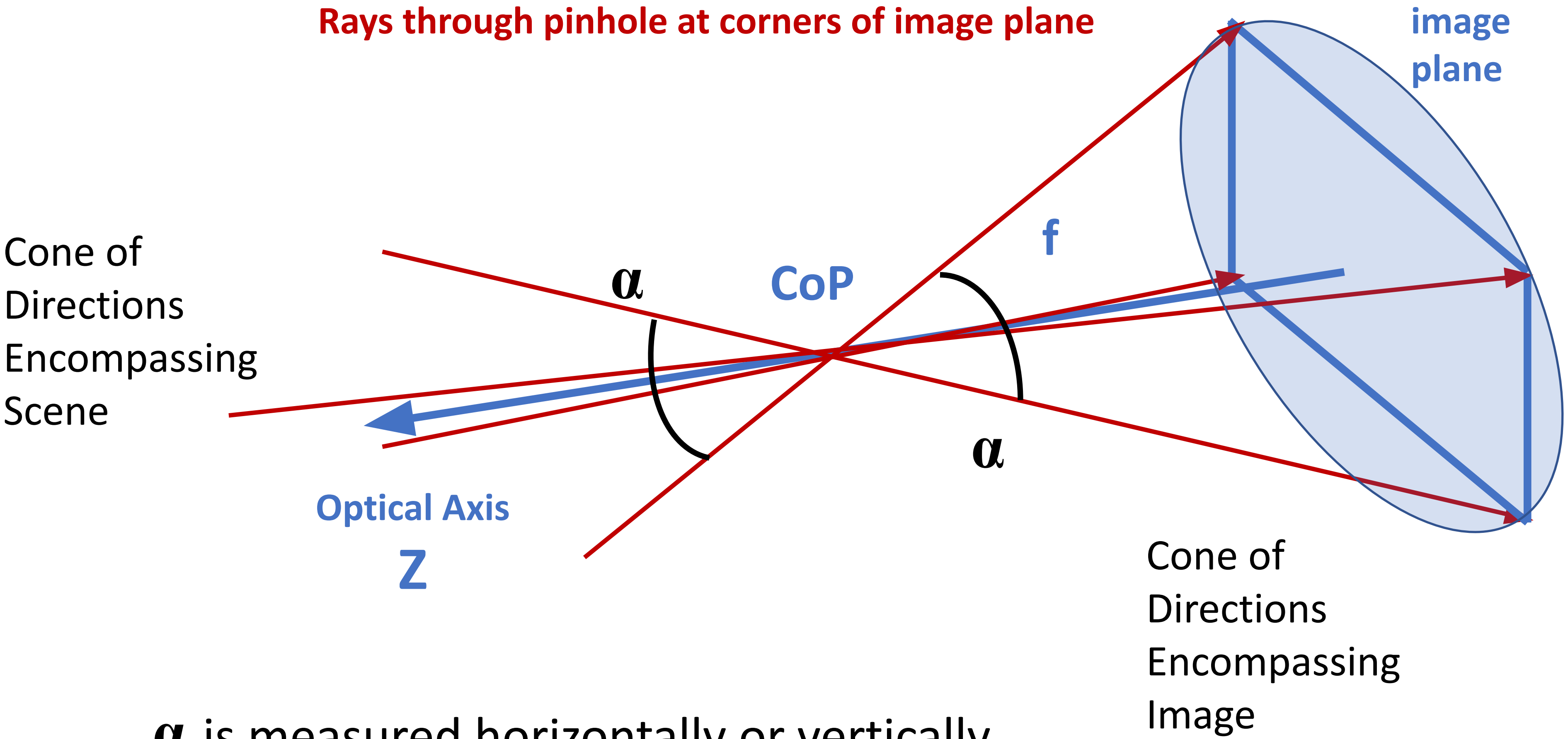
Field of View



Field of View



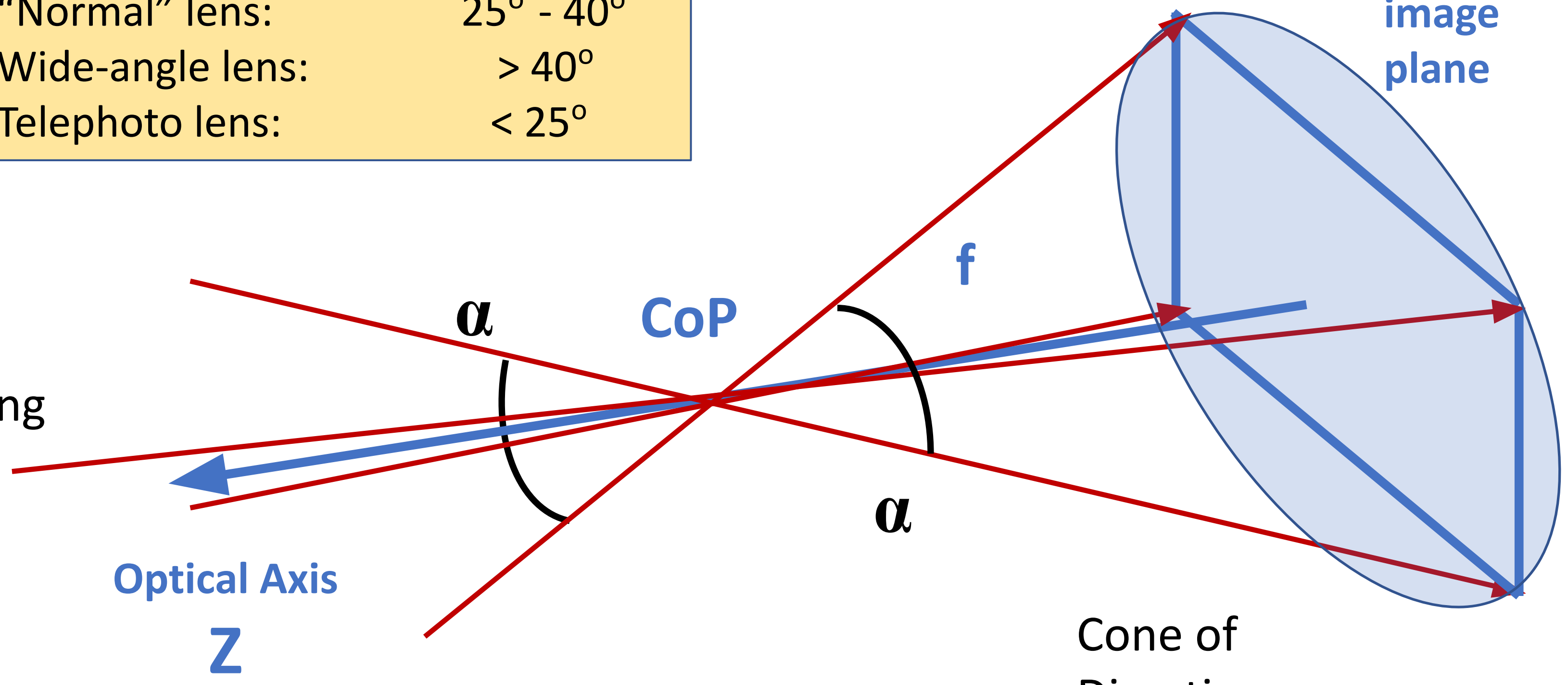
Field of View α



α is measured horizontally or vertically

Field of View of Lenses

"Normal" lens:	25° - 40°
Wide-angle lens:	> 40°
Telephoto lens:	< 25°



α is measured horizontally or vertically

“Normal lenses”

Wide-angle lens

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Image credits: Ashley Pomeroy, fir0002flagstaffotos, Wikipedia

