CS 511 Formal Methods, Fall 2018	Instructor: Assaf Kfoury
Lecture 3	
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(These lecture notes are **not** proofread and proof-checked by the instructor.)

- For those who were absent: It's possible to propose problems in any CS field in order to formally model and implement them using Z3 during this course.

1 Universe of PL

- Tautologies are WFF's that are always valid regardless of input. For example: $p \vee \neg p$
- Contraditions are WFF's that are not satisfiable at all. For example: $p \wedge \neg p$
- To semantically verify membership of a WFF, one can examine the truth-table. WFF is a tautology if the last column (containing the WFF) is T in every row, or a contradiction if it's F for every valuation of the corresponding atoms. WFF's which are T in some rows and F in others, are called contingents.
- Properties of different WFF membership groups:
  - $Tautology \cap Contradition = \varnothing$
  - $Satisfiables = Tautologies \cup Contingents$
  - $Falsifiables = Contradictions \cup Contingents$
- Although it's preferred to determine membership of WFF's semantically, but since turth-tables grow exponentially with the number of propositional atoms, proof rules are utilized as the more efficient alternative. Natural deduction is not the only approach that falls under the category of proof rule methods.

2 CNF, DNF, Horn Formulas, and other special forms

- A **canonical** form of a mathematical object is a standard way of presenting that object as a mathematical expression in a unique way.
- DNF and CNF both are canonical forms and a WFF in such forms is unique up to the commutativity of $\vee$ (for DNF) and $\wedge$ (for CNF).
- **Remember that:**
  - The logical operators $\vee$, $\wedge$, and $\leftrightarrow$ are **commutative**, but $\rightarrow$ is not:
    * $\phi \vee \psi \equiv \psi \vee \phi$
    * $\phi \rightarrow \psi \not\equiv \psi \rightarrow \phi$

- The logical operators $\vee$, $\wedge$, and $\leftrightarrow$ are **associative**, but $\rightarrow$ is not:
  - $(\phi \vee \psi) \vee \xi \equiv \phi \vee (\psi \vee \xi)$
  - $(\phi \rightarrow \psi) \rightarrow \xi \neq \phi \rightarrow (\psi \rightarrow \xi)$
- The logical operators $\vee$ and $\wedge$ can **distribute** over one another.
  - $\phi \wedge (\psi \vee \xi) \equiv (\phi \wedge \psi) \vee (\phi \wedge \xi)$
  - $\phi \vee (\psi \wedge \xi) \equiv (\phi \vee \psi) \wedge (\phi \vee \xi)$
- Regarding $\perp$ and $\top$:
  - $\perp \equiv (x \wedge \neg x)$
  - $\top \equiv (x \vee \neg x)$

- A note on HO6 - slide 3: Pay attention to the syntax of each form. For example $\neg(x \vee y)$ is not a valid CNF expression as $\neg$ is only allowed behind a propositional atom!
- A note on HO6 - slide 15: SAT solvers do not overcome the NP-Completeness problem, they rather convert the WFF in a more efficient way w.r.t the task's objective.
- A note on HO6 - slide 17: In NNF, there's no preference between $\wedge$ and $\vee$ when it comes to parse tree as opposed to CNF and DNF where the level at which these two operators appear in the tree is fixed.