CS 630 - Fall 2021
Homework 4

Due: Tuesday, November 23 by 9:00pm - submit via Gradescope

Reading : Read Chapter 35.3, pages 1117-1122 on the set cover problem,

and Chapter 34, the introduction and sections 2 and 3, pages 1061 to 1070.

Problems: Do only problems 1, 2 and 3 for this assignment. Problem 4 will appear again in the last homework, HW 5.

1. What is the probablilty of finding a min-cut in each of the following two graphs G1 and G2 ? Note. We run the min cut algorithm loop only once, so it carrie out n-2 contractions before outputting a cut of G1 and G2.

(i).

```
G1=


1------2-------6
        \        \
         \        \
    3-----4--------5
```

(ii).

```
G2=
     2-----5
    /  \   / \
   /    \ /   \
  1------3-----4
```

2. Use the ideas from Frievalds algorithm to design a Monte Carlo algorithm which, given $\times$ n matrices A and B, decides if B is the inverse of A or not.

Explain when your algorithm may make an error and when not, and also determine a bound on the probability of an error occurring when you run the algorithm.

3. Do problem 35-1 on page 1134, only using the best fit heuristic instead of first-fit.

The best fit heuristic takes each object in turn and places it into the bin which it fills closest to full as possible, otherwise if the object doesn't fit in a bin it opens a new one.

Do parts b, c, d and e. only.

NOTE: Problem 4 below will be moved to HW 5 which will appear right after the Thanksgiving break.

4. Let M be a an n by n matrix of 0's and 1's. M(i,j) is the entry in row i column j.

We call M switchable if there is a sequence of row switches and column switches of M which result in all 0's along the diagonal of the matrix. Elements not on the main diagonal can be either 0 or 1.

a. Give an example of a 3 x 3 M which is not switchable but which has at least one 0 in every row and in every column. Explain briefly why your example is correct.

b. Write an efficient (polynomial number of steps) algorithm to decide if a matrix M is switchable.

One way to construct such an algorithm is to use M to define a bipartite graph G = (L,R,E) with n vertices in its L set and n vertices in its R set and E as its edges.
Now prove that M is switchable iff its graph G has a perfect matching.

Note: If you use a different proof than the one I've suggested, explain why it is correct.

Then use this fact to conclude that there is a polynomial time algorithm as asked for in this problem. State and explain what the big-0 complexity of your algorithm is.

c. Show how your algorithm from part b. works and what answer it gives on the 4 by 4 matrix M given by

```
1 0 1 0

0 0 1 1

1 1 1 0

1 0 0 1
```