Telephoto lenses

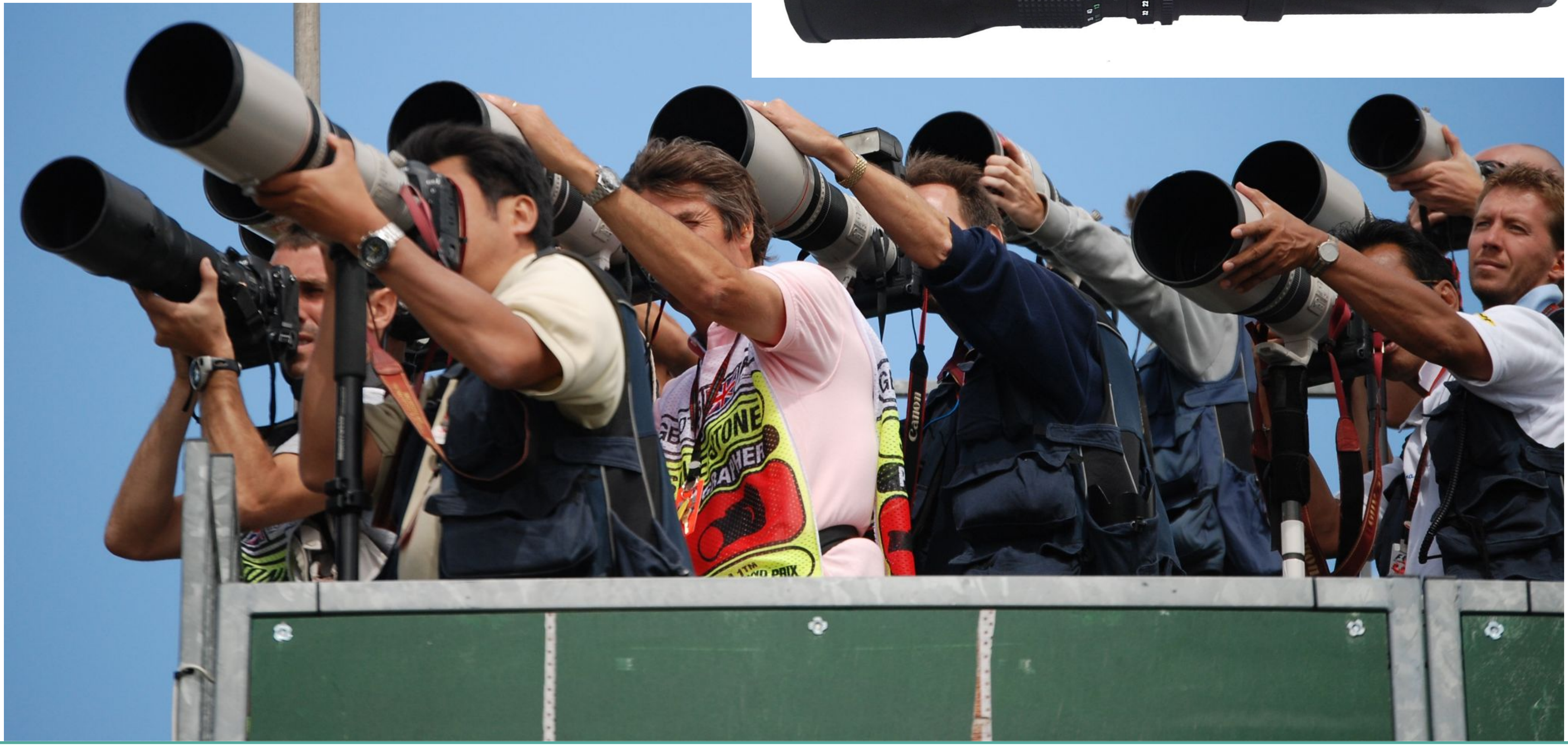

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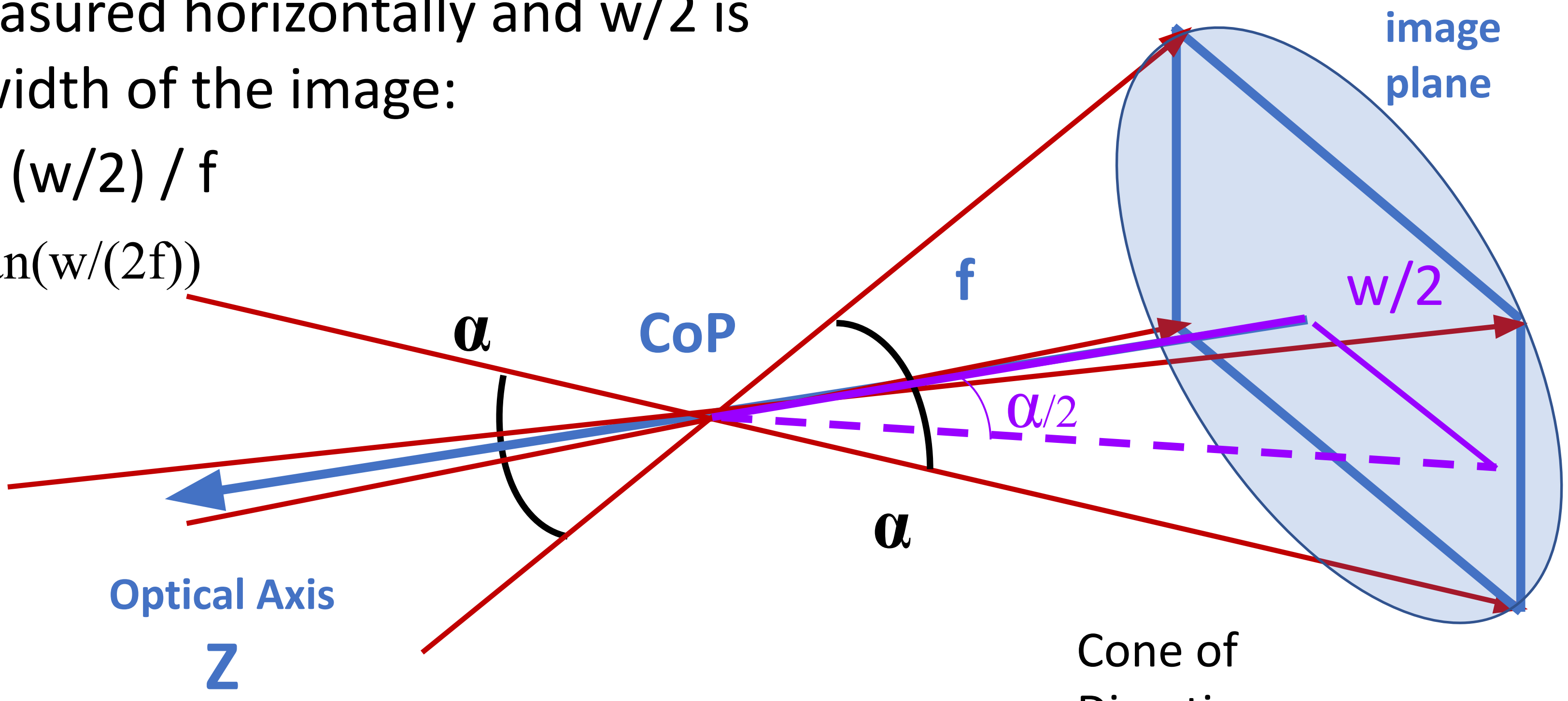
Image credits: Ann64, Dlohcierekim, Wikipedia

Conversion Field of View & Focal Length

If α is measured horizontally and w/2 is half the width of the image:

$\tan \alpha/2 = (w/2) / f$

$\alpha = 2 \arctan(w/(2f))$

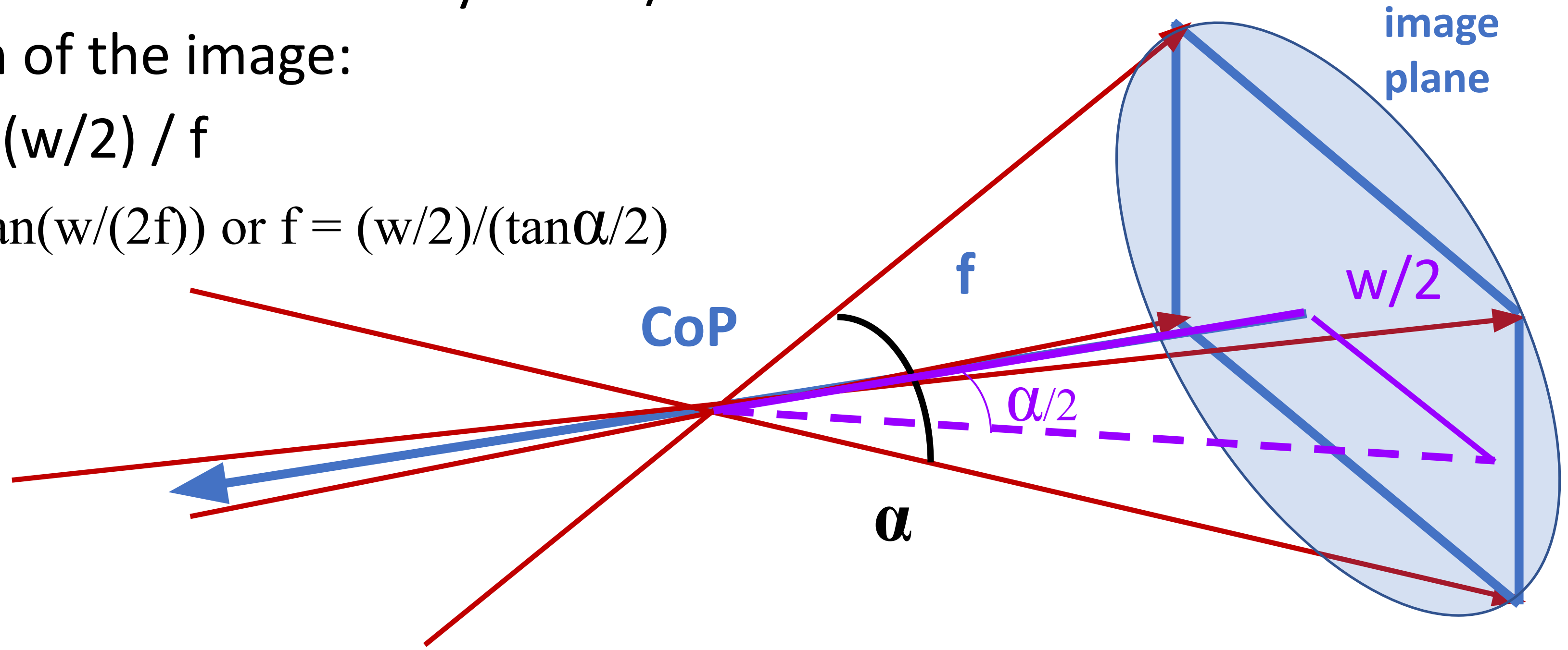


Conversion Field of View & Focal Length

If α is measured horizontally and w/2 is half the width of the image:

$\tan \alpha/2 = (w/2) / f$

$\alpha = 2 \arctan(w/(2f))$ or $f = (w/2)/(\tan\alpha/2)$

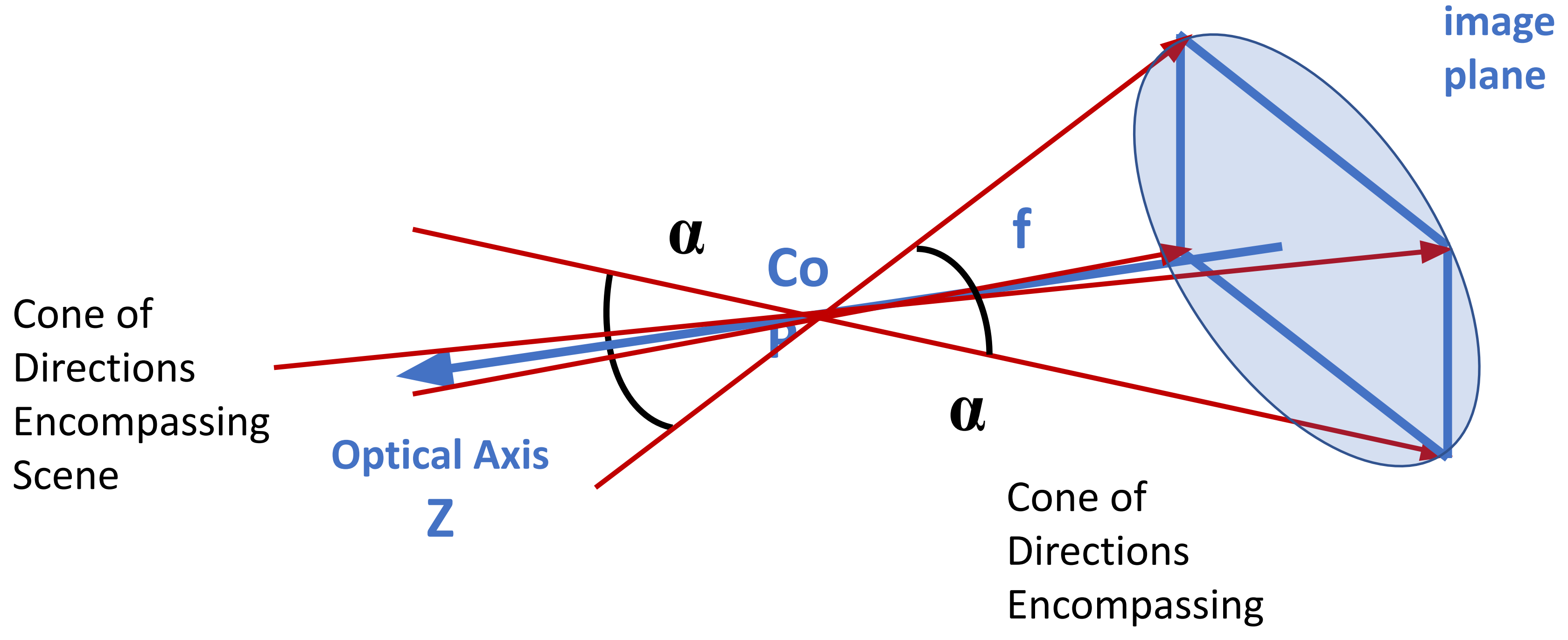


"Normal" lens:	25° - 40°	(f: 48 mm - 79 mm for w=35 mm film)
Wide-angle lens:	> 40°	(< 48 mm)
Telephoto lens:	< 25°	(> 79 mm)

Field of View of Lenses

"Rule of thumb:"

"Normal" lens:	25° - 40°	
Wide-angle lens:	> 40°	Use perspective projection (f << image size)
Telephoto lens:	< 25°	Use orthographic projection (i.e., rays parallel to optical axis)



α is measured horizontally or vertically

Images Captured with Wide-angle Lens

The distance between bottles was kept constant.

