Model Checking: Translating (Propositional) LTL into First-Order Logic

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translating propositional LTL into FOL

▶ consider FOL models \( \mathcal{M} \) over \( \mathbb{N} = \{ 0, 1, 2, \ldots \} \) with \( \mathcal{F} = \emptyset \) and \( \mathcal{R} = \{ < \} \cup \{ \text{propositional variables used as unary predicates} \} \)

(Sometimes this is called FOMLO = “First-Order Monadic Logic of Linear Order”)

▶ translation function \( \llbracket - \rrbracket (t) : \{ \text{LTL formulas} \} \times \mathbb{N} \to \{ \text{FOL formulas} \} \)

▶ \( \llbracket p \rrbracket (t) = p(t) \)

▶ \( \llbracket \varphi \land \psi \rrbracket (t) = \llbracket \varphi \rrbracket (t) \land \llbracket \psi \rrbracket (t) \)

▶ \( \llbracket \neg \varphi \rrbracket (t) = \neg \llbracket \varphi \rrbracket (t) \)

▶ \( \llbracket X \varphi \rrbracket (t) = \llbracket \varphi \rrbracket (t + 1) \)

▶ \( \llbracket F \varphi \rrbracket (t) = \exists t' [ t' \geq t \land \llbracket \varphi \rrbracket (t') ] \)

▶ \( \llbracket G \varphi \rrbracket (t) = \forall t' [ t' \geq t \rightarrow \llbracket \varphi \rrbracket (t') ] \)

▶ \( \llbracket \varphi \mathbf{U} \psi \rrbracket (t) = \exists t' [ t' \geq t \land \llbracket \psi \rrbracket (t') \land \forall t'' [ t \leq t'' < t' \rightarrow \llbracket \varphi \rrbracket (t'') ] ] \)

▶ \ldots
Theorem. Let $M$ be a transition system, i.e., $M$ is a LTL model, and $\pi$ a path in $M$. The following are equivalent assertions, for every WFF $\varphi$ of LTL and every $i \geq 0$ (not $i \geq 1$ as in the book):

1. $\pi^i \models_{LTL} \varphi$
2. there is a FOL model $N$ such that $N \models_{FOL} [\varphi](i)$

where $N$ is over the vocabulary $F = \emptyset$ and $R = \{<\} \cup \{\text{propositional variables used as unary predicates}\}$

Corollary. The following are equivalent, for every WFF $\varphi$ of LTL:

1. $\models_{LTL} \varphi$, i.e., $\varphi$ is semantically valid in LTL.
2. $\models_{FOL} \forall t \ (\llbracket \varphi \rrbracket(t))$, i.e., $\forall t \ (\llbracket \varphi \rrbracket(t))$ is semantically valid in FOL.
Question:

Is there a translation in the opposite direction, from FOL to LTL?

More precisely, is it the case that for every WFF $\varphi$ of “first-order monadic logic of linear order” we can define a WFF $\psi$ of LTL such that:

$\varphi$ is semantically valid iff $\psi$ is semantically valid?

Answer:

YES, by Kamp’s Theorem.