CS 630 - Fall 2021
Homework 1 - Brief Answers

Due: Friday, September 24 by 5:00pm - submit via Gradescope

Reading : For matrices read pages 75-82 about Strassen's algorithm, and also look through Appendix D.
For polynomials start reading Chapter 30 on the FFT in the textbook, pages 898 - 915.

Problems: Please limit your answer to the following problems to at most 1/2 a pages each.

1. i. You are given the point-value form of a polynomial consisting of the 3 points (1,-7), (-2, -7), and (-1, 7).

Use the interpolation formula of Lagrange (found on page 902, equation 30.5) to find a polynomial A of degree 2 which goes through those 3 points.
Show your work.

Answer: The polynomial A(x) =-7x^2 -7x + 7.

You should show some of the Lagrange interpoltaion formula on the 3 points give. Enough to get at least part of the answer.

ii. Is the polynomial A you found in (i). the unique polynomial of degree 2 which goes through the 3 points ? Why or why not?

Answer: Yes, the three given points of A and not on any line and a degree 2 polynomail is uniquely characterized by 3 non-collinear points.

iii. Could you find degree a one polynomial which goes through these same 3 points ? How about a degree four polynomial ? Why or why not?

Answer: A degree 3 or degree 4 polynomial which goes through these same 3 points is A(x)$(x-t)$ or A(x)$(x-t)^2)$.

2. i. Prove that for any positive even n, ${\omega_n}^{n/2} = \omega_2 =$ -1.

ii. List all the principal 6^{th} roots of unity, and 7^{th} roots of unity.

iii. Show that if p is prime then every p^{th} root of unity other than 1 is principal.

3. i. Recall the usual algorithm we use to multiply two 4×4 matrices of integers.
Exactly how many regular integer multiplications does this take?
How many integer additions ?

ii. Now do the same problem as in problem i. but this time use Strassens algorithm and divide and conquer to do the 4×4 multiplication. Make sure you use Strassen's algorithm at all places of the divide and conquer tree where you do the multiplications.

Answer the same two questions as in part i.

Answer: i. If we do the 4 by 4 matrix multiplication we use $4^3 = 64$ multiplications and $4^2 3 = 48$ additions.

ii. Strassen, on the other hand, takes 7 multiplications and 18 +'s to multiply two 2×2 matrices. So to multiply two 4×4 matrices we use 7x7 = 49 mults and 40+126+32 = 198 +'s.
The work to justify all these numbers is not shown here. However, as a hint, the number of additions for Strassen's algorithm is $198 = 40+126+32$ where $40 = 10 \times 4$, $126 = 7 \times 16$ and $32 = 8 \times 4$..