Assignment 5

Date Due: Thursday, March 25 at 5:00

Reading: Chapters 10, pages 201-212, and
and Chapter 11, page 223-239

Problems:

1. Let M be the ring of all 2 by 2 matrices whose entries are 0 and 1, where all arithmetic is carried out (mod 2).

   (i). How may different elements are there in this ring ?

   (ii). Which elements are units in this ring ?

2. Let R is a ring (possibly non-commutative).

   Define C to be the set of elements from R which commute with all elements in R.
   That is a $\in$ C if and only if ($\forall$ x $\in$ R) ax = xa.

   Prove that C is a subring of R.

3. For C defined as in the previous problem, let R be the ring of all 2 by 2 matrices whose entries are 0 and 1, where all arithmetic is carried out (mod 2). What is C in this case ? Justify your answer.

4. What is the smallest sized ring R you can think of that is not one of the mod m rings Z_m ? Note: The ring has to have more than 1 element in it (so 0 is not equal to 1).

   Write down the set R and its addition and multiplication tables ? Is R a field ? Why or why not ?

5. In class we proved that for a commutative ring R, a 0 divisor cannot be a unit. (This is Prop 11 on page 137). Now prove the converse: In R, a unit cannot be a 0-divisor.

6. Prove that for any finite commutative ring R and any a in R, where a is not 0, then a is either a unit or a 0 divisor in R.

   Hint: Look at page 138. Read and understand the proof but don't copy it. Instead explain it intuitively and in your own words.

7. Page 167, problem 18.

8. Consider the set F of all real numbers of the form a + $\sqrt{3}$b, where a and b are rational numbers. Is F, using the usual rules of real number addition and multiplication, a subfield of the real numbers ? Why or why not ?