1 Exercise 1: NCC (10 pts)

1.1 1. Proof (4 pts)

\[
r(m, as+b) = \frac{1}{n} \sum \frac{(as_i + b) - (as + b)}{\sigma_{as+b}\sigma_m} (m_i - \bar{m})
\]

where

\[
(as + b) = \frac{1}{n} \sum_i (as_i + b) = a \times \frac{1}{n} \sum_i s_i + b = a \bar{s} + b
\]

\[
\sigma_{as+b} = \sqrt{\frac{1}{n} \sum ((as_i + b) - (as + b))^2}
\]

\[
= \sqrt{\frac{1}{n} \sum ((as_i + b) - (a \bar{s} + b))^2}
\]

\[
= \sqrt{\frac{1}{n} \sum (a(s_i - \bar{s}))^2} = a \sqrt{\frac{1}{n} \sum (s_i - \bar{s})^2} = a \sigma_s.
\]

therefore

\[
r(m, as+b) = \frac{1}{n} \sum \frac{(as_i + b) - (as + b)}{\sigma_{as+b}\sigma_m} (m_i - \bar{m})
\]

\[
= \frac{1}{n} \sum \frac{(as_i + b) - (a \bar{s} + b)}{a \sigma_s \sigma_m} (m_i - \bar{m})
\]

\[
= \frac{1}{n} \sum \frac{a(s_i - \bar{s})(m_i - \bar{m})}{a \sigma_s \sigma_m}
\]

\[
= \frac{1}{n} \sum \frac{(s_i - \bar{s})(m_i - \bar{m})}{\sigma_s \sigma_m}
\]

\[
= r(m, s)
\]
1.2 2. (2 pts)
The coefficient is invariant to brightness change, which is useful in template matching under
different illumination conditions.

1.3 3. (2 pts)
It is useful when we compare two images. NCC = 1 means a perfect match, NCC = -1
means an “antimatch,” where bright pixels match dark pixels and vice versa dark pixels
match bright pixels.

1.4 4. (2 pts)
NCC = 0 means two images are not correlated.

2 Exercise 2: Circularity (12 pts)

2.1 Compactness: \((\text{Perimeter})^2/\text{Area}\) (4 pts)
Generally, the area of a binary image is defined as the total number of object pixels, and the
perimeter of a binary image is defined as the number of object pixels that have at least one
background pixel as its 4-neighbor (and we call those the border pixels).

You will not loose points if you used other definitions of perimeter and made
no mistakes in your calculation.

2.1.1 Object 1: 3x3
This object has 8 border pixels. Thus its perimeter is equal to 8. And clearly its area is
3 \times 3 = 9. Thus we have \(\text{Compactness} = (\text{Perimeter})^2/\text{Area} = \frac{8^2}{9} \approx 7.1\).

2.1.2 Object 2: 2x4
For this object, every pixel is a border pixel. Thus \(\text{Area} = \text{Perimeter} = 2 \times 4 = 8\). Finally,
\(\text{Compactness} = (\text{Perimeter})^2/\text{Area} = 8\).

2.1.3 Object 3: a diamond
Note that, we only consider 4-neighbor for border pixels. For this diamond-shaped object,
there are 8 border pixels. In total, it has 13 pixels. Thus,

\[
\text{Compactness} = (\text{Perimeter})^2/\text{Area} = (8^2)/13 \approx 4.92.
\]

2.1.4 Object 4: 1x10
We have \(\text{Perimeter} = \text{Area} = 10\), and then \(\text{Compactness} = (\text{Perimeter})^2/\text{Area} = 10\).
2.2 Second Moment Measure: $E_{\min}/E_{\max}$ (4 pts)

A detailed tutorial of this measurement could be found [here](#). We will use similar notations as this tutorial.

2.2.1 Object 1: 3x3

The center of mass of this object is its center pixel. Thus, for our new coordinate system $x' - y'$, its origin (0,0) is this center pixel.

Thus, we have

\[
\begin{align*}
a &= \int \int x'^2 b(x',y') dx' dy' = 6, \\
b &= 2 \int \int x'y'b(x',y') dx' dy' = 0, \\
c &= \int \int y'^2 b(x',y') dx' dy' = 6. 
\end{align*}
\]

Substitute these values into $I$, we have:

\[
I = a \sin^2 \theta - b \sin \theta \cos \theta + c \cos^2 \theta = 6(\sin^2 \theta + \cos^2 \theta) = 6.
\]

This means $I_{\min} = I_{\max} = 6$.

