CS520 Problems for Assignment 07

Posted: 22 October 2004
Due: 1 November 2004

Problem 4: Consider the language presented in [Pierce, Fig 15-1, Fig 15-2, and Fig 15-3] on pp 186-187. This is the simply-typed lambda calculus with subtyping, augmented with records.

1. Give two types T and U such that

   - $T <: U$
   - $U <: T$
   - $T \neq U$

   Thus, if two types T and U are subtypes of each other, it does not mean the two are the same (syntactically). Nevertheless, we would like to treat them as "equivalent" types – and this is what we do in parts 2 and 3 below.

2. Define a new binary relation on types, denoted $\equiv$, satisfying the following equivalence:

   (‡) $T \equiv U$ if and only if (both $T <: U$ and $U <: T$)

   We want you to define $\equiv$ without any mention of $<:$ and by means of rules (just as $<:$ was defined by rules).

3. Using your definition of $\equiv$ in part 2 above, prove equivalence (‡).