CS511 - Fall 2018

Notes from 9/20 lecture

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Note: These notes contain mostly material discussed in class, but not presented in the handouts

Propositional Logic

There are several approaches to reason about a WFF in PL.

- **Semantic approach** using a Truth Table
- **Formal deduction approach**, which inludes several ways as:
  - **Natural deduction**
    Handout also includes approaches like: Hilbert-style formal proof system, or Gontzen-style formall proof system
  - **Analytic tableaux** which was discussed during the previous lecture
  - **Resolution** which is the content of today's lecture

Resolution in PL

Note that the book only presents natural deductions and extends it only to 1^{st} order logic.
Resolution is **refutation-based** & r**efutation complete**.
It can also be used to decide any semantic entailment $\Gamma \models \phi$, and can also be used to decide satisfiability.

A short note about Intutionistic PL: LEM, equivalently $\neg\neg e$ and PBC are all forbidden in Intutionistic propositional logic.
Every proof in I.P.L. is a proof in classical P.L., but not the other way.

In resolution we assume that WFF is in CNF, like:

$\phi = (p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee \neg r) \wedge (r \vee s)$.
We exploit the associativity of $\wedge$ and $\vee$ operators, there is no need to make a tree binary associative.

ϕ: is a formula that does modelling of a system or application
$\phi \sim \psi$ ψ, ϕ are equisatisfiable, but not equivalent. ϕ is not in CNF in general, but ψ has to be. (see more through links in the handout)
So, previous formula can be written as:
$\phi \sim \{C_1, C_2, C_3\}$, where: $C_1 = p \vee \neg q \vee \neg r$ $C_2 = \neg p \vee q \vee \neg r$ $C_3 = r \vee s$
From page 4 in handout:
In step 1 we have $2n$ variables, or $4n - 1$ symbols and $2n$ parenthesis.
Variables represent natural instances, but new variables z_i do not.
From function in page 5:
$CNF(\phi, \Delta)$,
where *phi* is a single WFF of PL and Δ is a set of WFFs (actually a set of clauses that correspond to CNF). It is defined recursively:
$BNF : \phi := \chi | \neg\phi | \phi \wedge \psi | \phi \vee \psi |$, we can add $\rightarrow$, $\leftrightarrow$, but we do not need them. χ stands for a propositional atom.
As the function is defined recursively we have to consider all cases in BNF definition.
Initial call $CNF(\phi, \{\})$ Equisatisfiability: First WFF is satisfiable, iff second is.
As already mentioned, there is no need to handle $\wedge, \vee$ as binary, but because of associativity we can process them in multi-arity.

Remark: Analytic tableaux requires less creativity than natural deduction (not many guesses and there are more heuristics to apply).
In resolution there us only 1 rulem, totally deterministic (we only choose two clauses). Even less guesswork than analytic.
No method of course is perfect for everything.

Clauses have to satisfy resolution condition, one clause should have a literal, the other its negation.
What we do? We take all literals except the cases that satisfy the condition.
Example
$(x \vee \neg p \vee r)$ and $(\neg y \vee p \vee s \vee r)$.
We scan from left to right, $(\neg p, p)$ appears. So i can apply resolution and I can create a new clause $(x \vee r \vee s \vee r)$.
A heuristic for improvement is when a literal is repeated, remove the duplication.

Question: Can I extract from resolution process an assignment, if a formula is satisfiable? Answer is yes. Not as simple as in tableaux method.