Syntax of Propositional Logic

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Syntax of the WWF’s of Propositional Logic

▶ Reading: [LCS, Section 1.3]
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- The WWF’s of propositional logic are obtained by applying the construction rules below, and only these finitely many times:
  
  1. every propositional atom (i.e., propositional variable) \( p \) is a WFF
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4. if \( \varphi \) and \( \psi \) are WFF’s, then so is \( \varphi \lor \psi \)
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4. if $\varphi$ and $\psi$ are WFF’s. then so is $(\varphi \lor \psi)$

5. if $\varphi$ and $\psi$ are WFF’s. then so is $(\varphi \rightarrow \psi)$
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More succintly, in BNF (Backus Naur Form):

\[ \phi ::= p \mid (\neg \phi) \mid (\phi \land \phi) \mid (\phi \lor \phi) \mid (\phi \rightarrow \phi) \]

This is the same as in [LCS, page 33].
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- Or, in **Extended BNF**:

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- Or, in **Extended BNF**:

\[ \phi ::= p \mid (\neg \phi) \mid (\phi \land \psi) \mid (\phi \lor \psi) \mid (\phi \to \psi) \]

- Or, more abstractly by omitting parentheses, in **Extended BNF**:

\[ \phi ::= p \mid \neg \phi \mid \phi \land \psi \mid \phi \lor \psi \mid \phi \to \psi \]

Parentheses are used only to set an order of precedence among logical connectives \{\neg, \land, \lor, \to\}. 
Parse Trees of WFF’s

- A fully-parenthesized WFF:

\[
\left( \neg((\neg P) \lor (Q \land (\neg P))) \right)
\rightarrow \left( \neg((\neg P) \rightarrow (Q \lor (\neg R))) \right)
\]
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\rightarrow \left( \neg \left( \left( \neg P \right) \rightarrow \left( Q \lor \neg R \right) \right) \right)
\]

- Same WFF with all parentheses omitted:

\[
\neg \neg P \lor Q \land \neg P
\rightarrow \neg \neg P \rightarrow Q \lor \neg R
\]

(an incomprehensible mess!)
Parse Trees of WFF’s

- A fully-parenthesized WFF:

\[
\left( \neg \left( \neg P \lor (Q \land \neg P) \right) \right) \\
\rightarrow \left( \neg \left( \neg P \rightarrow (Q \lor \neg R) \right) \right)
\]

- Same WFF with all parentheses omitted:

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\neg \neg P \lor Q \land \neg P \\
\rightarrow \neg \neg P \rightarrow Q \lor \neg R
\]

(an incomprehensible mess!)

- Same WFF minimally parenthesized:

\[
\neg \left( \neg P \lor (Q \land \neg P) \right) \\
\rightarrow \left( \neg P \rightarrow (Q \lor \neg R) \right)
\]
Parse Trees of WFF’s

▷ A fully-parenthesized WFF:

\[
\left( \neg \left( \neg ((\neg P) \lor (Q \land (\neg P))) \right) \rightarrow \neg \left( \neg ((\neg P) \rightarrow (Q \lor (\neg R))) \right) \right)
\]

▷ Same WFF with all parentheses omitted:

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\[
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\]