The wider the lens (= shorter f),
the more scene is included and
the smaller the blue bottle appears.

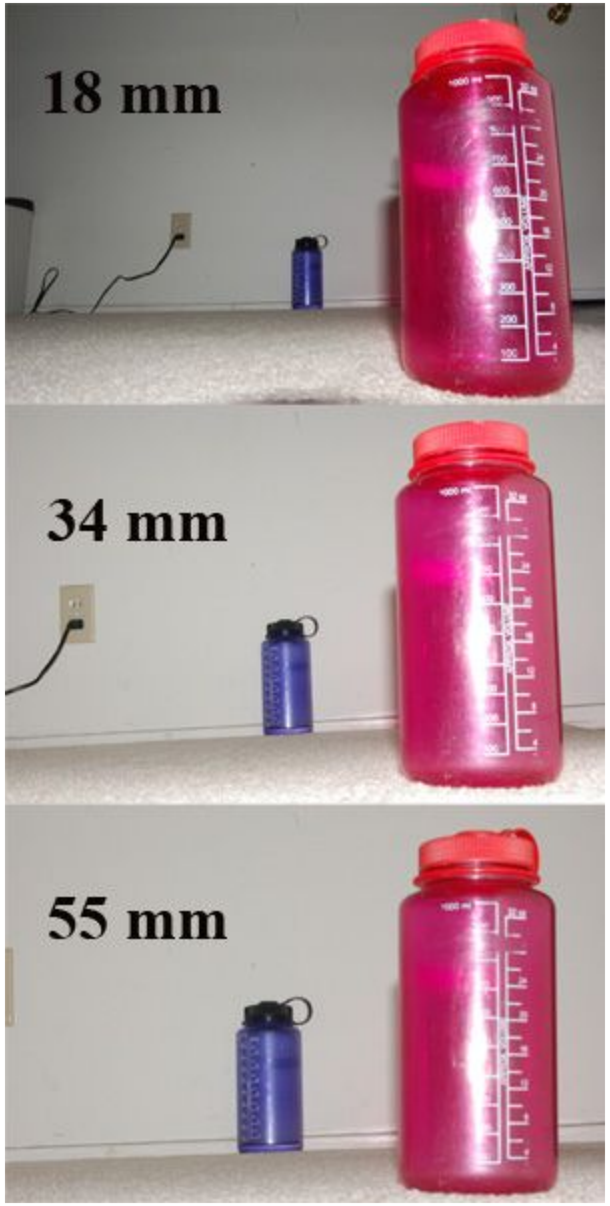


Image credits: Jcbrooks, Wikipedia

Images Captured with Wide-angle, “Normal,” and Telephoto lenses

Which focal length belongs to which image? Is A, B, or C correct?




A:	28 mm	50 mm	70 mm	210 mm
B:	210 mm	70 mm	28 mm	50 mm
C:	210 mm	28 mm	70 mm	50 mm

Image credits: Koyaanis Qatsi, Wikipedia

Images Captured with Wide-angle, “Normal,” and Telephoto lenses

28 mm

50 mm

70 mm

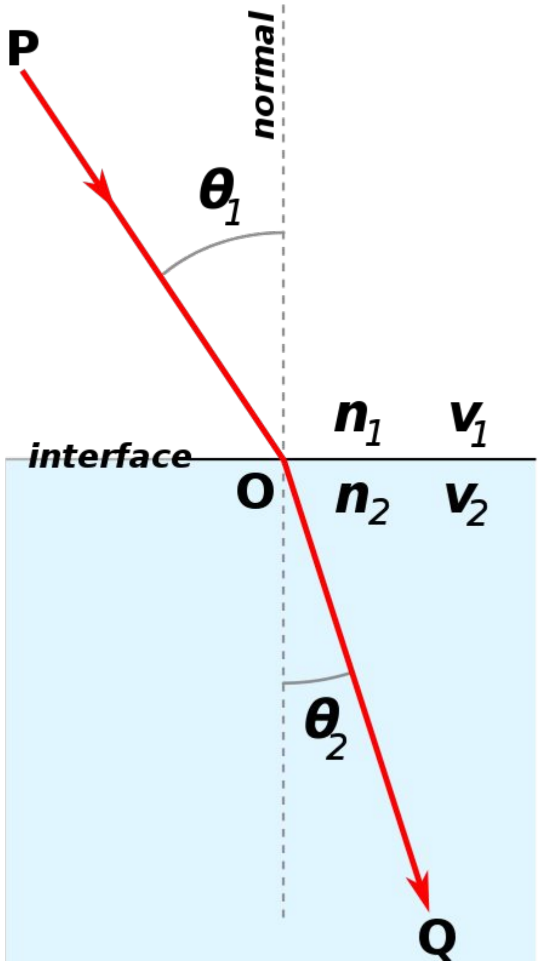
210 mm

Image credits: Koyaanis Qatsi, Wikipedia

What is a lens?

A simple lens is an object made of transparent material that focuses a light beam by means of refraction. Refraction is the phenomenon that traveling waves (here light) change their direction.

Law of Refraction



Refractive indices n_1, n_2 of two media

Velocities v_1, v_2

Incident angles θ_1

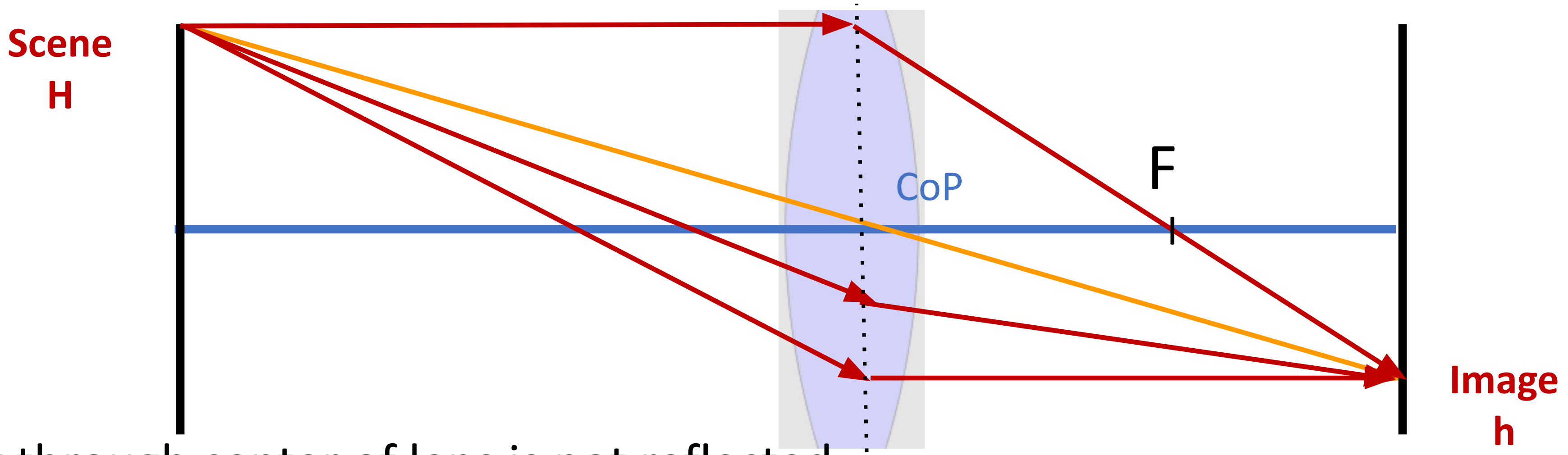
Refracted angle θ_2:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \frac{v_2}{v_1}$$

Also called Snell's Law

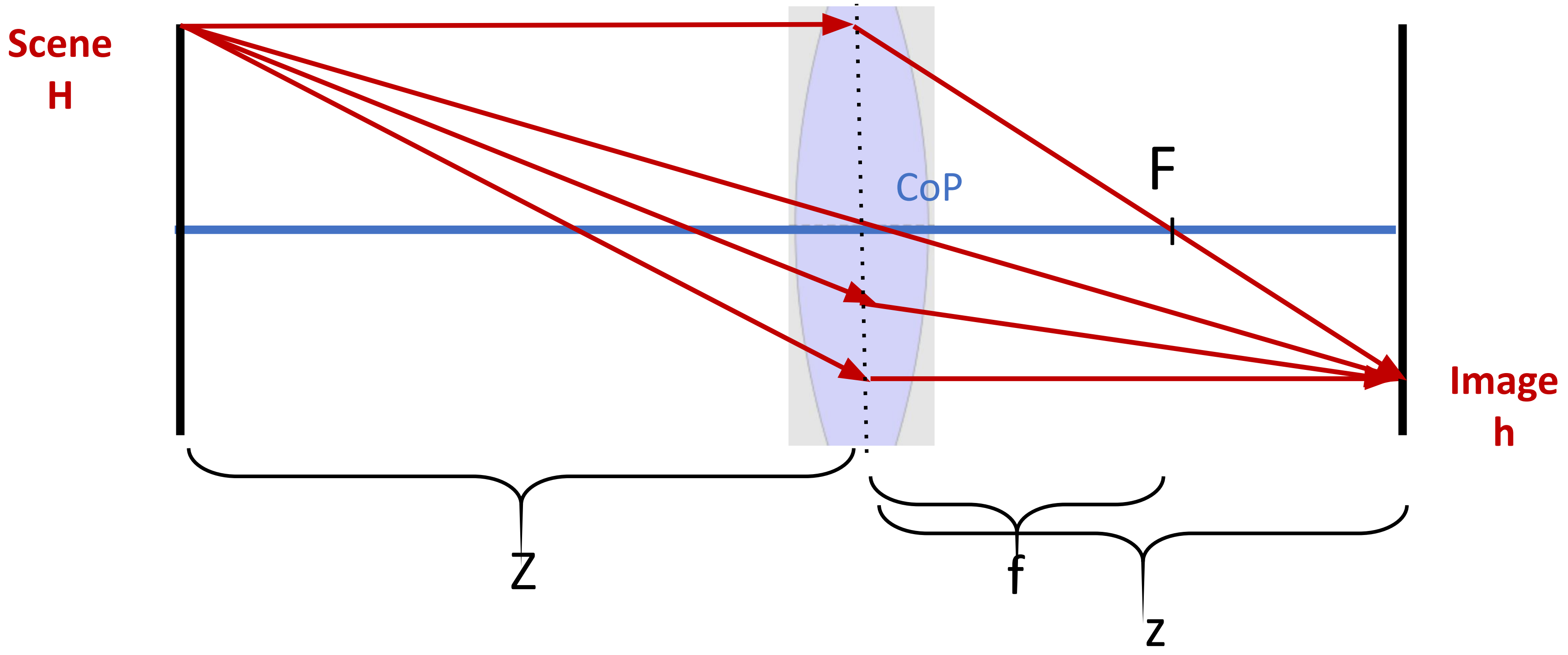
Drawing credit: sawmis, Wikipedia

Ideal Lens: Same projection as pinhole camera but gathers more light



1) Ray through center of lens is not reflected
2) Parallel ray intersects optical axis at F from CoP
3) Spherical shape of lens -> well-focused images only at particular distance
4) Well-focused system = all rays from scene point H reach same image point h as central ray