Finally, we have

\[
E_{\min}/E_{\max} = I_{\min}/I_{\max} = 6/6 = 1.
\]

2.2.2 Object 2: 2x4

Note that, each block represents a pixel. Thus, the center of mass (or centroid) for this object is not a pixel, but rather a point between four pixels in the center area. Let this point be (0,0), therefore the coordinates for these four pixels would be

\[
(-1/2, 1/2), (1/2, 1/2), (-1/2, -1/2), (1/2, -1/2).
\]

We can calculate $a, b, c$ respectively: $a = 4*(1/2)^2 + 4*(3/2)^2 = 10$, $b = 0$, and $c = 8*(1/2)^2 = 2$. Thus, $I = 10 \sin^2 \theta + 2 \cos^2 \theta = 2 + 8 \sin^2 \theta$. Obviously, $I_{\min} = 2$, and $I_{\max} = 10$. Finally,

\[
E_{\min}/E_{\max} = I_{\min}/I_{\max} = 2/10 = 0.2.
\]

2.2.3 Object 3: a diamond

The center of mass if the center pixel. Let it be (0,0) and we could obtain that: $a = c = 14$, and $b = 0$. Thus, $I = 14 = I_{\min} = I_{\max}$. Therefore,

\[
E_{\min}/E_{\max} = I_{\min}/I_{\max} = 14/14 = 1.
\]

2.2.4 Object 4: 1x10

In this case, the center of mass is also not a pixel. But note that all pixels have $y' = 0$, therefore $c = 0$. And due to its symmetry, we have $b = 0$.

Thus, we have $I = a \sin^2 \theta$. And $I_{\min} = 0$, $I_{\max} = a$. Therefore,

\[
E_{\min}/E_{\max} = I_{\min}/I_{\max} = 0.
\]

2.3 $\mu/\sigma$ (4 pts)

For this problem, we just list all the distances and directly calculate the mean and standard deviation. We use $D$ to represent the set of distances from all border pixels to the centroid.
2.3.1 Object 1: 3x3

\[ D = \{1, 1, 1, \sqrt{2}, \sqrt{2}, \sqrt{2}, \sqrt{2}\}. \] Thus, \( \mu \approx 1.207106 \), and \( \sigma \approx 0.207106 \), and \( \mu/\sigma \approx 5.828427 \).

2.3.2 Object 2: 2x4

\[ D = \{\sqrt{2}, \sqrt{2}, \sqrt{2}, \sqrt{2}, \sqrt{10}, \sqrt{10}, \sqrt{10}, \sqrt{10}\}. \] Thus, \( \mu \approx 1.144122 \), and \( \sigma \approx 0.437016 \), and \( \mu/\sigma \approx 2.618034 \).

2.3.3 Object 3: a diamond

\[ D = \{\sqrt{2}, \sqrt{2}, \sqrt{2}, \sqrt{2}, 2, 2, 2, 2\}. \] Thus, \( \mu \approx 1.707107 \), and \( \sigma \approx 0.292893 \), and \( \mu/\sigma \approx 5.828427 \).

2.3.4 Object 4: 1x10

\[ D = \{0.5, 0.5, 1.5, 1.5, 2.5, 2.5, 3.5, 3.5, 4.5, 4.5\}. \] Thus, \( \mu = 2.5 \), and \( \sigma \approx 1.414213 \), and \( \mu/\sigma \approx 1.767767 \).

3 Exercise 3: Confusion Matrix (8 pts)

3.1 Filling the blanks (2 pts)

From left to right: 36, 56

3.2 Computing the recall, precision, and accuracy (6 pts)

True Positive: 64; False Negative: 36; False Positive: 24; True Negative: 56.

Recall = \( \frac{TP}{TP + FN} = \frac{64}{64 + 36} = 0.64 \);

Precision = \( \frac{TP}{TP + FP} = \frac{64}{64 + 24} = 0.73 \);

Accuracy = \( \frac{TP + TN}{TP + TN + FP + FN} = \frac{64 + 56}{64 + 56 + 24 + 36} = 0.67 \).