CS 630 - Fall 2021
Homework 5

Due: Wednesday, December 8 by 9:00pm - submit via Gradescope

Last date to turn in any work or regrading requests (accept possibly homework 4): Thursday, December 9 at midnight.

Reading : Read Chapter 34, the introduction and sections 2 and 3, pages 1061 to 1070.

Problems: Pick three of the first four problems below to turn in. Do not do all 4. If you do then the last one will not be graded. You should insert the statement "Do not grade, not doing this problem" at the place in your answers where the solution would have been given.

1. (i). Give an example of an instance I of the unweighted set cover problem where the greedy algorithm for set cover uses more than OPT(I)+2 many sets for its cover.

(ii). Give an example of an instance I of the unweighted set cover problem where OPT(I) ≥ 2 and where the greedy algorithm for set cover uses more than 2OPT(I) many sets for its cover.
(I'm not sure but this one may be hard.)

2. Each of the following 3 problems are in NP (non-deterministic polynomail time). For each problem prove they are in NP by by showing they satisfy the conditions of the definition of NP given on page 1064 of the textbook. Specifically they are decision problems which have verification algorithms.

i. The Eulerian path problem:
Given a graph, is there an Euler cycle of its edges.

ii. The independent set problem: Given a graph and a number k, does G have an independet set of k or more vertices. (See page 1101, problem 34-1 in our book for the definition of independent set.)

iii. A version of bin packing: Given a finite collection of objects each of size ≤ 1. Can all of these objects be fit into k bins of size 1?

3. Do parts a, b, and c of problem 35-4, page 1135 (Maximum Matching) from our textbook.

4. Let M be a an n by n matrix of 0's and 1's. M(i,j) is the entry in row i column j.

We call M switchable if there is a sequence of row switches and column switches of M which result in all 0's along the diagonal of the matrix. Elements not on the main diagonal can be either 0 or 1.

a. Give an example of a 3 x 3 M which is not switchable but which has at least one 0 in every row and in every column. Explain briefly why your example is correct.

b. Write an efficient (polynomial number of steps) algorithm to decide if a matrix M is switchable.

One way to construct such an algorithm is to use M to define a bipartite graph G = (L,R,E) with n vertices in its L set and n vertices in its R set and E as its edges.
Now prove that M is switchable iff its graph G has a perfect matching.

Note: If you use a different proof than the one I've suggested, explain why it is correct.

Then use this fact to conclude that there is a polynomial time algorithm as asked for in this problem. State and explain what the big-0 complexity of your algorithm is.

c. Show how your algorithm from part b. works and what answer it gives on the 4 by 4 matrix M given by

```
1  0  1  0

0  0  1  1

1  1  1  0

1  0  0  1
```

Problems 5 and 6 will not be graded. Together with the other problems in this homework, they may be taken as practice problems for the final exam.

5. Consider the polynomial h(x) = $(x-2)(3x+12)(-x)(7x-14)(x)$.

a. True or False: h(x) has exactly 5 roots.

b. True or False: h(x) has exactly 1 negative root

c. True or False: h(x) has degree 4

d. Let S = {-2, -1, 0, 1,2,3,4,5}. How many of these numbers is a root of h ?

e. If you choose two different random numbers uniformly at random from S = {-2, -1, 0, 1,2,3,4,5 }, what is the probablility that at least one of the two random numbers is a root of h ?

6. Classify the following 3 randomized algorithms as being Monte Carlo algorithms or Las Vegas algorithms, and give a short (2 or 3 sentence) reason for your answers. Note: The algorithms may not be either one.

a. Tha Frievalds algorithm for matrix multiplication checking.

b. The FFT algorithm

c. The max-cut algorithm which flips fair coin for each vertex of a graph with m edges to determine a cut in the graph whose expected value is 1/2m.