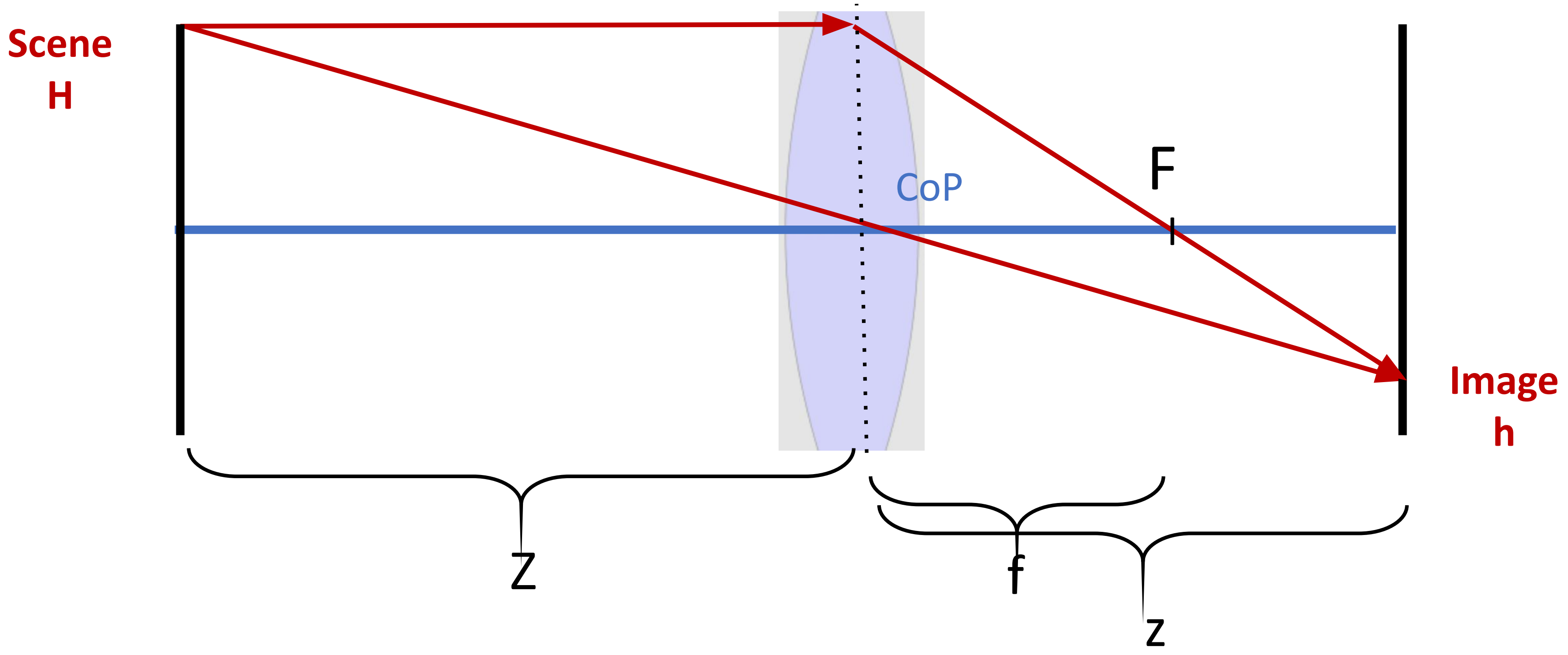
Ideal Lens: Same projection as pinhole camera but gathers more light



z = principal distance
Z = depth

F = Focal point
f = focal length

Derivation of the Lens Equation



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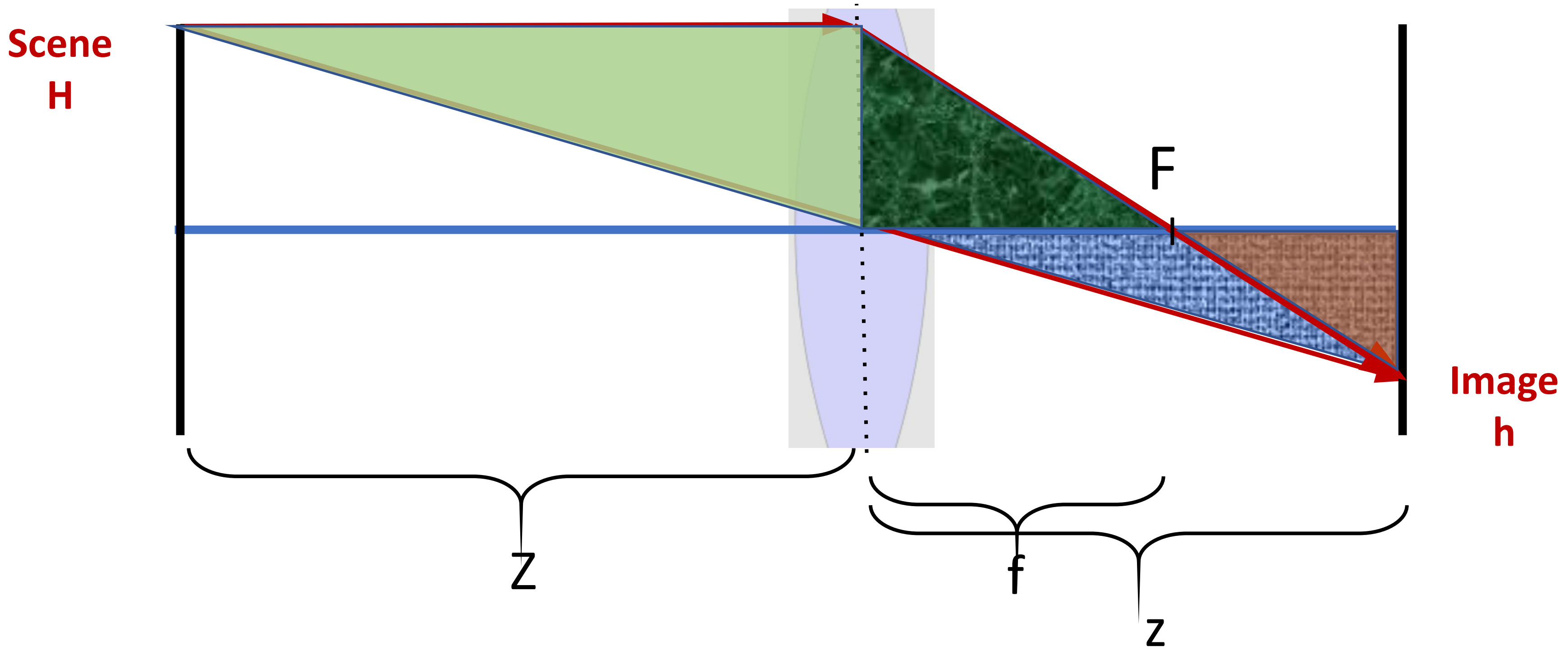

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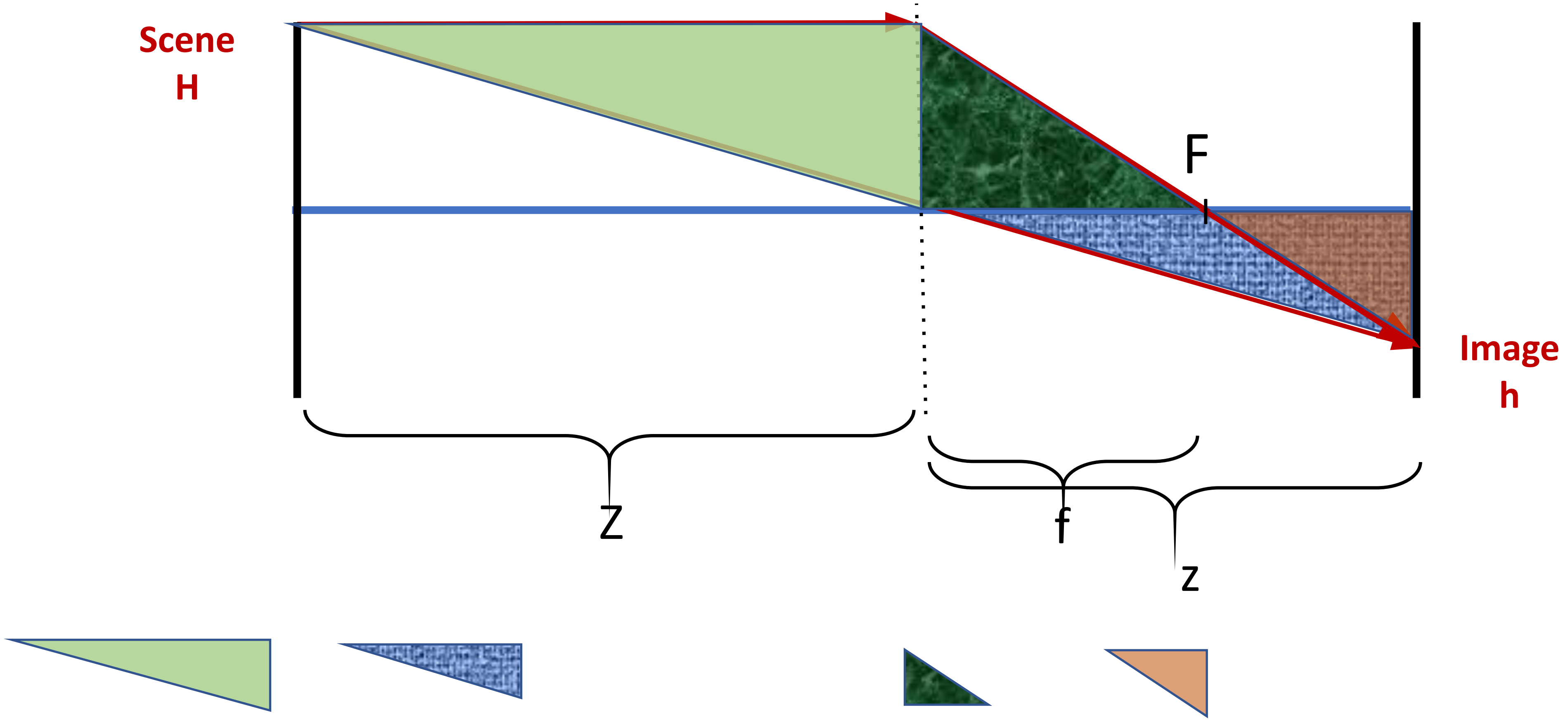
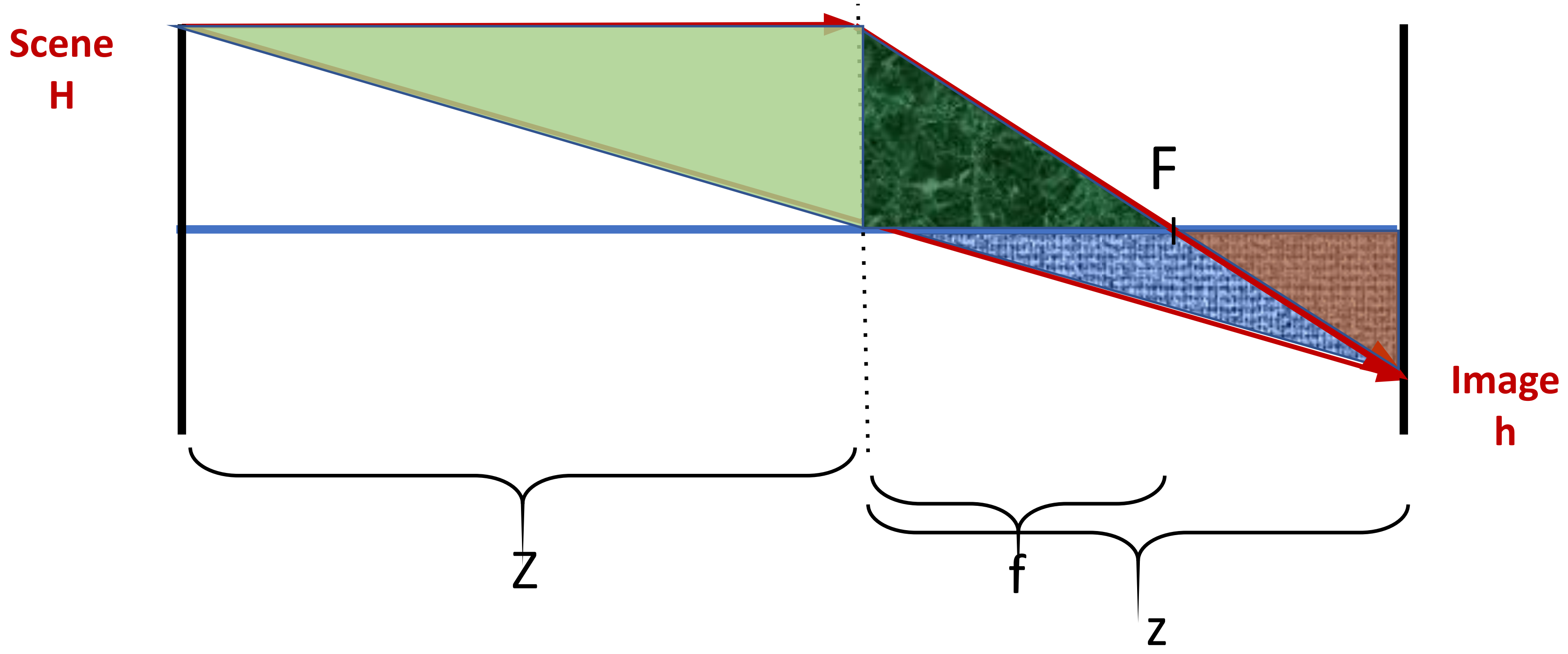
Derivation of the Lens Equation



