CS 235
Spring 2010

Assignment 2

Date Due: Thursday, February 4 at 5:00

Reading: Chapter 6, pages 93-119 (Chapter called Congruence Classes)

Problems:

1. Find the following residues.
   i. $5^{11} \pmod 7$
   ii. $(6^{44} + 13^{14}) \pmod{12}$
   iii. $(140 * 27) + (2^{19} + 1) + 83 \pmod{5}$

2. Prove that is x ≡ y (mod m) then (x,m) = (y,m).

3. Find all numbers b between 1500 and 1550 which are congruent to a mod n when:
   (1) a=1, n = 13
   (2) a=1515, n = 15

4. Prove than for all a, b and n, ab ≡ (a mod n) * (b mod n) (mod n)

5. Assume that a and b are positive integers and that $a|b$.

   (i). Prove that $\forall\ x\ (\ x\ mod\ b)\ mod\ a\ =\ x\ mod\ a$

   (ii). $\forall x, y\ x \equiv y\ (mod\ b)$ implies $x\ \equiv\ y\ (\ mod\ a)$

6. Find all $a \geq 0$ such that 10a ≡ 0 (mod 36). Justify your answer.

7. From all these rules about mod, it is tempting to make the following similar claim:
   $\forall k, n \in N\ \ a^k \equiv a^{(k\ mod\ n)}\ (mod\ n)$
   Is this true? Prove your answer.

8. Prove that $\Sigma_{i=1}^{i=n-1} i = 0 \ (\ mod\ n)$ if $n > 0$ is odd.

   (Hint: One way is to think about negative numbers mod n.)

9. Define a sequence of natural numbers by, $a_1 = 2$, and for $n > 1$, $a_{n+1} = a_n(a_n - 1)+1$.

   Prove that a_n=$a_{n-1}a_{n-2}...a_1 + 1$ and that each of the a_i's are relatively prime.

10. You are given 2 jars, one holds exactly 6 quarts and one holds exactly 11 quarts, and one bowl which holds more than 60 quarts. You want to put 13 quarts of water into the bowl using the jars. How do you do it ?