CAS CS 585 Image and Video Computing - Spring 2024

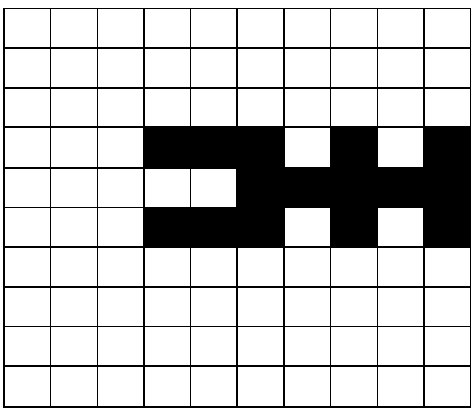
Assignment 1

Due on GradeScope, Wednesday, January 31, 2024, 11:59 pm

Include a "readme file" with acknowledgements of any help you may have received in solving this assignment**.**

Exercise 1

Consider the following binary image:



(a) Compute the coordinates of the centroid of the binary object using pencil and paper. Hint: You can select a coordinate system origin that simplifies your computation.

(b) Draw the axes of least and most inertia of the object. To distinguish them, use a dashed line for the axis of most inertia.

Exercise 2

Consider the following three lines:

$(-1,1)^T \mathbf{x} - 7 = 0$

$(1,0)^T \mathbf{x} - 5 = 0$

$(2,1)^T \mathbf{x} - 4 = 0$

(a) Draw the lines in a 2D coordinate system using pencil and paper.

(b) Rewrite the line equations using the $x \sin \alpha - y \cos \alpha + g = 0$ notation of a line.

(c) For each line, sketch a binary object for which the line is its axis of least inertia.

Exercise 3

Second moments can be used to evaluate how longish or how circular the shape of a binary object is. This has been used, for example, to evaluate blood samples in microscopy images for sickle cell disease.

(a) Write down the relevant mathematical expression as a function of the second moments of the binary image B(x,y).
(b) Is the expression larger or smaller for sickle cells compared to healthy blood cells?
(c) Propose another computer vision problem where object circularity is a property that could aid in image interpretation.

Exercise 4

The minimum and maximum values of the moment of inertia can be written as

$E = 0.5\,(a+c) \pm 0.5\,\mathrm{sqrt}(b^2 + (a-c)^2)$,

where a, b, and c are defined as in class.

(a) Prove that $E \geq 0$.
(b) When is $E = 0$?

Exercise 5

Prove that the axis of most inertia of a binary object goes through the object centroid.

Exercise 6

When we want to represent a binary object in an image with an object that has a simpler shape, we can use a region that has the shape of an ellipse with the same zeroth, first, and second moments. An ellipse can be defined by the equation

$$(x/\alpha)^2 + (y/\beta)^2 = 1,$$

where α is the semi-major axis long the x-axis and β is the semi-minor axis along the y-axis.

(a) Prove that the minimum and maximum second moments of the region about an axis through the origin are $\pi/4\ \alpha\ \beta^3$ and $\pi/4\ \beta\ \alpha^3$, respectively.

The second moment of any region about an axis inclined at an angle γ can be written in the form
$E = a \sin^2\gamma - b \sin\gamma \cos\gamma + c \cos^2\gamma$.

(b) Compute the major and minor axes of an equivalent ellipse, which means an ellipse that has the same second moment about any axis through the origin.