1 Dijkstra Ordering - Isabelle. Extract dijkstra.als from the Alloy jar file. Reimplement in Isabelle.

2 Process Priorities - Alloy. Implement an Alloy model that has a numerical priority value for each process (for example, starting from dijkstra.als). Create a test for priority inversion, and create two examples: one that displays priority inversion, and one that does not. Choose any of the standard tactics for preventing priority inversion, and demonstrate that the bad behavior of your first example has been corrected.

3 Process Priorities - Isabelle. Implement project 2 using Isabelle.

4 Circuit Properties - Isabelle. A boolean circuit is an arbitrary collection of AND, OR and NOT gates. First, develop a method for representing any such circuit in Isabelle. For an arbitrary boolean circuit with $N$ inputs and $M$ outputs prove that this boolean circuit is stateless: if $I_1$ and $I_2$ are input vectors, and $O_1$ and $O_2$ are the corresponding output vectors, prove that if $I_1 = I_2$ then $O_1 = O_2$. Second, extend your circuit with NAND and NOR gates, then create a one−bit register with cross−coupled NAND gates. Prove that the register behaves the way one would expect. Also construct a circuit with one register that has state by finding inputs $I_1 = I_2$ such that the corresponding outputs are not equal.

5 Circuit Properties - Alloy. Reimplement project 4 in Alloy.

6 Factorization - Isabelle Use Isabelle to prove or refute the statement:

$$gcd(N, M) \times lcm(N, M) = N \times M$$

for all positive integers $N$ and $M$. If the statement is false, produce a counterexample.

7 Fibonacci Numbers - Isabelle. Let $F_n$ denote the Fibonacci numbers. Use Isabelle to prove the following statements:

$$gcd(F_{n+1}, F_n) = 1$$

$$gcd(F_{n+2}, F_n) = 1$$

8 Fibonacci Numbers - Alloy. Using a scope of at least 8, prove the two statements of Project 7 in Alloy. Then find an Alloy counterexample to the statement:

$$gcd(F_{n+3}, F_n) = 1$$
9 A Paradox of Set Theory - Alloy. Consider a system where there are 31 branch libraries, 31 branch librarians, and one head librarian. At the end of each year, the branch librarians compile a catalog of all books at their library. They send these catalogs to the head librarian. This year, the head librarian has discovered an inconsistency: some catalogs list themselves, while others do not. The head librarian sorts the catalogs into two piles. Pile G contains the catalogs that list themselves, while pile B contains the catalogs that do not list themselves. Create an Alloy model of this system.

The head librarian files all the catalogs in pile G, and then decides to create a new catalog U that lists all the catalogs that do not list themselves. First, show that the assertion "U lists itself" is inconsistent with your Alloy model of the catalog system. Second, show that the assertion "U does not list itself" is also inconsistent with your Alloy model. Finally, make a simple change to your model that makes it consistent with the second assertion.

10 Bernoulli Polynomials - Isabelle. Use Isabelle to prove the following two statements:

\[
\sum_{x=1}^{N} x^0 = N
\]

\[
\sum_{x=1}^{N} x^1 = \frac{N(N + 1)}{2}
\]

for all positive \(N\).

The right hand sides of these statements are the first two Bernoulli polynomials. Note that the index 0 polynomial is \(O(N)\) (order 1), while the index 1 polynomial is \(O(N^2)\) (order 2). Assume that the index \(k\) polynomial has order \(k + 1\) and can be written as

\[
\sum_{x=1}^{N} x^k = \sum_{i=0}^{i=k+1} c_i x^i
\]

Assume further that all the \(c_i\) are rational numbers. Use Isabelle to prove that \(c_0 = 0\).