H/Z = h/z and H/f = h/(z-f)

Derivation of the Lens Equation



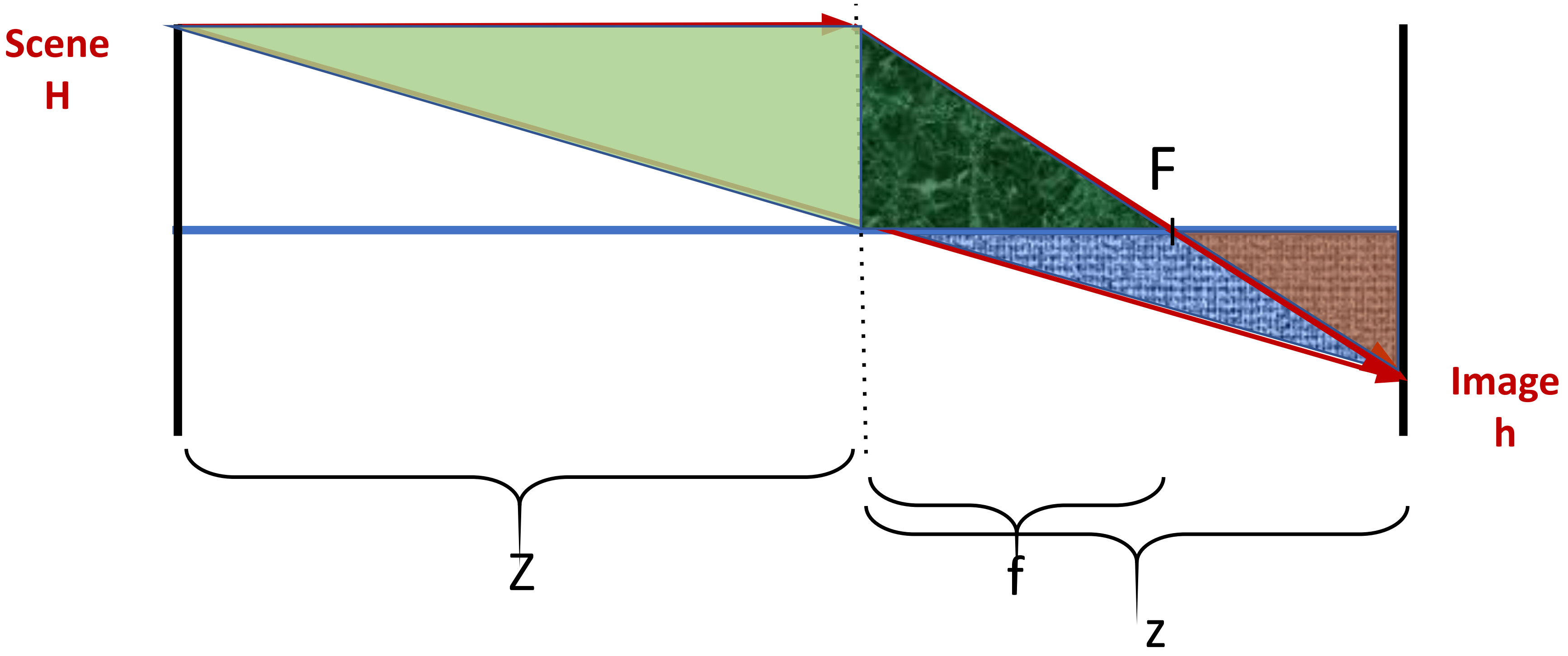
H=h Z/z   substitute into:

f/H = (z-f)/h

f/(h Z/z) = (z-f)/h

H/Z = h/z   and   H/f = h/(z-f)

Derivation of the Lens Equation



H=h Z/z   substitute into:

f/H = (z-f)/h

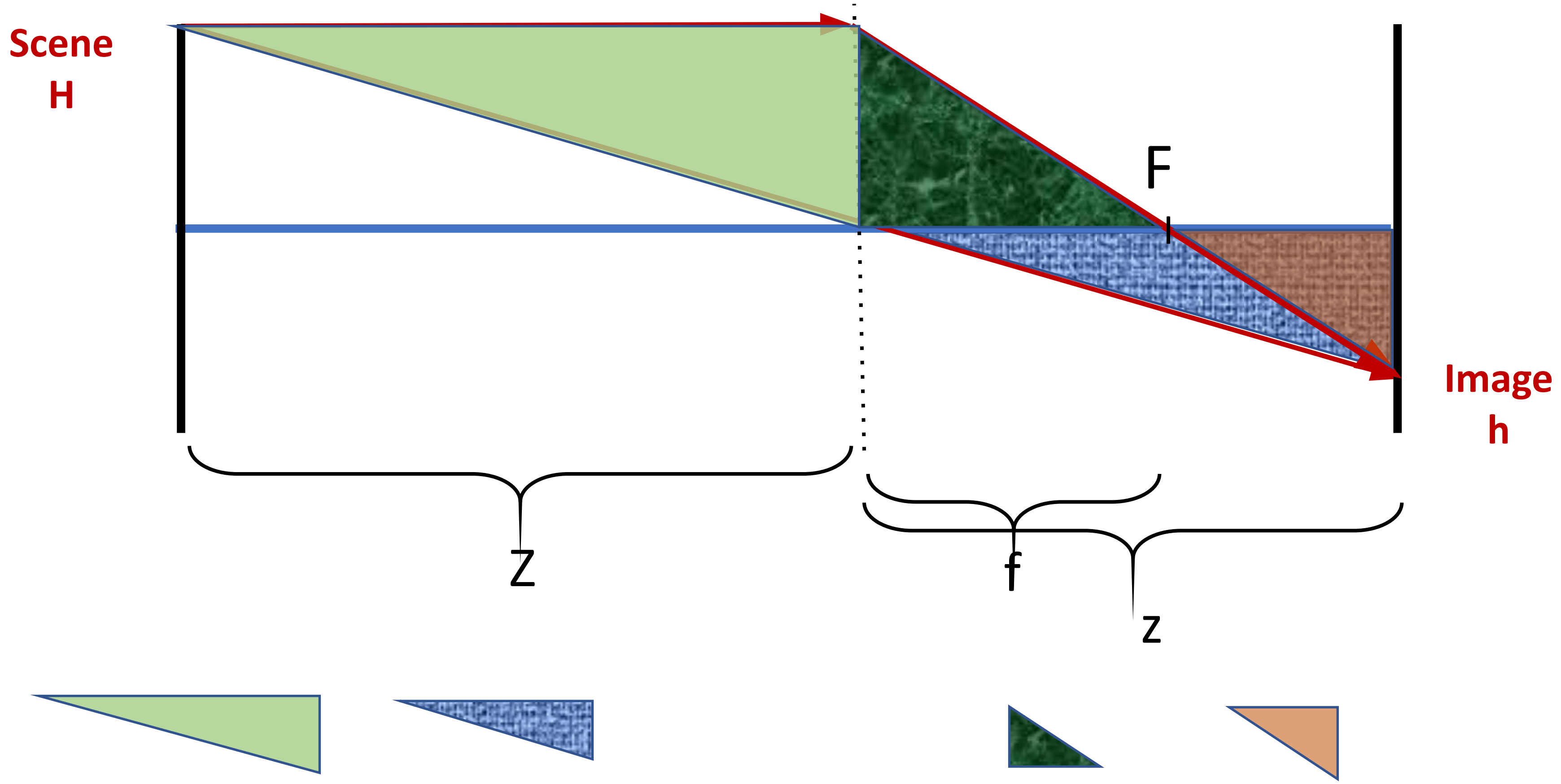
f/(h Z/z) = (z-f)/h

Multiply by h:

f z/Z   = z-f

H/Z = h/z   and   H/f = h/(z-f)

