Problem Set Eight Answer Key

Solutions taken from various students

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Problem 1: Credit to Jennifer Collins

a) \( A \not\models_{LTL} \varphi_1 \). We know this to be the case via the following counterexample. Let \( \pi' = (s_0s_3s_4)^* \) (i.e. a continuous loop through states \( s_0, s_3, s_4 \)). Since we are always moving between \( p \) and \( \neg p \) and we never reach a state where \( \neg p \) is continuously true, we do not have a situation of \( FG \neg p \). Therefore, \( A \not\models_{LTL} \varphi_1 \).

b) \( A \models_{LTL} \varphi_2 \). We can see that this is true by looking at the possible path options for \( \pi \). One option is to perpetually cycle through \( s_0s_3s_4 \), the second is to go \( s_0s_1 \) and then \( s_2 \) forever, or we can have our first option eventually lead into the second cycle. In every situation, it is always true that at some point we will reach a state where \( \neg p \) is true. Therefore, \( A \models_{LTL} \varphi_2 \).

c) \( A \models_{LTL} \varphi_3 \). We can see that this is true by once more looking at the possible path options and see if there is a future state where our present state is \( p \) and our next state is also \( p \). If our path goes through the \( s_0s_3s_4 \) loop, we can see that at \( s_4 \) we have \( p \) and our next state \( (s_0) \) will also be \( p \). Therefore this path option satisfies the condition. Our other path option is to follow \( s_0s_1s_2 \). In this case, when we are at \( s_0 \) our state is \( p \) and our next state will be \( s_1 \) which will also have a state of \( p \). Therefore, \( A \models_{LTL} \varphi_3 \).

Problem 2: Credit to Rebecca Graber

a) \( \varphi'_1 \triangleq \forall F \forall G \neg p \)  
No: \( A, s_0 \not\models \varphi'_1 \)

b) \( \varphi'_2 \triangleq \forall G \forall F \neg p \)  
Yes: From every state, every infinite path will include \( s_3 \) or \( s_2 \), where \( p \) is false.

c) \( \varphi'_3 \triangleq \forall F(p \land \forall X p) \)  
No: \( A, s_1 \not\models \varphi'_3 \)
Problem 3: Credited to Jennifer Collins

Note: there are multiple correct answers to this problem. I just chose one such correct example.

As was seen in the earlier problems, consider the LTL WFF $\varphi \triangleq F(p \land Xp)$. Through problems 1 and 2 we demonstrated that $A \models_{LTL} \varphi$ but $A \not\models_{CTL} \forall[\varphi]$

Problem 4: Credited to Jennifer Collins, Professor Kfoury, and Rebecca Graber respectively

Note: there are multiple correct answers to these problems. I just chose one such correct form.

a) $L(E) = (abb)^*ab(a^*)$.
This can be determined by combining the two possible loops: $s_0s_3s_4 \rightarrow ((abb)^*)$ and $s_0s_1(s_2)^* \rightarrow (ab(a)^*)$

b) $L(G) = (abb)^*aba^\omega + (abb)^\omega$

c) Yes there is an infinite path $\pi$ in $A'$ with the specified conditions. We will define the path and $\text{trace}(\pi)$ below:
$\pi = (s_0s_3s_4)^\omega$ (as described by $(abb)^\omega$ above).
$\text{trace}(\pi) = (p \neg p \overline{p})^\omega$ as can be seen in Figure 2.

Alternatively, the following answer is also valid:
Yes: $\pi = ab(b^\omega)$
$\text{Trace}(\pi) = (p)(p)(\overline{p}^\omega)$