

CS 630 - Fall 2021  
Homework 4

Due: Tuesday, November 23 by 9:00pm - submit via Gradescope

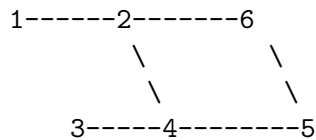
Reading : Read Chapter 35.3, pages 1117-1122 on the set cover problem,  
and Chapter 34, the introduction and sections 2 and 3, pages 1061 to 1070.

Problems: Do only problems 1, 2 and 3 for this assignment. Problem 4 will appear again in the last homework, HW 5.

1. What is the probability of finding a min-cut in each of the following two graphs G1 and G2 ?  
Note. We run the min cut algorithm loop only once, so it carries out  $n-2$  contractions before outputting a cut of G1 and G2.

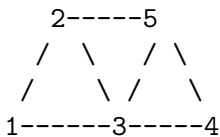
(i).

G1=



(ii).

G2=



2. Use the ideas from Friedvalds algorithm to design a Monte Carlo algorithm which, given  $n \times n$  matrices A and B, decides if B is the inverse of A or not.

Explain when your algorithm may make an error and when not, and also determine a bound on the probability of an error occurring when you run the algorithm.

3. Do problem 35-1 on page 1134, only using the best fit heuristic instead of first-fit.

The best fit heuristic takes each object in turn and places it into the bin which it fills closest to full as possible, otherwise if the object doesn't fit in a bin it opens a new one.

Do parts b, c, d and e. only.

NOTE: Problem 4 below will be moved to HW 5 which will appear right after the Thanksgiving break.

4. Let  $M$  be a an  $n$  by  $n$  matrix of 0's and 1's.  $M(i,j)$  is the entry in row  $i$  column  $j$ .

We call  $M$  switchable if there is a sequence of row switches and column switches of  $M$  which result in all 0's along the diagonal of the matrix. Elements not on the main diagonal can be either 0 or 1.

a. Give an example of a  $3 \times 3$   $M$  which is not switchable but which has at least one 0 in every row and in every column. Explain briefly why your example is correct.

b. Write an efficient (polynomial number of steps) algorithm to decide if a matrix  $M$  is switchable.

One way to construct such an algorithm is to use  $M$  to define a bipartite graph  $G = (L,R,E)$  with  $n$  vertices in its  $L$  set and  $n$  vertices in its  $R$  set and  $E$  as its edges. Now prove that  $M$  is switchable iff its graph  $G$  has a perfect matching.

Note: If you use a different proof than the one I've suggested, explain why it is correct.

Then use this fact to conclude that there is a polynomial time algorithm as asked for in this problem. State and explain what the big-0 complexity of your algorithm is.

c. Show how your algorithm from part b. works and what answer it gives on the 4 by 4 matrix  $M$  given by

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1 0 1 0
0 0 1 1
1 1 1 0
1 0 0 1
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