Derivation of the Lens Equation



$H/Z \quad = \quad h/z$ and $H/f \quad = \quad h/(z-f)$

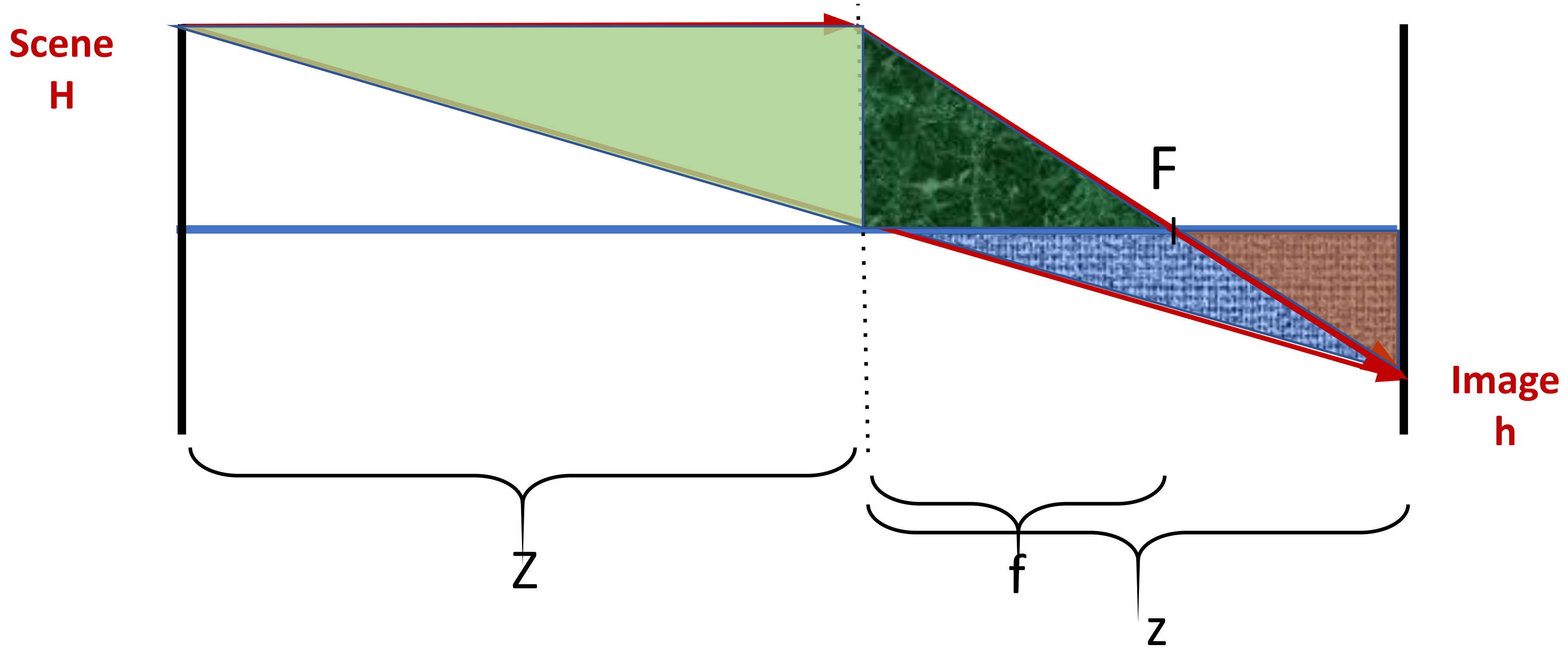
$H = h\,Z/z$ substitute into:
$f/H = (z-f)/h$

$f/(h\,Z/z) = (z-f)/h$

Multiply by h:
$f\,z/Z = z-f$

Add f:
$f\,z/Z + f = z$

Derivation of the Lens Equation



$H/Z = h/z$ and $H/f = h/(z-f)$

$H = h\,Z/z$ substitute into:
$f/H = (z-f)/h$

$f/(h\,Z/z) = (z-f)/h$

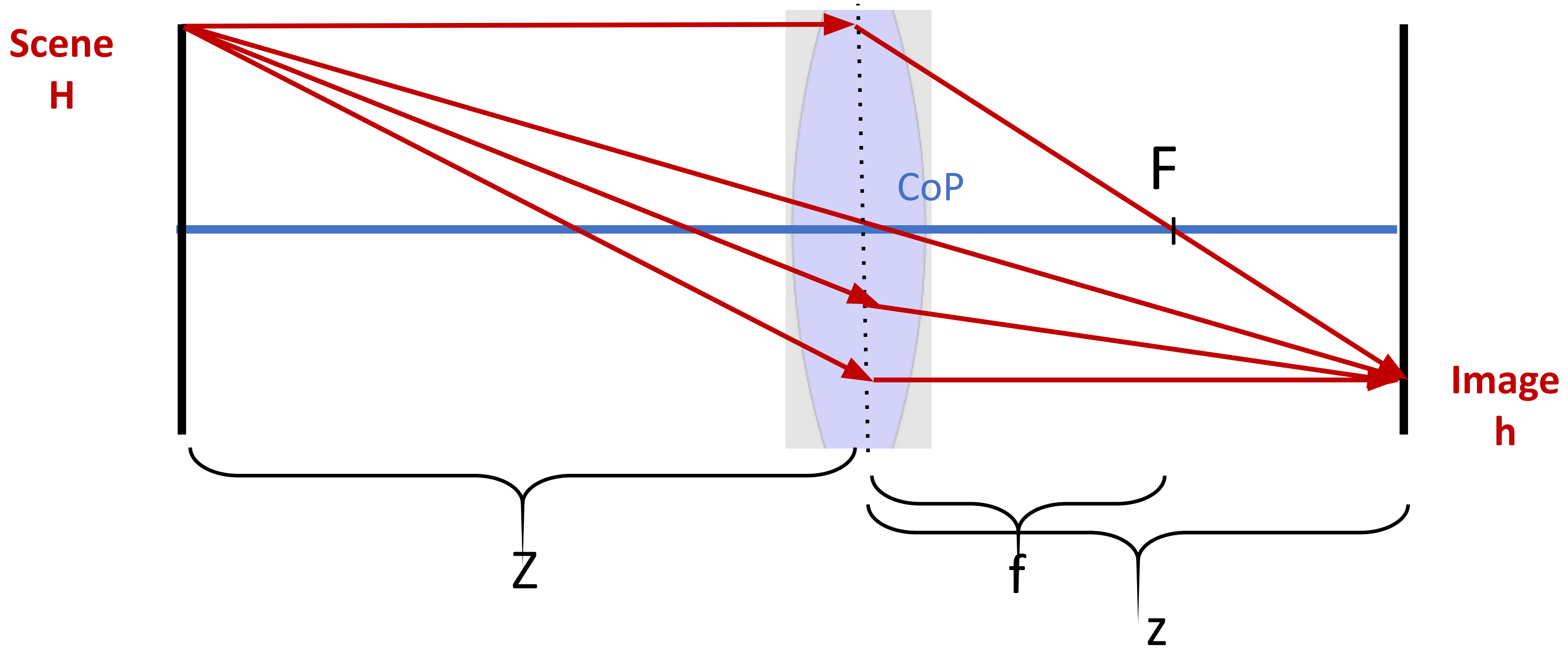
Multiply by h:
$f\,z/Z = z-f$

Add f:
$f\,z/Z + f = z$

Divide by f and z:

$$1/Z + 1/z = 1/f$$

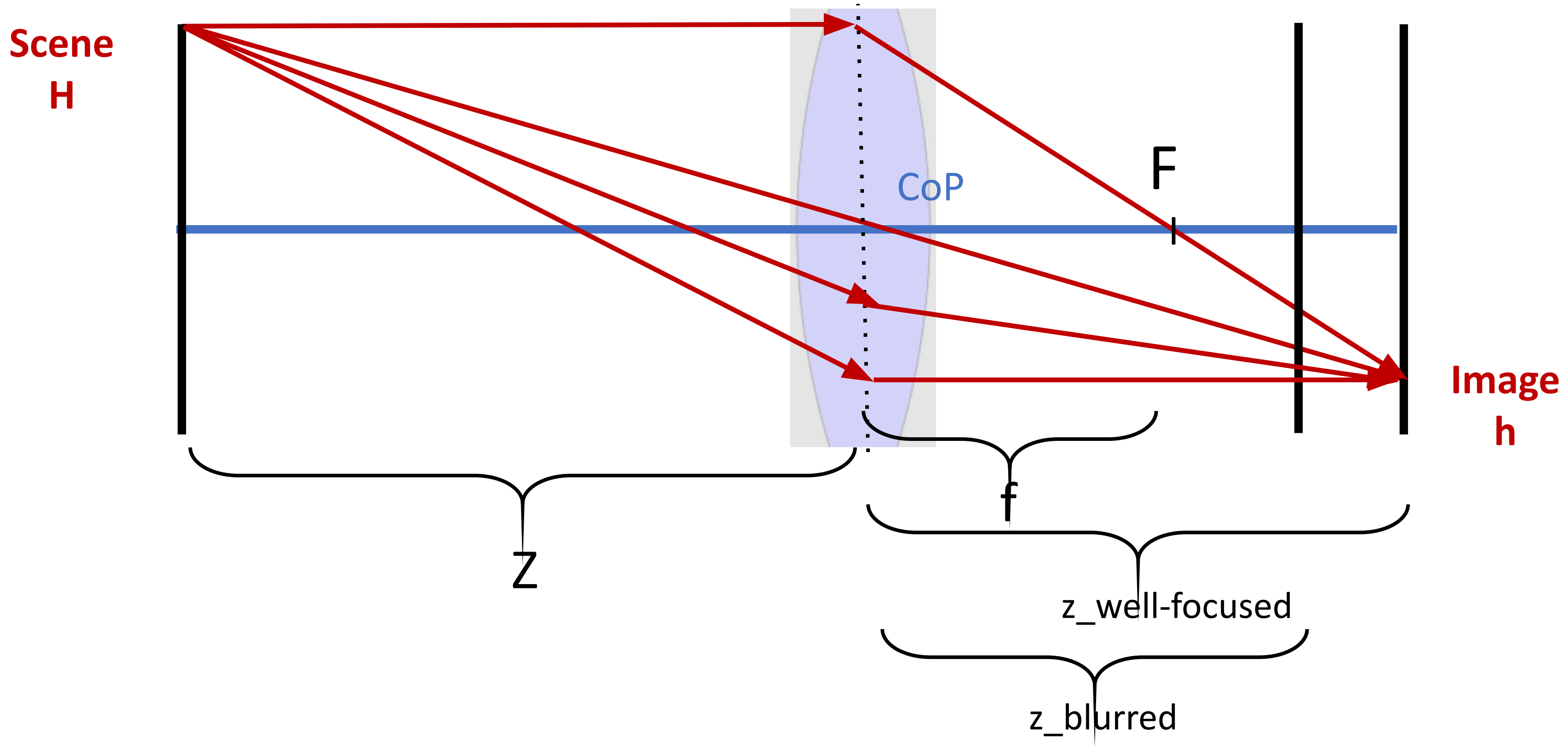
Interpretation of the Lens Equation



$1/Z + 1/z = 1/f$

Lens equation determines how far image plane can be placed to have image in focus

