CS 235

Spring 2010

Assignment 0

Date Due: Thursday, Jan. 21

Reading: Chapter 1, pages 1-7
Chapter 3, pages 25-40
Chapter 5, pages 63-75
Review Chapter 2 and Chapter 4, pages 8-20 and 47-59

This homework will not be graded.

Please Note: We will only use Chapters 2 and 4 as needed. You should look through these two chapters, reading parts of chapters 2 and 4 which are unfamiliar.

Problems:

1. Prove by induction that:

For all integers $n \geq 25,\ 5^n \geq n^{10}$

2. Find the error in the following proof by induction.

Claim: In any set of h horses, h $\geq$ 1, all horses in the set are the same color.

Proof: By induction on h.

Base Case: For h = 1. In any set containing just one horse, all horses clearly are the same color.

Inductive Step: For $k > 0$ assume the claim is true for h= k and prove that it is true for h = k + 1.

Take any set H of k + 1 horses. we will show that all horses in this set are the same color. Remove one horse from this set to obtain the set H1 with just k horses. By the induction hypothesis, all the horses in H1 are the same color. Now replace the removed horse and remove a different one to obtain the set H2 . By the same argument, all the horses in H2 are the same color. Therefore, all the horses in H must be the same color and the proof is complete. (Exercise due to Mike Sipser)

3. Let R be the relation on N defined by, R=$\{(x, y) \mid x + y \in N$ and $x + y$ is even $\}$

(i) Prove that R is an equivalence relation.

(ii) How many equivalence classes are there for this relation and what are they ?

4. For each of the following 3 statements about Z, state if they are true or false and say why.

a. $\forall x, y\ \exists z\ (x + z = y)$

b. $\forall x, y\ \exists z\ (x * z = y)$

c. $\exists n\ \forall m\ \exists t\ (nm + mt > 0)$

5. Write the negation of the statements in problem 4.