BU CLA CS 530: Analysis of Algorithms
14 March 1995
Mid-Term Test

Reminder: This is a closed-book test. You are allowed one 8.5 by 11 crib sheet which you have to hand in with your test book.

Read carefully all questions before you start, organize your time, start with what you know best.

Problem 1 (5 points).

Design a linear-time algorithm to determine if a text T is a cyclic rotation of another string T'. For example, *arc* and *car* are cyclic rotations of each other. (Hint: The KMP-MATCHER is a linear-time algorithm.)

Problem 2 (5 points).

Design an efficient randomized algorithm (the Monte-Carlo variety) to decide whether two rational polynomials with n variables are equal.

Problem 3 (15 points).

Consider a set S of $n \geq 2$ distinct numbers given in unsorted order. Each of the following 3 questions asks you to give an algorithm satisfying a stated condition. In as few words as possible, describe your algorithm and justifiy its running time. (We measure the running time by counting the number of comparisons performed.)

1. In $\mathcal{O}(n \log n)$ time, determine whether a given number $x \notin S$ is the sum of two numbers in S.
2. In $\mathcal{O}(n)$ time, find $v, w \in S$ such that $|v - w| \geq |x - y|$ for all $x, y \in S$.
3. In $\mathcal{O}(n \log n)$ time, find $v, w \in S$ such that $v \neq w$ and $|v - w| \leq |x - y|$ for all $x, y \in S$ such that $x \neq y$.

Problem 4 (15 points).

Let $G = (V, E)$ be a flow network with source s and sink t, and suppose that each edge $e \in E$ is restricted to capacity $c(e) = 1$. Assume also, for convenience, that $|E| = \Omega(V)$.

1. Suppose we run the basic FORD-FULKERSON maximum-flow algorithm on G. What is its worst-case running time? Justify briefly. (Hint: The total running time of FORD-FULKERSON is at most $\mathcal{O}(E \cdot |f^*|)$ in general, where f^* is the maximum flow found by the algorithm. But here, with the stated restriction, we have a tighter bound.)

2. Suppose a maximum flow for G has been computed, and a new edge with capacity = 1 is added to E. Describe how the maximum flow can be efficiently updated. Analyze your algorithm. (Hint: It is not only the value of the flow that must be updated, but the flow itself.)

3. Suppose a maximum flow for G has been computed, but an edge is now removed from E. Describe how the maximum flow can be efficiently updated. Analyze your algorithm.

Created: 95.03.13