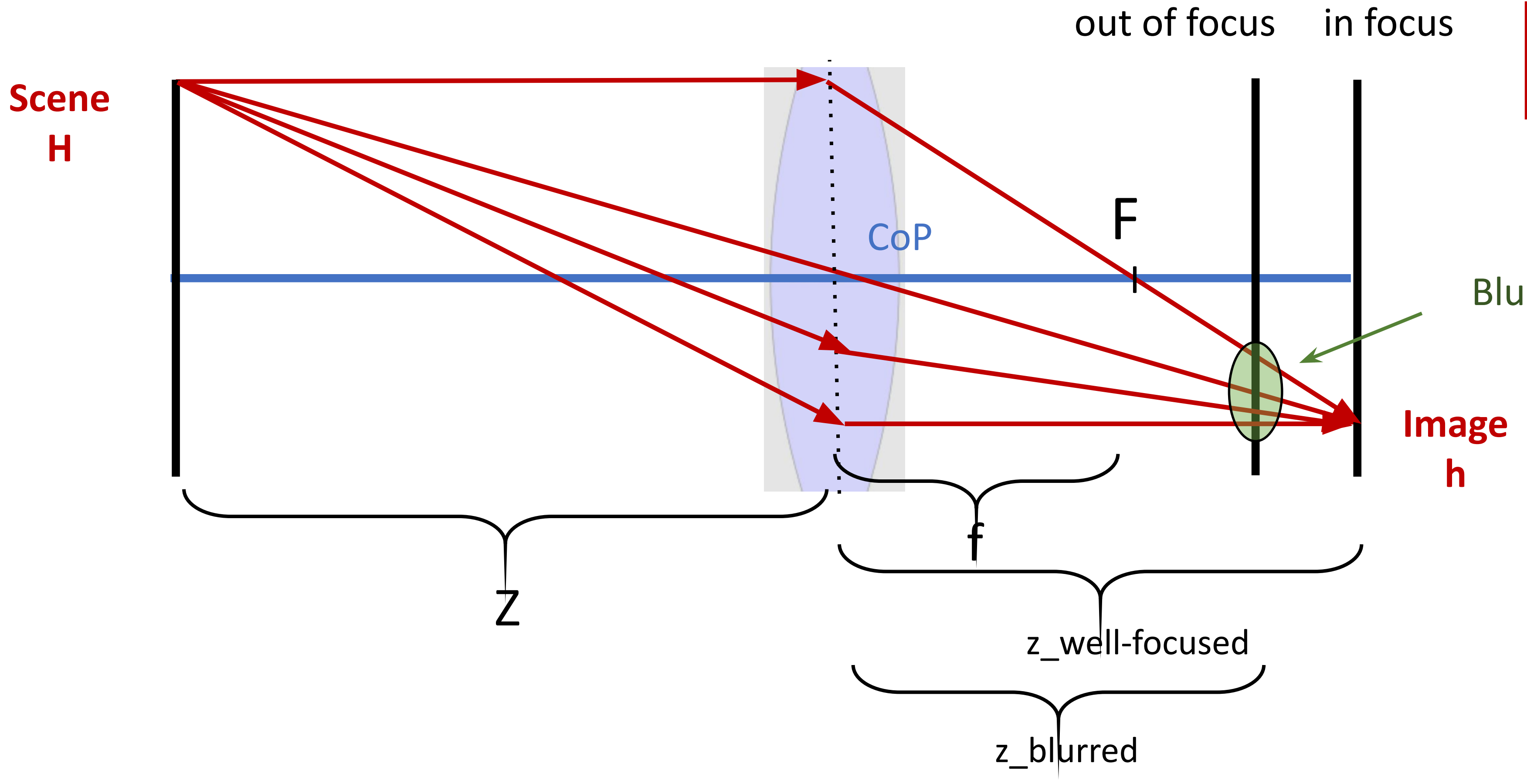
Interpretation of the Lens Equation



$$1/Z + 1/z = 1/f$$

Lens equation determines how far image plane can be placed to have image in focus

Interpretation of the Lens Equation

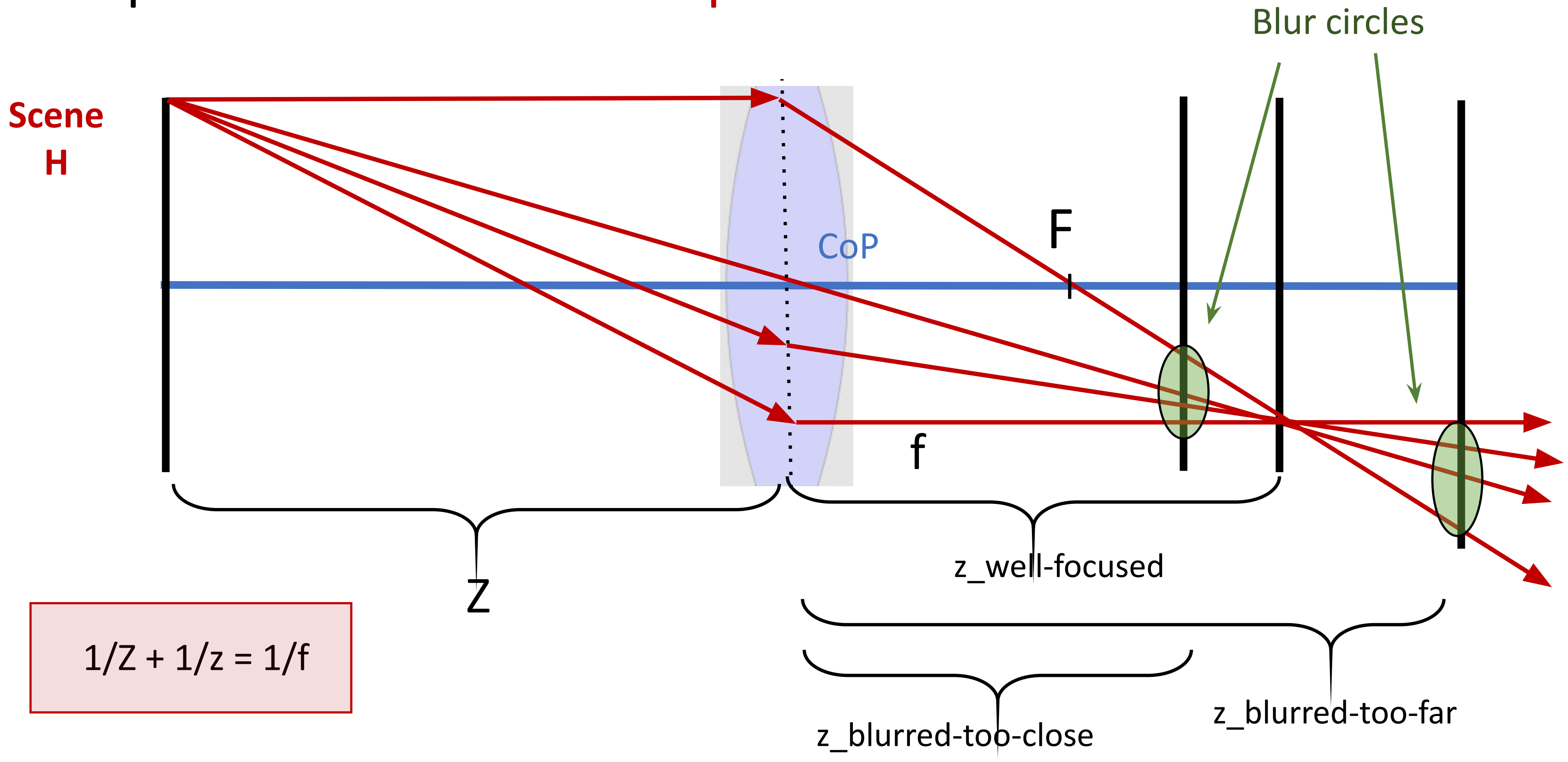


1/Z + 1/z = 1/f

Blur circle: Light from scene point is not bundled in a single spot on the image plane

Lens equation determines how far image plane can be placed to have image in focus

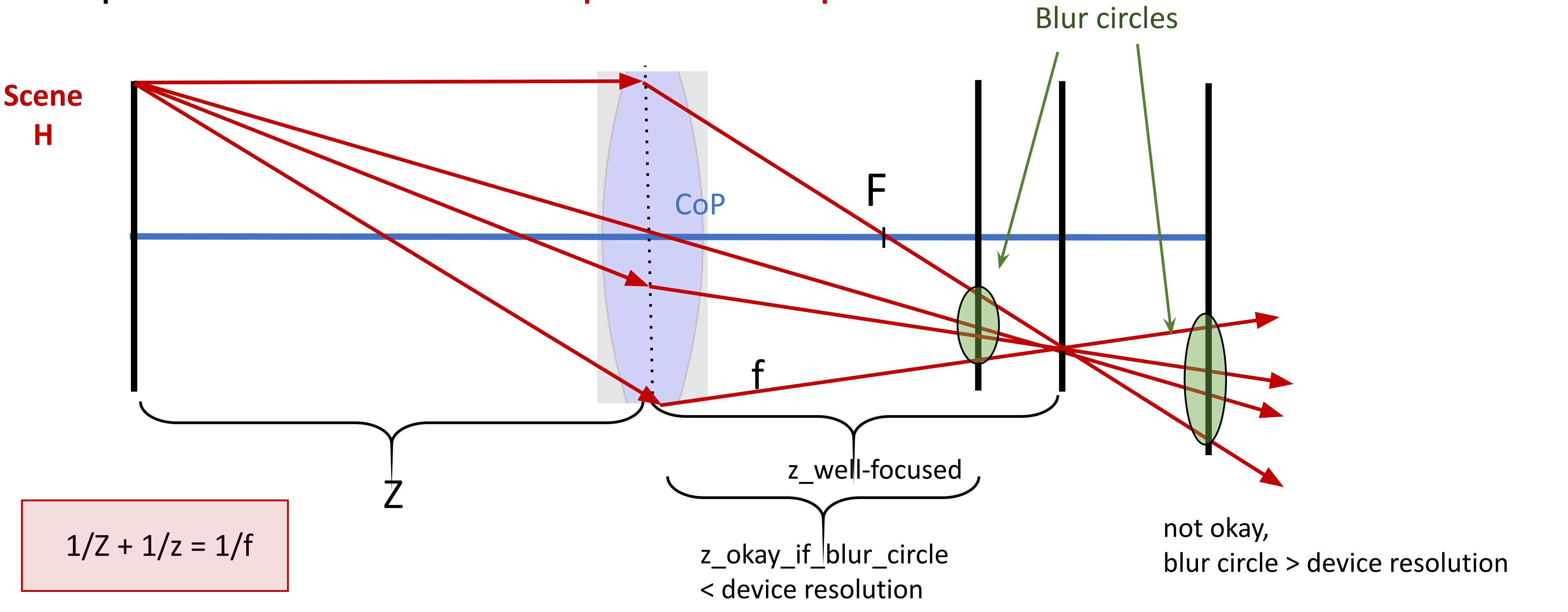
Interpretation of the Lens Equation



$1/Z + 1/z = 1/f$

Lens equation determines how far image plane can be placed to have image in focus

