| CS 511 Formal Methods, Fall 2018 | Instructor: Assaf Kfoury |
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| November 13th, 2018 | Isidora Chara Tourni |

(These notes contain mostly material discussed in class, but not presented in the handouts.)

## - Notes on Handout 28

pages 3,5
$\overline{\mathrm{A} w f f}$ in $\mathrm{CNF} \phi$ is a conjunction of clauses. Examples:
$\phi \triangleq(\ldots) \wedge(\ldots) \wedge \ldots(\ldots)$ is a propositional wff, for MaxSAT problem.
$\phi \triangleq(\ldots)^{4} \wedge(\ldots)^{3} \wedge \ldots(\ldots)^{2}$ is a weighted wff, for weighted maxSAT problem.
In the latter case, a truth value assignment $\sigma$ satisfying the first two clauses will be different from a truth value assignment $\sigma^{\prime}$ satisfying the last three.
pages 6-7
$\overline{\mathrm{D}}$ is a finite set of events.
In the DAG of p.7, $\left\{x_{1}, \ldots, x_{8}\right\}$ are binary random variables. The arrows provide information about the network. The random variables do not depend on others.

MPE: Most Probable Explanation. Given what we observe, what is the most probable explanation of this observation? What values should we assign/ have at the beginning, so that we can have the resulting values we see? What will be the MPE which will maximize the probability of the occurence we see in the end?

## - Notes on Handout 30

Example: Suppose we have a set of variables $X=\{x, y, z\}$
The signature of the vocabulary is: $\Sigma=\{c, d, f(), g(),, \ldots\}$.
The parse trees for terms $g(c, g(f x, y))$ and $g(c, z)$ are given below:


Matching means that we want to apply a substitution so that both trees look the same. It is done at the syntactic level, we substitute into the variables while symbols in $\Sigma$ are fixed. Suppose we substitute:


Then we obtain the matching of the two trees before. One more example is given below, for terms $g(c, g(f x, y))$ and $g(c, g(f x, f y))$ :


At the top, we have the same symbol.
Then starting from the bottom, I sect the tree to smaller pieces, and all these pieces should be unified. With $x \mapsto c$, I unify x,c. Then, I sect the left (sub)part and going up I see that I have the same symbol, and substitute $y \mapsto f c$.
When substitution is done to two given terms, in order to unify them, we have unification. When substitution is done to one term, so that we match both sides, we have matching.

## - Notes on Handout 29

pages 3,4
$\overline{\sigma(x)=c}, \sigma(y)=f c$, the substitution $\sigma:\{x, y, x\} \rightarrow \mathcal{T}$. We extend $\sigma$ to all terms, following the substitution definition.
$\mathcal{T}$ is the smallest set s.t. $\mathcal{T} \supseteq \mathcal{X} \cup\{c, d\} \cup\{f t \mid t \in \mathcal{T}\} \cup\left\{g\left(t_{1}, t_{2}\right) \mid t_{1}, t_{2} \in \mathcal{T}\right\}$
Given the substitution we wrote before, we have
$\sigma(x)=c, \sigma(y)=f c$ with $\sigma: \mathcal{X} \rightarrow \mathcal{T}$
I want to extend it to $\sigma: \mathcal{T} \rightarrow \mathcal{T}$. So I will have:
$\sigma(t)= \begin{cases}\mathrm{t}, & \text { if } t \in\{c, d\} \\ \sigma(t), & \text { if } t \in\{x, y, z\} \\ \mathrm{f}\left(\sigma\left(t^{\prime}\right)\right), & \text { if } \mathrm{t}=\mathrm{f}\left(\mathrm{t}^{\prime}\right) \\ \mathrm{g}\left(\sigma\left(t_{1}\right), \sigma\left(t_{2}\right)\right), & \text { if } \mathrm{t}=\mathrm{g}\left(\mathrm{t}_{1}, t_{2}\right)\end{cases}$
In this way we extended the variables in $\mathcal{X}$ to all possible terms in $\mathcal{T}$.

If I can do one unification, then I can do infinitely many.
Example:


I can do $x \mapsto y$ or $y \mapsto x$ to unify.
But I can also do $x \mapsto f^{10} d$ and $y \mapsto f^{10} d$, but this is not MGU.

