CS 512 Formal Methods, Fall 2018 — Instructor: Assaf Kfoury

Lecture 4

Sep 13, 2018 — Donghyun Kim

(These lecture notes are **not** proofread and proof-checked by the instructor.)

1 Precedence of Propositional Logic

1. Implication in propositional logic is right associative:

$$\vdash (\phi \to \psi \to \theta) \to \phi \to \psi \to \theta$$
$$:= \ \vdash (\phi \to (\psi \to \theta)) \to (\phi \to (\psi \to \theta))$$

Proof:

1	$\phi \to \psi \to \theta$	assume
2	ϕ	assume
3	$\psi \to \theta$	$\to$e1, 2
4	ψ	assume
5	θ	$\to$e3, 4
6	$(\psi \to \theta)$	$\to$i4 − 5
7	$\phi \to (\psi \to \theta)$	$\to$i2 − 6
8	$(\phi \to \psi \to \theta) \to (\phi \to (\psi \to \theta))$	

2 Relating Truth Tables and Proof Rules

- $\phi_1, \phi_2, ..., \phi_n \vDash \psi$
  We say that "$\phi_1, \phi_2, ..., \phi_n$" semantically entails ψ (e.g., by a truth table).
- $\phi_1, \phi_2, ..., \phi_n \vdash \psi$
  We say that "$\phi_1, \phi_2, ..., \phi_n$" formally entails ψ (e.g., by proof rules).

3 Soundness of propositional logic

- If $\phi_1, \phi_2, ..., \phi_n \vdash \psi$, then $\phi_1, \phi_2, ..., \phi_n \vDash \psi$.
- If we can deduce ψ from $\phi_1, \phi_2, ..., \phi_n$ with proof rules, then $\phi_1, \phi_2, ..., \phi_n$ implies ψ is true.

4 Completeness of propositional logic

- If $\phi_1, \phi_2, ..., \phi_n \vDash \psi$, then $\phi_1, \phi_2, ..., \phi_n \vdash \psi$.
- If we can derive ψ from $\phi_1, \phi_2, ..., \phi_n$ with a truth table, then there is a deduction for $\phi_1, \phi_2, ..., \phi_n$.