Problem 1: Handout

It is simple to answer with free terms:
Γ = {y : T}
t = (λ x. y)z
u = y
$\Gamma \nvdash t : T$
$\Gamma \vdash$ u : T

$t \to u$ and u is well typed, but t is not because $\Gamma \nvdash$ z : T. Thus the converse of the preservation thm. does not hold.

A stronger example involves only closed terms:
t = (λ x. λ y. y) (λ z. zz)
u = λ y. y

Again, $t \to u$, and u is well-typed, but t is not because (zz) is not well typed.

Problem 2: Handout

$$\begin{aligned}
\mathsf{fibfn} \;=\; \lambda f.\,\lambda n.\quad & \mathsf{if}\ (\mathsf{iszero}\ n) \\
& \mathsf{then}\quad 0 \\
& \mathsf{else}\quad \mathsf{if}\ (\mathsf{iszero}\ (\mathsf{pred}\ n)) \\
& \qquad\quad \mathsf{then}\ \ \mathsf{succ}\ 0 \\
& \qquad\quad \mathsf{else}\ \ \mathsf{plus}\ (f\,(\mathsf{pred}\ n))\ (f\,(\mathsf{pred}\ (\mathsf{pred}\ n)))
\end{aligned}$$

$$\mathsf{fib} \;=\; \mathsf{fix}\ \mathsf{fibfn}$$

$$\begin{aligned}
\mathsf{plusfn} \;=\; \lambda p.\,\lambda n.\,\lambda m.\ & \mathsf{if}\ (\mathsf{iszero}\ n) \\
& \mathsf{then}\quad m \\
& \mathsf{else}\quad \mathsf{succ}(p\ (\mathsf{pred}\ n)\ m) \\
\mathsf{plus} \;=\; \mathsf{fix}\ \mathsf{plusfn} &
\end{aligned}$$

Problems 3 $\to$ 5

See solutions in back of [Pierce].