Notes on "Handout 16 Second Order Logic (continued)".

Consider the following problem given graph($A, R$):

- $A$ is a set of nodes and $R$ is a binary relation representing edges
- A Hamiltonian path is a path that visits every node exactly once.

Can we model whether a graph has a Hamiltonian path? (A graph with $n$ nodes has $n!$ possible orderings of nodes which is an inefficient algorithm.)

You can express it in second order as follows:

$$\phi \equiv \exists P["P is a linear order" \lor \forall x \forall y("y = x + 1" \rightarrow R(x, y))]$$

*Naturally the above is not a complete answer as we have English placeholders that are not part of the syntax.*

Note: In dense linear ordering you cannot talk about successors, this is only allowed in discrete linear ordering.

Make $P$ represent a discrete ordering (so every element has a successor):

$$\exists z \forall y P(x, y) : \text{there is a largest element}$$

$$\exists y \forall z P(z, y) : \text{there is a smallest element}$$

*Note: Even if there is a largest and smallest element that does not necessarily mean the ordering is finite if it is not discrete.*

$$\forall x \forall y ((P(x, y) \land \neg(x = y)) \rightarrow \exists z (P(x, y) \land \neg(x = z) \land \forall w (P(x, w) \rightarrow (w = x \lor P(z, w)))))$$

2-colorability in second order:

$$\phi \equiv \exists P \forall x \forall y [\neg(x = y) \land R(x, y) \rightarrow (P(x) \leftrightarrow \neg P(y))]$$

*Three colorability discussed in handout 16.*
After the break:
Handout 17 Beyond Propositional Logic

Ch3

1. Linear Temporal Logic (LTL)
2. Branching Time Logic (BTL)
3. Computation Tree Logic (CTL)
4. CTL* = LTL + CTL

Plan to skip Ch4

If we have time we will begin Ch5.