CS 520 - Programming Languages
Solutions for Problem Set 6

Problem 1. Exercise 15.2.3 in [Pierce, page 186]

[Pierce, pages 517-518]

Problem 2. Exercise 15.2.5 in [Pierce, page 187]

[Pierce, page 518]

Problem 3. Exercise 15.3.2 in [Pierce, page 188]. part (1) only

[Pierce, page 519]

Problem 4. Exercise 15.5.2 in [Pierce, page 198]

(a)
From

$$\frac{S <: T}{Ref\ S <: Ref\ T}$$

we can derive

$$\frac{Nat <: Real}{Ref\ Nat <: Ref\ Real}$$

Consider the following program:

line 1	$x\ =\ ref\ 5;$
line 2	$x\ :=\ 10.5;$
line 3	$iseven(!x)$

According to our program, $x : Ref\ Nat$. Thus, $!x : Nat$. Note that the program phrase $iseven(!x) : Bool$ is typable, even though $iseven(!x) \rightarrow iseven(10.5)$ is a stuck term.

(b)
From

$$\frac{T <: S}{Ref\ S <: Ref\ T}$$

we can derive

$$\frac{Nat <: Real}{Ref\ Real <: Ref\ Nat}$$

Consider the following program:

line 1	$x \ = \ ref\ 10.5;$
line 2	$iseven(!x)$

According to the rules, $x : Ref\ Real <: RefNat$. Thus, $!x : Nat$, hence $iseven(!x) : Bool$ is typable, but $iseven(!x) \rightarrow iseven(10.5)$ is a stuck term.

$\boxed{\textbf{Problem 5}}.$

$t \ = \ (\lambda x : Top \rightarrow T.x\ x)$ is typable as follows:

$$\dfrac{\dfrac{x{:}Top \rightarrow T \vdash x{:}Top \rightarrow T \qquad \dfrac{x{:}Top \rightarrow T \vdash x{:}Top \rightarrow T \qquad Top \rightarrow T <: Top}{x{:}Top \rightarrow T \vdash x{:}Top}}{x{:}Top \rightarrow T \vdash x\ x{:}T}}{\vdash (\lambda x : Top \rightarrow T.x\ x) : (Top \rightarrow T) \rightarrow T}$$