Interpretation of the Lens Equation: Depth of Focus

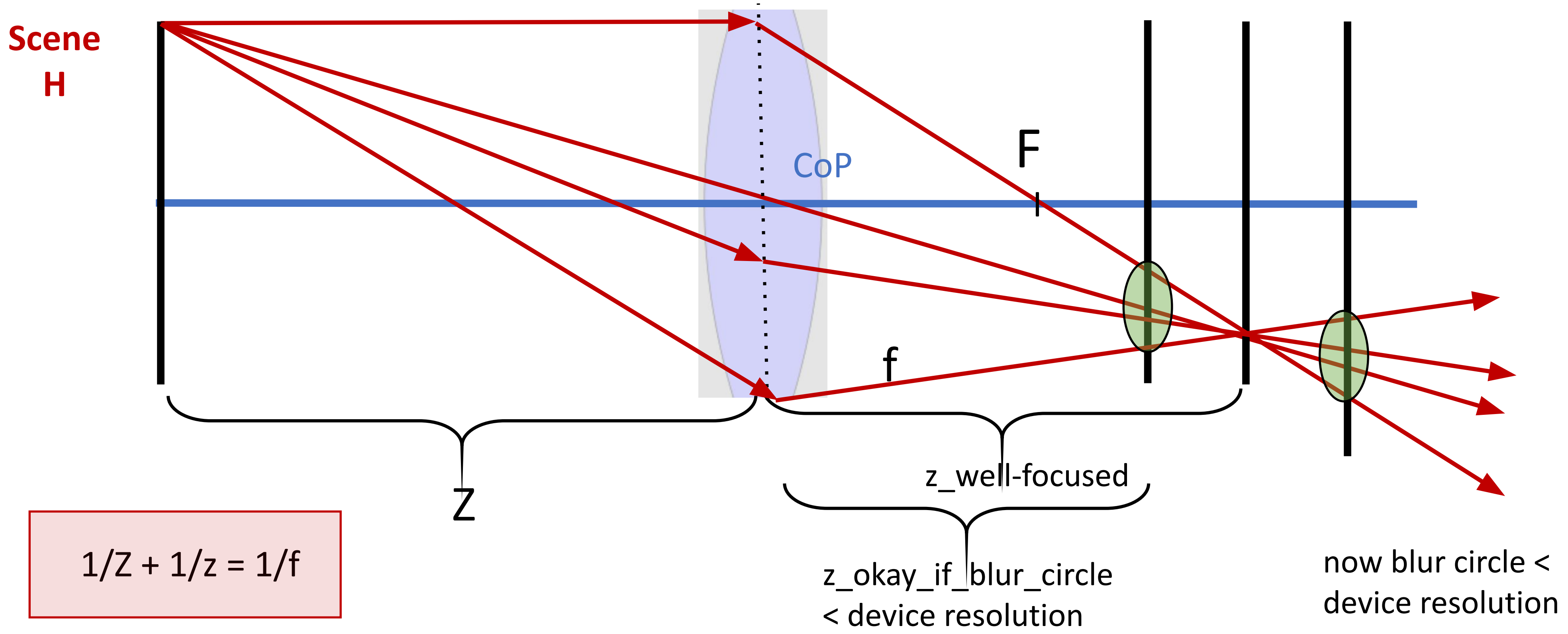


$1/Z + 1/z = 1/f$

Depth of focus = range of image plane placement so that objects are focused sufficiently well

Blur circle must be < resolution of the image device

Interpretation of the Lens Equation: Depth of Focus

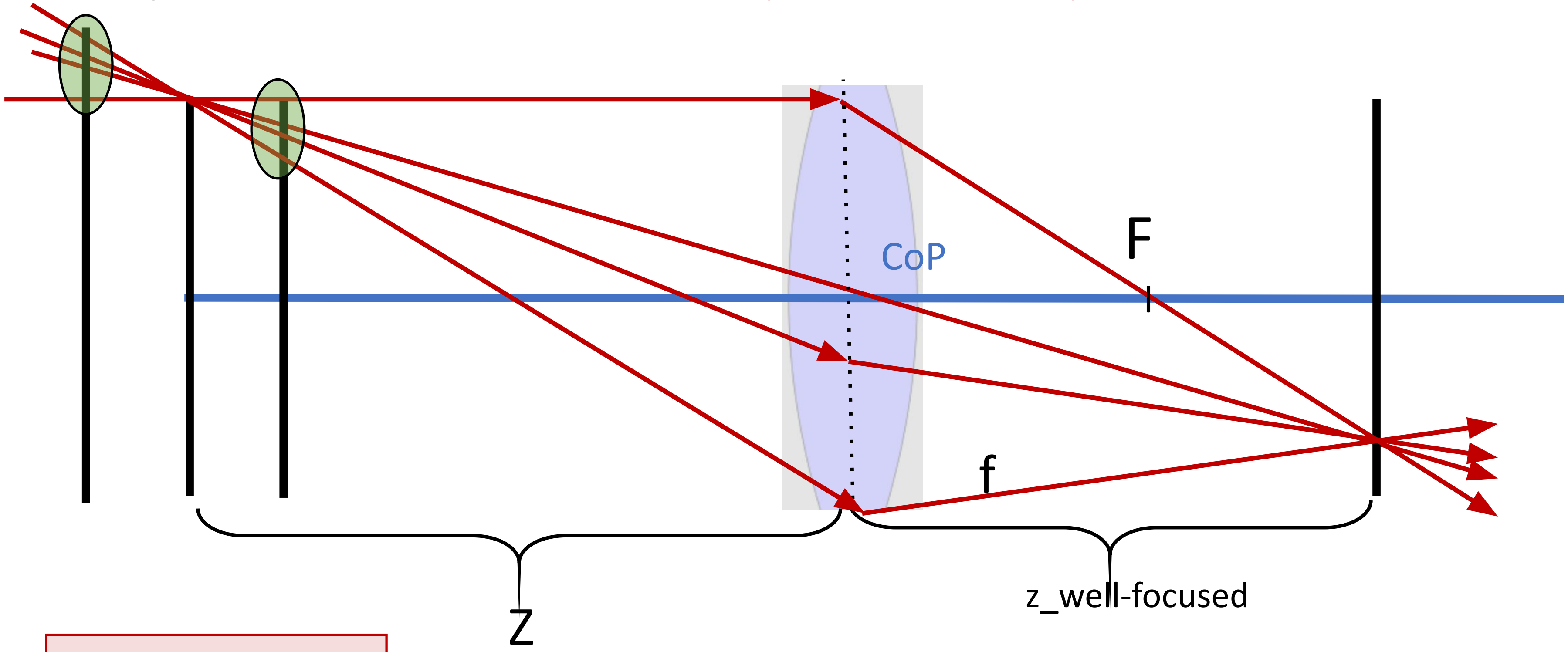


$$1/Z + 1/z = 1/f$$

Depth of focus = range of image plane placement so that objects are focused sufficiently well

Blur circle must be < resolution of the image device

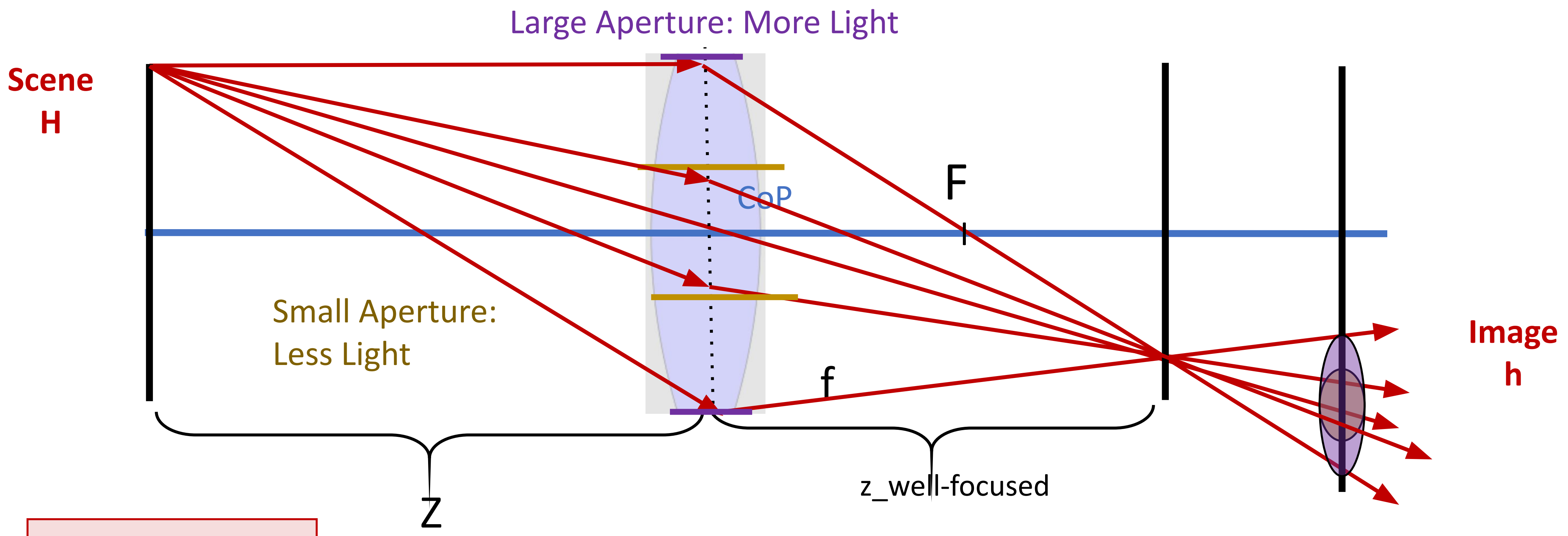
Interpretation of the Lens Equation: Depth of Field



$1/Z + 1/z = 1/f$

Depth of field = range of distances over which objects are focused sufficiently well

Interpretation of the Lens Equation: Aperture of Lens



$1/Z + 1/z = 1/f$

Large Aperture:
- ⇒ Large blur circle
- ⇒ Small depth of field

Small Aperture:
- ⇒ Small blur circle
- ⇒ Large depth of field

Watch Steve Seitz' Video:

https://www.youtube.com/watch?v=F5WA26W4JaM&list=PLWfDJ5nla8UpwShx-lzLJqcp575fKpsSO&index=11

after 2:58

Summary of Concepts: Ideal Thin Lens

- Field of View
- Law of Refraction
- Imaging Rules for lenses
- Focal Point
- Lens Equation
- Depth of Focus
- Depth of Field
- Aperture

Learning Objectives

Be able to explain and use:

- Use of perspective versus orthographic projection
- One, two, and three-point perspective
- Vanishing points to analyze paintings
- Cross-ratio rule for projected lines and vanishing points for image interpretation
- Law of refraction
- Ideal thin lens model (concepts on previous slide)
- Differences of wide-angle, “normal,” and telephoto lenses
- Trade-off of small versus large aperture of a lens model

Reading materials:

Carlbom and Paciorek, 1978:

http://www.cs.uns.edu.ar/cg/clasespdf/p465carlbom.pdf

Criminisi et al., 2016:

https://www.microsoft.com/en-us/research/wp-content/uploads/2016/02/criminisi_chart2002.pdf