CS520 Problems for Assignment 4

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Due: 8 October 2004

Problem 1: Let t and u be terms of the simply-typable pure lambda-calculus $\boldsymbol{\lambda}_{\rightarrow}$ (this is not $\boldsymbol{\lambda}\mathbf{NB}_{\rightarrow}$, i.e., no booleans and natural numbers allowed). Suppose that $t \Rightarrow u$. You do not need to assume the evaluation is carried out in call-by-value, and it can be full beta-reduction.

We know from the Preservation Theorem that if $\Gamma \vdash t : T$ for some context Γ and type T, then $\Gamma \vdash u : T$, i.e., if T is a valid type for t then T is also a valid type for u. (The Preservation Theorem also holds for full beta-reduction, although this is not mentioned in [Pierce].)

Show that the converse does not hold in general. That is, give an example of terms t and u of $\boldsymbol{\lambda}_{\rightarrow}$ such that $t \Rightarrow u$, together with a context Γ and a type T, for which it holds that:

- $\Gamma \vdash u : T$, but
- $\Gamma \not\vdash t : T$.

Problem 2: Define the fibonacci function using `fix` according to the material presented in [Pierce, Section 11.11]. You will find it useful to look at Exercise 11.11.1 in [Pierce, page 144].